EXPERIMENTAL RESEARCH OF THE INFLUENCE OF PARTICLE SIZE AND FLUIDIZATION VELOCITY ON ZEOLITE DRYING IN A TWO-COMPONENT FLUIDIZED BED

by

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This paper presents the results of the kinetics research into the drying of fine grained material in a two-component fluidized bed. A review of theoretical and experimental investigations of aerodynamics of the fluidized bed is given, with a special insight into two-component fluidized beds, as well as the basics of heat and material transfer through a fluidized bed. Apart from the theoretical basis of convective drying of wet materials in a stagnant fluidized bed, the paper also emphasizes different approaches to fine grained material drying kinetics. Based on the experimental investigations, where zeolits used as a representative of fine grained material and polyethylene as a representative of inert material (another component), an analysis of the influence of working parameters on drying in a two-component fluidized bed is performed. It is established that, apart from the influence of the considered parameters, such as fluidization velocity, diameter of fine grained material particles and drying agent temperature, on the drying curve, the participation of inert material can considerably increase the intensity of heat and material transfer in the fluidized bed. A comparison of the experimental drying curves of fine grained material in the two-component fluidized bed with the results from the studies by other authors shows satisfactory agreement.

Key words: fluidization, bed, experiment, particle, zeolite, drying, heat, transfer

Introduction

The attractiveness of the application of fluidization in various technological operations comes from the fact that it provides features that can be viewed as: intensive mixing of solid particles in a fluidized bed, easy supply and drainage of material, a large contact surface between the gas and solid particles and nearly constant temperature all over the bed. These characteristics have been crucial to fluidization finding wide application in many industrial processes such as drying of fine-grained material, transportation of powdered materials, coal combustion [1], freezing of food products, *etc.* Due to the fact that the application of fluidization in the industry requires as precise and simple methods as possible so as to calculate the relevant parameters a large number of papers have been published in recent years that, mainly through experimental research, deal with this area [2-5].

Many operations in a fluidized bed are characterized by the processes of heat exchange between the gas and solid particles which include the mass transfer. One of these

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processes is the material drying in a fluidized bed, which is a technological operation used to change the properties of natural raw materials as a result of application for the use, conservation, new structure, or simply, reduction of the original mass.

The process of drying granular material in a fluidized bed with an infill or the socalled inert material, has been of particular interest to many researchers, starting from the 1970s. The purpose of introducing the second component into the fluidized bed was to reduce the time of drying at a defined humidity. An inert medium fluidized bed dryer can be defined as a fluidized bed of fine and coarse particles for fine-grained material drying whereby the coarse particles serve as inert material for heat transfer media. Therefore, this type of dryer has a higher overall heat transfer coefficient than those without an inert medium. This also leads to energy savings. The absence of reliable data necessary for calculation in the available literature, imposes a need for a special approach to the research problems of fine-grained material drying (zeolite) in a two-component fluidized bed. It is considered that zeolite is the mineral of the future, which is frequently used in ecology and environmental protection. It has already demonstrated high effectiveness in many agricultural industries, for example, drying wet grain.

The results of these studies have direct application in the processes of heat and mass transfer in the drying of fine-grained material in a fluidized bed, especially in the pharmaceutical and food industry, which can lead to a qualitative progress in creating conditions for increasing the intensity and quality of drying.

Fluidization of two-component mixtures

In the case of two-component mixture fluidization, the transition to fluidized condition occurs gradually because the finer particles pass into a fluidized state at lower fluid ve-



Figure 1. Fluidization curve of a two-component fluidized bed [6]; (a) completely mixed, (b) completely layered, (c) partially mixed/layered

locities. On the other hand, coarser, heavier particles transit more slowly to the fluidization state. That leads to delamination (separation) of the bed. The transitional area size depends on the physical characteristics of particles and increases with an increasing difference in the physical properties of the particles.

The fluidization curve which is obtained at the transition to the fluidized state of twocomponent mixtures is presented in fig. 1 [6]. According to Rowe [6], the component with lower fluidization velocity is marked with F(the fluid component) – particles of smaller equivalent diameter, and the component with

a higher minimum fluidization velocity is marked with 0 (the packed component) – particles of larger equivalent diameter.

Heat and mass transfer in a fluidized bed

As the mechanisms of heat exchange in a fluidized bed are very complex, it is necessary to analyze them separately: interphase heat transfer (gas-solids) and heat transfer between the fluidized bed and the walls of the apparatus or the surface immersed in the fluidized bed [7, 8]. Also, a very important role in these processes is played by the effective thermal conductivity of the fluidized bed and the emulsion phase. Experimental results of most previous research in mass transfer are given in the dimensionless form as a function of Sherwood's, Schmidt's, and Reynolds's number or directly as a function of operating characteristics such as gas velocity, particle diameter, *etc.* A quite reliable expression for the Sherwood number was provided by Stojanovic [8] and Kunii and Levenspiel [9] for the use in the particles with diameters of less than 1 mm:

$$\operatorname{Sh} = \frac{\delta_b}{1 - \varepsilon_f} \left[\gamma_b \eta_d \, \operatorname{Sh}_t + \frac{Y^* \varphi_s d_p^2}{6D_s} \left(K_{bc} \right)_b \right] \tag{1}$$

For the application of a certain correlation of the coefficient for mass transfer, it is very important that the conditions of the experiment correspond to the process.

Drying in a fluidized bed

The fluidized bed is used for drying of various kinds of granular materials in the industry. The particles in the fluidized bed mix intensely, in contrast to the stagnant fluidized bed. The drying agent flows around the material particles, there by intensifying the exchange of heat and mass transfer between the drying agent and the solid particles. It is considered that the distribution of temperature and moisture content of particulate materials is practically uniform in the fluidized bed.

Mostafa [10] analyzed the drying of silica gel in a fluidized bed with a diameter of 0.1 m. Measurements of bed temperature and humidity as a function of height above the distributor plate show that in the period of constant and falling rate of fluidization velocity, drying occurs only at a depth of 0.02 m. A height increase in the fluidized bed of 0.05 m to 0.20 m does not result in significant differences in the degree of air moistening. The results of the influence of the fluidized bed height obtained by Mostafa [10] have been confirmed by Reay and Allen's [11] experiments with synthetic resin and iron ore. By using wheat as the material for fluidization, Reay and Allen showed that the level of drying is totally independent of the gas velocity. This is in accordance with a relatively low internal resistance of the transmission of moisture, which controls the drying process. Lee and Kim [12] investigated drying characteristics of starch in a fluidized bed with inert particles. Experiments were carried out in a column of stainless steel with a diameter of 0.083 m and height of 0.80 m. A mixture of glass beads as inert material ($d_p = 0.4$; 1.0 mm) and wet particles of starch $(d_p = 16.3 \,\mu\text{m})$ were supported by a perforated plate with 19 evenly spaced holes with a diameter of 3 mm. The minimum air velocity for fluidization of beds of glass particles were 0.13 m/s and 0.54 m/s, respectively. It is possible to determine the condition of the fluidized bed, at any time, using the following relations:

$$\frac{\Delta P}{\Delta L} = (\varepsilon_{\rm g} \,\rho_{\rm g} + \varepsilon_l \,\rho_l + \varepsilon_{sf} \,\rho_{sf} + \varepsilon_{sc} \,\rho_{sc}) \mathrm{g} \tag{2}$$

$$\varepsilon_{\rm g} \rho_{\rm g} + \varepsilon_l \rho_l + \varepsilon_{sf} \rho_{sf} + \varepsilon_{sc} \rho_{sc} = 1 \tag{3}$$

They determined the influence of inlet air temperature (25 °C to 100 °C) on the degree of drying in the bed. Their conclusion was that the degree of drying in a fluidized bed with inert material increases with temperature and velocity of incoming gas. Degrees of drying were ten times higher than a conventional drying process. In addition, the maximum degree of drying was at the optimum porosity layer. Scala [5] determined the coefficient of mass transfer in a laboratory reactor around the active particles that move freely in a bubbling fluidized bed with inert material. The author applied the technique that involved the oxidation of carbonmonoxide at a temperature of 450 °C over one or several platinum catalyst spheres inserted into an inert layer of sand. Through out the experiment, the fluidization velocity (0.15-0.9 m/s), the size of active particles (1.0-10.0 mm) and the size of inert particles (0.1-1.4 mm) were varied. The analysis of the results was conducted through the Sherwood number for particles, where it was concluded that the velocity fluidization and change of its regime do not have any influence on the Sherwood number which increases with the square root dependence of the minimum fluidization velocity and the size of active particles:

$$Sh = 2.0\varepsilon_{mf} + K \sqrt{\frac{Re_{mf}}{\varepsilon_{mf}}} \sqrt[3]{Sc}$$
(4)

Ramakers *et al.* [13] analyzed the behavior of a mixture of wood/sand drying process in a fluidized bed. Ramakers *et al.* concluded that the fluidization of various types of particles was more efficient with the presence of sand as the inert material. Their experimental installation was composed of a glass tube with a diameter of 7.56 cm and a height of 111 cm. Superficial velocity of cold air ranged within the values of 0 m/s to 2.2 m/s. Beech cylinders of 9 mm in length and a diameter of 6 mm were used in the experiment.

Experimental research

The laboratory apparatus was designed and manufactured to suit the specific requirements of this experimental research (fig. 2). Special attention was directed towards enabling a continuous measurement of operating parameters. The apparatus consists of the following com-



Figure 2. Schematic layout of the pparatus

ponents: variable speed fan (4), sections for air flow measuring with a metering station, electric heating (2), fluidized bed (1), the device for measurement, regulation and registration process (3, 5). Fluidization bed has a circular cross-section with the inner diameter of 120 mm and the height of 600 mm. It was made of plexiglass, which allowed for the visual monitoring of the process. At the bottom of the column is a distributor for air with 2800 holes with a diameter of 0.9 mm evenly distributed over the whole surface. Under the distributor, a canvas prevents degradation of the material and provides for an even distribution

of air in the cross-section of the column. The openings for thermocouples setting are located at the same height as the fluidized bed. An opening for the sampling of material to be dried is positioned on the side. Immediately above the distributor there is an opening for discharging the apparatus. The measurement of air flow was performed using a standard metering station that was previously calibrated and installed in a pipe line with a diameter of 50 mm. The pressure drop was registered by a micromanometer. An electrical heater was used for air heating.

The measurement of temperature was performed with a chromel-alumel thermocouple (0.2 mm in diameter). The inlet air temperature was measured directly under the distribution plate; the temperature of the fluidized bed was measured at half the height of the bed. All thermocouples were connected to a data acquisition system. The regulation of all operating parameters was performed manually.

Before the examination of the drying kinetics of the material in the fluidized bed, a specific mass of the inert material was poured into the fluidization column, followed by the

adjustment of the working parameters of the drying process: the inlet air flow and temperature. After heating the inert material and the column, *i. e.* reaching the stagnant state, the examination process began and a measured mass of the wet material was introduced into the bed. At pre-determined time intervals, which depended on the period of drying, samples of the dried material were taken from the bed through an opening on the column wall, and after their mass was determined, they were placed in the electric chamber dryer for further drying. Following the completion of the examination and the end of the time estimated for the drying of samples in the electric dryer, the mass of the dry material of each sample was determined, *i. e.* the absolute humidity of each sample was calculated.

The fluidization air flow was set manually for each experiment, by changing the number of fan revolutions, and determined using the standard method on the basis of the pressure drop measured in the measuring orifice.

Series of 29 experiments were conducted.

To calculate the air flow eq. (5), verified by the previous calibration, was used in the examination:

$$L = 1.111 \cdot 10^{-5} \sqrt{\rho_1 \,\Delta p_b} \tag{5}$$

For the cross-sectional area of the apparatus of 0.011304 m^2 , after measuring the temperature in the bed, the air velocity in the working part of the apparatus was determined using the expression:

$$U = 0.02357 \frac{\sqrt{\rho_1 \,\Delta p_b}}{\rho_v} \tag{6}$$

Experimental research results

In these experiments, zeolite was sifted through standard sieves and divided into fractions, whose middle particle diameters were: 0.3 mm, 0.5 mm, 0.7 mm, and 0.9 mm. Each fraction of zeolite was determined by the following characteristics: average particle diameter, actual density, bulk density, porosity of the bed at minimum fluidization, and minimum fluidization velocity (d_p , ρ_p , ρ_n , ε_{mf} , U_{mf}).

The inert material was polyethylene (PE) in the form of a cylinder with an oval base of a = 3.8 mm, b = 4.4 mm, and h = 4.0 mm in dimensions. The real and bulk density of zeolite and PE were determined experimentally; the values of the specific heat capacity and thermal conductivity of the material were taken from the reference literature, tab. 1. As the particles of zeolite of all fractions were monodisperse, the equivalent diameter of the particle was determined on the basis of the screen size, and the equivalent diameter of the particles of the inert material PE was calculated by the expression:

$$d_{\rm p} = \sqrt[3]{a \ b \ h} \tag{7}$$

The minimum fluidization velocity of all fractions of zeolite and PE was determined experimentally, fig. 3, on the basis of standard measurements, depending on the pressure drop in the bed of the fluidization velocity:

$$\Delta p_{sp} = \frac{\Delta p}{\frac{1}{A} m_{sl} g} = f(U_f)$$
(8)



Figure 3. Minimum fluidization velocity of the mixture of zeolite ($d_p = 0.5$ mm) and PE

The minimum velocity of the mixture can be calculated on the basis of the existing expressions in the literature. An expression by Goossens [14] was used for the calculation of the minimum fluidization velocity of the mixture, and it showed the best agreement with the experimental data. The comparative results of calculation and experimental determination of the minimum fluidization velocity of the mixture of zeolite and PE are given in tab. 1. The characteristics of materials used for the experimental research are presented in the same table.

Diameter	Density	Bulk density	$\begin{array}{c} \text{Porousity} \\ \text{at} \ U_{\rm mf} \end{array}$	Spec. thermal capacity	Thermal conductivity	Min. fluidiz. velocity $U_{\rm mf}$, [ms ⁻¹]	
d_{p} [mm]	$ ho_{p} [\mathrm{kgm}^{-3}]$	$\rho_n [\mathrm{kgm}^{-3}]$	<i>€</i> _{mf} [−]	$c_{p} [kJkg^{-1}K^{-1}]$	$\lambda_p [Wm^{-1}K^{-1}]$	Exper.	Calcul.
Zeolite							
0.5	1765	781	0.57	0.4 (dry)	0.65	0.11	0.12
0.3	1770	740	0.59			0.04	0.046
0.7	1815	782	0.58			0.19	0.21
0.9	1870	780	0.58			0.27	0.30
Polyethylene							
4.06	905	580	0.37	2.51	0.32	0.85	0.833

Table1. Characteristics of zeolite and polyethylene

Analysis of the influence of operating parameters on the drying kinetics

The experimental research of the drying kinetics of fine-grained material in a fluidized bed included the variation of certain operating parameters with the greatest influence on the process, as follows: the particle size of fine-grained material (zeolite), fluidization velocity, air temperature, and the drying ratio of the mass fractions of zeolites and the inert material.

The influence of particle size

The basic experimental investigations of drying kinetics of zeolite in the fluidized bed with the inert material (polyethylene) were conducted with the particles of the average diameter of 0.5 mm. The other granulations of zeolite (0.3 mm, 0.7 mm, and 0.9 mm) were used to investigate the impact of particle size on the drying process. The primary objective of the experimental study was to determine the influence of the mass fractions of zeolites and the inert material (*Z/I*) on the zeolite drying process. Experimental investigations were conducted for the following values: Z/I = 20/80, 30/70, 40/60, and 50/50, as well as pure zeolite at a constant fluidization velocity. The limit values of the weight of fractions (relations) were determined with regard to the possibility of segregation in the bed. Experiments were conducted with three temperatures of the fluidized bed: $42 \,^{\circ}$ C, $55 \,^{\circ}$ C, and $65 \,^{\circ}$ C. To investigate the impact of particle size of zeolite on the change of the intensity of the drying, the measurements were performed with all sizes of particles -100% of zeolite (without the inert material) and with the ratio of zeolite to the inert material Z/I = 50/50. Figures 4 to 8 show the changes in the drying kinetics.



Figure 4. Drying curves of zeolites in with the ratio of Z/I = 100 and Z/I = 50/50, and $d_p = 0.3$ mm

Figure 5. Drying curves of zeolites with the ratio of Z/I = 100 and Z/I = 50/50, and $d_p = 0.7$ mm

Figure 4 clearly shows that the layer without the inert material (Z/I = 100) has the maximum drying velocity with the smallest particle diameter $d_p = 0.3$ mm. For each larger diameter of zeolite particles, the drying velocity decreased. Since this is a relatively small range of particle sizes in the experimental research, the differences of intensity in drying are not great, but the drying velocity still differed by about 18% in the initial period. In all of the cases of different fractions of zeolite the drying time was reduced. Djani [15] showed a comparison between conventional dryers and dryers using zeolite for dehumidification of air. Drying temperatures were in the range of 52 °C to 70 °C. The published results indicate that the dryers that use zeolite to absorb the air moisture are 10-18% more efficient than conventional dryers.

The rate of removal of moisture from the fine-grained material in the first drying period was higher with the participation of the inert material than in the case without it, since the mixing of the two components of the bed at the same time led to a more intensive heat and mass exchange in the bed.

The influence of fluidization velocity

During one group of measurements, the drying kinetics of zeolite ($d_p = 0.5$ mm) was determined for several different fluidization velocities. The measurements were performed with 100% zeolite (the layer without the inert material) and with the ratio of zeolite to the inert material Z/I = 50/50 at fluidization velocities of 0.73-1.30 m/s. The results of measurements are shown in figs. 6-8.

Based on the obtained moisture curves of zeolite for a certain period, conclusions can be drawn about the influence of fluidization velocity on the drying process. One of them relates to the fact that the increase in fluidization velocity generally expedites the drying process, and reduces the time required to dry the material. As can be seen from figs. 7-9, the effect of changing fluidization velocity was more pronounced in the bed without the inert material, *i. e.* the 100% zeolite bed. The diagrams clearly show that in the bed with the inert material the drying time changes less with the changes in fluidization velocity than in pure zeolite. Specifically, in the case of the ratio Z/I = 50/50, the change in fluidization velocity had less influence on the intensity of drying, which means that the other factors in the process of heat and mass exchange more dominantly influenced the drying process. In analyzing the impact of certain fluidization velocity to the drying curve for the cases Z/I = 100 and



Figure 6. Drying curves of zeolites with the ratio of Z/I = 100 and Z/I = 50/50, and $d_p = 0.9$ mm

Figure 7. Drying curves of the mixture of zeolite $(d_p = 0.5 \text{ mm})$ and PE at different fluidization velocities

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τ [min]

Z/I = 50/50, it can be seen that at low velocity of fluidization the inert material has a significant influence (fig. 8), while at certain sufficiently high fluidization velocities the influence of the inert material on the drying kinetics is almost non-existent (fig. 9).





Figure 8. Drying curves of the mixture of zeolite $(d_{\rm p} = 0.5 \text{ mm})$ and PE at the fluidization velocity U = 0.73 m/s



Conclusions

Based on the obtained results and the conducted analysis, the following conclusions can be made.

- The results of the experiments clearly indicate that the inert material in the fluidized bed accelerates the drying process, and exhibits positive effects on the intensification of the process; in all of the cases of zeolite ratios the drying time is reduced.
- It is important to find an adequate expression for determining the minimum fluidization velocity of this mixture based on the relevant literature. The adopted Goossens [14] expression gives satisfactory agreement with the experimental values.
- During the drving in the layer of zeolite with the inert material in the ratio of Z/E = 50/50. the effects of the particle size on the velocity of drying, compared to pure zeolite, decreased in the investigated range by about 32%. This is certainly a consequence of the intensification of the contact of the particles with the drying agent and the inert material.
- In the mixture in the ratio of Z/I = 50/50, with fluidization velocity changing between 0.73-1.3 m/s, there was a reduction in the drying time by about 35%, which is less than in the layer without the inert material.
- The processed experimental results lead to the conclusion that the drying time was the shortest for the 20/80 ratio of zeolites to the inert material.

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Nomenclature

- Α - cross section of layer, $[m^2]$
- diffusion coefficient of particles, [m²s⁻¹] D_{\circ}
- inert material T
- K_{bc} coefficient of gas exchange, [s⁻¹]
- L air flow, [kgm⁻²s⁻¹]
- Δp_b pressure drop through the apparatus, [Pa]
- Re Reynolds number, [–]
- Sc Schmidt number, [–]
- real fluidization velocity, [ms⁻¹] $U_{\rm f}$
- Ŷ - logarithmic ratio of inert components in the mixture, [-]
- Ζ - zeolite
- Greek symbols

 ε_q – ratio in eq. 2

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 ε_{sf} – ratio in eq. 2 ρ_1 – density of the air in front of the apparatus, [kgm⁻³] $\rho_{\rm g}$ – density of gas phase in eq. 2, [kgm⁻³] ρ_1 – density of liquid phase in eq. 2, [kgm⁻³] ρ_{sc} – density of inert particles in eq. 2, [kgm⁻³] ρ_{sf} – density of fine particles in eq. 2, [kgm⁻³] Subscripts

- gas phase g

 $\varepsilon_{\rm I}$ – ratio in eq. 2

 ε_{sc} – ratio in eq. 2

- liquid phase
- sc inert particles
- sf fine particles