

PARTIAL SLIP AND THERMAL RADIATION EFFECTS ON HYDROMAGNETIC FLOW OVER AN EXPONENTIALLY STRETCHING SURFACE WITH SUCTION OR BLOWING

by

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This paper is devoted to analyze computational simulation to study the partial slip and thermal radiation effects on the flow of a viscous incompressible electrically conducting fluid through an exponentially stretching surface with suction or blowing in presence of magnetic field. Using suitable similarity variables, the nonlinear boundary layer partial differential equations are converted to ordinary differential equations and solved numerically by Runge-Kutta fourth order method in association with shooting technique. Effects of suction or blowing parameter, velocity slip parameter, magnetic parameter, thermal slip parameter, thermal radiation parameter, Prandtl number and Eckert number are demonstrated graphically on velocity and temperature profiles while skin friction coefficient and surface heat transfer rate are presented numerically. Moreover, comparison of numerical results for non-magnetic case is made with previously published work under limiting cases.

Key words: *partial slip; thermal radiation; hydromagnetic flow; exponentially stretching surface; suction or blowing*

1. Introduction

The phenomenon of laminar flow and heat transfer of a viscous incompressible fluid driven by a linearly stretching surface has received great appreciation due to its applications in several technological processes. These applications involve paper production, hot rolling, annealing of copper wires and glass blowing. It is also important in geothermal areas because the shallow surface layers are being stretched with a small velocity. It is worth mentioning that the hydromagnetic flows over a moving surface have been extensively studied in the past few decades, because of its increasing applications in various manufacturing processes, such as the enhanced recovery of petroleum resources, spinning of metals and extrusion of plastic sheets. In all of the above engineering processes, to get the desired thickness the mixture issued from a slit is subsequently stretched. Crane [1] was the first to study an analytical solution of the steady two-dimensional flow over linearly stretching surface in a quiescent incompressible fluid. Later, many researchers such as Chakrabarti and Gupta [2], Carragher and Crane [3], Kumaran and Ramanaiah [4], Ishak et al. [5] Liu and Andersson [6], Jat and Chaudhary [7], Sahoo and Do [8], Mahapatra et al. [9] and Makinde et al. [10] have presented various aspects of linear stretching surface problem for non-magnetic and magnetic cases.

From practical point of view, a continuous surface stretched with a linear velocity is not appropriate for the problem of filaments from a die and continuous extrusion of polymer sheet. Therefore the stretching velocity is expected to be nonlinear. The boundary layer flow over exponential stretching surface under different situations were studied by Elbashbeshy [11], Parhta et al. [12], Sanjayanand and Khan [13], Sajid and Hayat [14], Bidin and Nazar [15], Nadeem et al. [16], Mukhopadhyay and Gorla [17] and Raju et al. [18].

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The central tenet of the boundary layer problems is no-slip boundary conditions. In this case the fluid velocity is zero at the surface. But in the existence of slip flow the fluid velocity is non-zero at the solid-fluid interface. In various technological processes, the assumption of no-slip is not applicable and must be substituted by partial slip boundary conditions. Such flow situations are encountered in a wide variety of industrial processes like foams and polymer solutions, polishing of artificial heart valves and internal cavities, and emulsion suspensions. Pursuing the pioneering studies of Hasimoto [19], the flow with partial slip boundary condition has been investigated by Wang [20], Ariel [21], Hron et al. [22] and Fang et al. [23]. Recently, Sajid et al. [24] and Das [25] studied about flow and heat transfer with different conditions and slip effects.

Forecasting of heat transfer characteristics of viscous incompressible flow with suction or blowing is very important in engineering and physics namely thermal oil recovery, design of radial diffusers and thrust bearings, prevent corrosion or scaling, reducing the drag and transition to turbulence. In chemical processes, suction can be used to remove reactants while to add reactants, blowing is used. The low energy fluid from the system is removed by suction, whereas blowing reduces the wall shear stress and hence the frictions drag. The boundary layer flow with suction or blowing was first presented by Gupta and Gupta [26]. Further, Chen and Char [27], Ali [28], Seddeek [29], Pantokratoras [30] and Cortell [31] studied the various aspects of the flow problems with suction or blowing.

Meanwhile, in most of the investigations, the thermal radiation effects on the flow and heat transfer have not been taken into the account. Boundary layer flow and heat transfer with radiation have a great importance in high temperature processes and space technology. It also plays an important role in many applications in engineering areas which occur at high temperature, like various propulsion devices or aircraft, design of reliable equipments, high temperature plasmas, liquid metal fluids, gas turbines, satellites, missiles and space vehicles. When the difference between the surface temperature and the ambient temperature is very large then thermal radiation effects become more important besides the convective heat transfer. The radiative heat flux is described by using the Rosseland approximations in the energy equations. The Thermal radiation effects on the flow with and without a magnetic field with several cases were presented by Bestman and Adjepong [32], Naroua et al. [33], Ouaf [34], Makinde and Ogulu [35], Pal and Mondal [36], Jat and Chaudhary [37]. Most recently, Elbashbeshy and Emam [38], Khan et al. [39], Chaudhary et al. [40] and Sandeep et al. [41] analyzed the radiation effects over viscous incompressible and MHD flow.

Keeping the above literature in view, and inspired by the research paper of Mukhopadhyay and Gorla [17], the main motive of this article is to describe the partial slip effects as well as the effects of thermal radiation on hydromagnetic fluid over an exponentially stretching surface with suction or blowing. The present study of the boundary layer flow will be highly beneficial in various engineering and technological processes such as magnetohydrodynamic flight, foodstuff processing, MHD-power generators and in the field of planetary magnetosphere. It is hoped that the current work will be extensively used over previous contents.

2. Formulation of the problem

Figure 1 describes the geometrical structure of two-dimensional flow of a viscous incompressible electrically conducting fluid past an exponentially stretching sheet with thermal radiation and partial slip boundary conditions. In order to study the considered problem, the following assumptions are made:

- (i) The exponentially stretching sheet is placed along the x – axis with the slot as the origin and is stretched along both ends of the sheet with the velocity $U_w(x) = U_0 e^{x/L}$ where U_0 is the reference velocity, x is the coordinate measured along the exponentially stretching sheet and L is the reference length.
- (ii) The flow is confined in half plane $y > 0$ and velocity components are u and v in the directions of x – and y – axes respectively.

- (iii) A uniform magnetic field of strength B_0 is assumed to be applied normal to the stretching surface.
- (iv) The magnetic Reynolds number is taken very small than unity so the induced magnetic field is negligible in comparison with the applied magnetic field.
- (v) Surface temperature along the exponentially stretching sheet is $T_w(x) = T_\infty + T_0 e^{x/2L}$ where T_∞ is the free stream temperature and T_0 is the reference temperature.
- (vi) The time independent suction or blowing at the surface is also considered.
- (vii) All the fluid properties are constant throughout the motion.

Under the above mentioned assumptions, the governing boundary layer equations are given as follows:

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 & (1) \\ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e B_0^2}{\rho} u & (2) \\ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} & (3) \\ &+ \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_e B_0^2}{\rho C_p} u^2 \end{aligned}$$

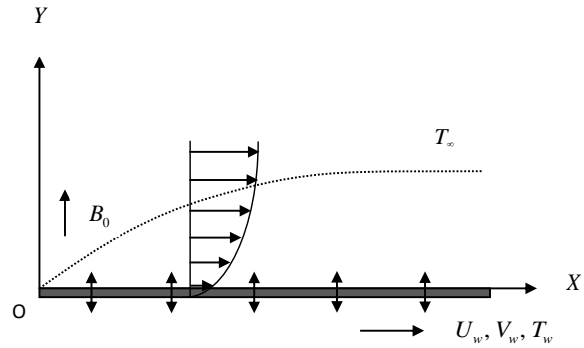


Figure 1. Schematic diagram of the problem

with the appropriate boundary conditions:

$$\begin{aligned} y = 0 & : \quad u = U_w + N(x) \nu \frac{\partial u}{\partial y}, \quad v = -V_w(x), \quad T = T_w + D(x) \frac{\partial T}{\partial y} \\ y \rightarrow \infty & : \quad u \rightarrow 0, \quad T \rightarrow T_\infty \end{aligned} \quad (4)$$

where y is the coordinate measured along normal to the exponentially stretching sheet, $\nu = \mu / \rho$ is the kinematic viscosity, μ is the coefficient of fluid viscosity, ρ is the fluid density, σ_e is the electrical conductivity, T is the temperature of the fluid, κ is the thermal conductivity, C_p is the specific heat at constant pressure, q_r is the radiative heat flux, $N(x) = N_1 e^{-x/2L}$ is the velocity slip factor, N_1 is the initial value of velocity slip factor, $V_w(x) = V_0 e^{x/2L}$ is the velocity of suction and blowing at the surface when $V_w(x) > 0$ and $V_w(x) < 0$ respectively, V_0 is the initial strength of suction, $D(x) = D_1 e^{-x/2L}$ is the thermal slip factor and D_1 is the initial value of thermal slip factor.

The radiative heat flux q_r can be written by using the Rosseland approximation for radiation (Brewster [42]) as:

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad (5)$$

where σ^* and k^* are the Stefan-Boltzmann constant and the absorption coefficient respectively.

Considering that the differences of temperature within the flow is such that the term T^4 can be expanded in a Taylor series about T_∞ and neglecting higher-order terms to yield:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (6)$$

In view of equations (5) and (6), equation (3) becomes:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3k^* \rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_e B_0^2}{\rho C_p} u^2 \quad (7)$$

3. Analysis

In order to investigate the heat transfer on exponentially stretching surface the following dimensionless similarity variables (Mukhopadhyay and Gorla [17]) are introduced:

$$\psi(x, y) = \sqrt{2\nu L U_0} e^{x/2L} f(\eta) \quad (8)$$

$$\eta = \sqrt{\frac{U_0}{2\nu L}} e^{x/2L} y \quad (9)$$

$$T = T_\infty + T_0 e^{x/2L} \theta(\eta) \quad (10)$$

where $\psi(x, y)$ is the stream function defined as $u = \partial\psi / \partial y$ and $v = -\partial\psi / \partial x$ which automatically satisfy the continuity equation (1), $f(\eta)$ is the dimensionless stream function, η is the similarity variable and $\theta(\eta)$ is the dimensionless temperature. Finally the momentum and energy equations (2) and (7) subject to the boundary conditions (4), can be transformed to nonlinear ordinary differential equations:

$$f''' + ff'' - 2f'^2 - Mf' = 0 \quad (11)$$

$$\left(1 + \frac{4}{3} R \right) \theta'' + \text{Pr} \left\{ f\theta' - f'\theta + Ec \left(f''^2 + Mf'^2 \right) \right\} = 0 \quad (12)$$

with the transformed boundary conditions:

$$\begin{aligned} \eta = 0 : \quad f = S, \quad f' = 1 + \lambda f'', \quad \theta = 1 + \delta \theta' \\ \eta \rightarrow \infty : \quad f' \rightarrow 0, \quad \theta \rightarrow 0 \end{aligned} \quad (13)$$

where the prime denotes differentiation with respect to η . $M = 2L\sigma_e B_0^2 / \rho U_w$ is the magnetic parameter, $R = 4\sigma^* T_\infty^3 / \kappa k^*$ is the thermal radiation parameter, $\text{Pr} = \mu C_p / \kappa$ is the Prandtl number, $Ec = U_w^2 / C_p T_0 e^{x/2L}$ is the Eckert number, $S = V_0 (2L / \nu U_0)^{1/2}$ is the suction or blowing parameter, $\lambda = N_1 (\nu U_0 / 2L)^{1/2}$ is the velocity slip parameter and $\delta = D_1 (U_0 / 2\nu L)^{1/2}$ is the thermal slip parameter.

4. Numerical solution

For computations of the equations (11) and (12) along with the boundary conditions (13), the Runge-Kutta fourth order method in the association with shooting technique is applied. First introducing the new set of dependent variables w_1, w_2, w_3, p_1 and p_2 , equations (11) and (12) with the boundary condition (13) are converted into the following simultaneous linear differential equations of first order

$$w_1' = w_2 \quad (14)$$

$$w_2' = w_3 \quad (15)$$

$$w_3' = -(w_1 w_3 - 2w_2^2 - M w_2) \quad (16)$$

and

$$p_1' = p_2 \quad (17)$$

$$p_2' = -\frac{3\text{Pr}}{3+4R} \left\{ w_1 p_2 - w_2 p_1 + Ec (w_3^2 + M w_2^2) \right\} \quad (18)$$

with the boundary conditions

$$\begin{aligned} \eta = 0 & : w_1 = S, \quad w_2 = 1 + \lambda w_3, \quad p_1 = 1 + \delta p_2 \\ \eta \rightarrow \infty & : w_2 \rightarrow 0, \quad p_1 \rightarrow 0 \end{aligned} \quad (19)$$

where $w_1 = f$, $w_2 = f'$, $w_3 = f''$, $p_1 = \theta$ and $p_2 = \theta'$.

To solve the equations (16) and (18) as an initial value problem, one requires a value for $w_3(0)$ and $p_2(0)$ but no such values are given at the boundary. Using shooting technique, the suitable estimated values for $w_3(0)$ and $p_2(0)$ are chosen randomly and the fourth order Runge-Kutta method is applied to obtain the solution. Comparing the calculated values for $w_2(0)$ and $p_1(0)$ for various values of different parameters at the far field boundary condition assuming $\eta \rightarrow \infty = 6$ with the given boundary conditions $w_2(6) \rightarrow 0$ and $p_1(6) \rightarrow 0$, the values of $w_3(0)$ and $p_2(0)$ are adjusted for a better approximation. As the criterion of convergence the step size is taken as $\Delta\eta = 0.001$ and accuracy of the six decimal places is considered.

5. Rate of shear stress and rate of heat transfer

The physical quantities of primary interest are the skin friction coefficient C_f and the local Nusselt number Nu_x , which are defined by:

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho U_w^2} \quad (20)$$

$$Nu_x = \frac{x q_w}{\kappa (T_w - T_\infty)} \quad (21)$$

where $\tau_w = \mu (\partial u / \partial y)_{y=0}$ is the wall shear stress and $q_w = -\kappa (\partial T / \partial y)_{y=0}$ is the heat transfer from the sheet. In the present case, the equations (20) and (21) can be expressed in the following forms:

$$C_f = \frac{2}{\sqrt{\text{Re}_x}} f''(0) \quad (22)$$

$$Nu_x = -\sqrt{\text{Re}_x} \theta'(0) \quad (23)$$

where $\text{Re}_x = U_w x / \nu$ is the local Reynolds number. The rate of shear stress $f''(0)$ and the rate of heat transfer $\theta'(0)$ are proportional to the skin friction coefficient C_f and the Nusselt number Nu_x respectively.

6. Validation of the proposed method

In order to validate the numerical method which was proposed in the previous section, the results of heat transfer rate $\theta'(0)$ for different values of the thermal radiation parameter R , the Prandtl number Pr and the Eckert number Ec are compared in the absence of the suction or blowing parameter S , the velocity slip parameter λ , the magnetic parameter M and the thermal slip parameter δ with the earlier researchers like Bidin and Nazar [15], Nadeem et al. [16] and Mukhopadhyay and Gorla [17] in Table 1. In this table, the comparison shows that the present results are very close to those researchers. It can also be claimed that the demonstrated results are reliable and efficient.

Table 1. Comparison of $-\theta'(0)$ for several values of R , Pr and Ec with $S = \lambda = M = \delta = 0$ and $f''(0) = -1.2821307$.

R	Pr	Ec	Bidin and Nazar [15]	Nadeem et al. [16]	Mukhopadhyay and Gorla [17]	Present Results
0.5	1	0.0	0.6765	0.680	0.6765	0.6859730
	2		1.0735	1.073	1.0734	1.0737274
	3		1.3807	1.381	1.3807	1.3805010
	1	0.2	0.6177			0.6270190
	2		0.9654			0.9655080
	3		1.2286			1.2282404
1.0	1	0.0	0.5315	0.534	0.5315	0.5527834
	2		0.8627	0.863	0.8626	0.8653065
	3		1.1214	1.121	1.1213	1.1214546
	1	0.2	0.4877			0.5094000
	2		0.7818			0.7843958
	3		1.0067			1.0066859

7. Discussion of the computed results

Figures 2 to 4 show the influence for the various values of the suction or blowing parameter S , the velocity slip parameter λ and the magnetic parameter M on the velocity distribution $f'(\eta)$ respectively, while the other parameters are constant. From these figures, it is observed that the velocity decreases with the increasing values of the suction or blowing parameter S , the velocity slip parameter λ and the magnetic parameter M while an opposite phenomenon occurs for the velocity slip parameter λ at $\eta > 2.7$ in figure 3.

The behavior of the temperature profiles $\theta(\eta)$ for several values of the suction or blowing parameter S , the velocity slip parameter λ , the magnetic parameter M , the thermal slip parameter δ , the

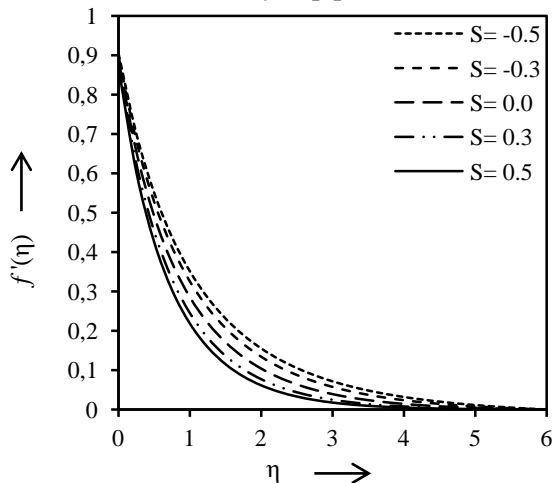


Figure 2. Influence of S on velocity against η for $\lambda=0.1$ and $M=0.1$.

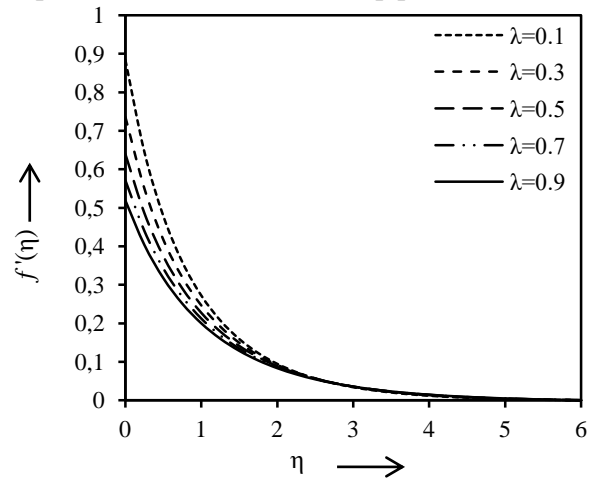


Figure 3. Influence of λ on velocity against η for $S=0.1$ and $M=0.1$.

thermal radiation parameter R , the Prandtl number Pr and the Eckert number Ec are presented in Figures 5 to 11 respectively, keeping other parameters constant. It is ascertained from these figures that the temperature decreases with the increasing values of the suction or blowing parameter S , the thermal slip parameter δ and the Prandtl number Pr but the reverse is true for the velocity slip parameter λ , the magnetic parameter M , the thermal radiation parameter R and the Eckert number Ec . When a uniform magnetic field is applied normal to the flow direction, a force is produced which acts in negative direction of flow. This force is known as Lorentz force. The increasing values of the magnetic parameter make this force stronger, which ultimately slows down the fluid flow and accelerate the temperature.

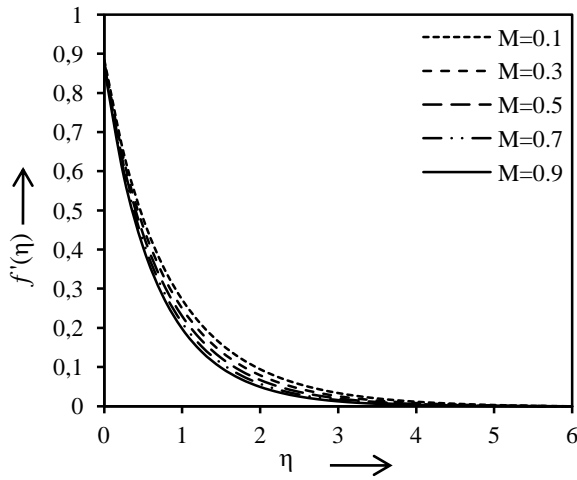


Figure 4. Influence of M on velocity against η for $S=0.1$ and $\lambda=0.1$.

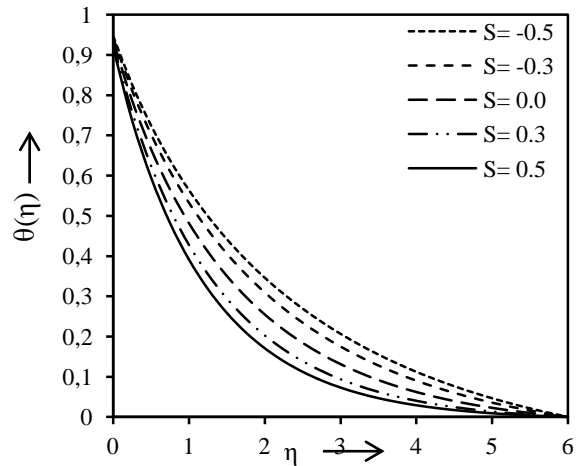


Figure 5. Influence of S on temperature against η for $\lambda=0.1$, $M=0.1$, $\delta=0.1$, $R=10$, $Pr=10$ and $Ec=0.01$.

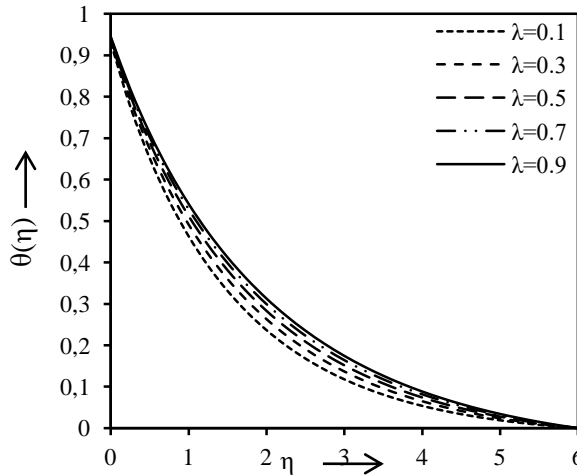


Figure 6. Influence of λ on temperature against η for $S=0.1$, $M=0.1$, $\delta=0.1$, $R=10$, $Pr=10$ and $Ec=0.01$.

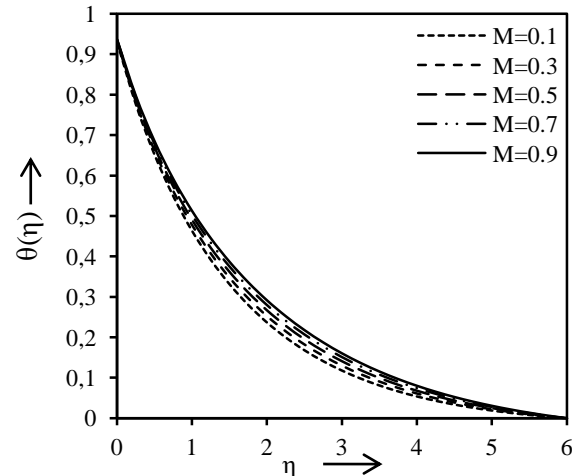


Figure 7. Influence of M on temperature against η for $S=0.1$, $\lambda=0.1$, $\delta=0.1$, $R=10$, $Pr=10$ and $Ec=0.01$.

Table 2 depicts the computations for the skin friction coefficient $f''(0)$ and the Nusselt number $\theta'(0)$ at the surface for different values of the suction or blowing parameter S , the velocity slip parameter λ , the magnetic parameter M , the thermal slip parameter δ , the thermal radiation parameter R , the Prandtl number Pr , and the Eckert number Ec . From the table, it is obvious that the local skin-

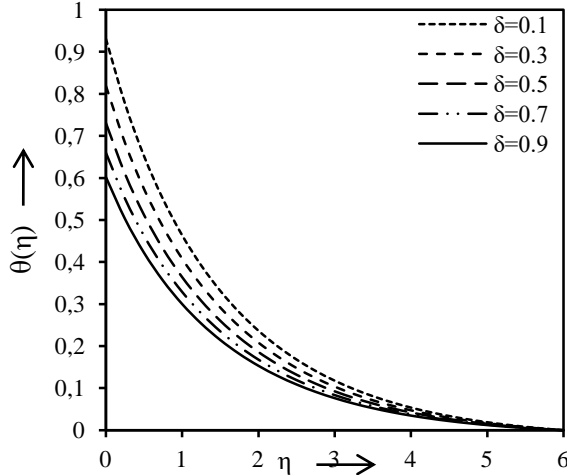


Figure 8. Influence of δ on temperature against η for $S=0.1$, $\lambda=0.1$, $M=0.1$, $R=10$, $Pr=10$ and $Ec=0.01$.

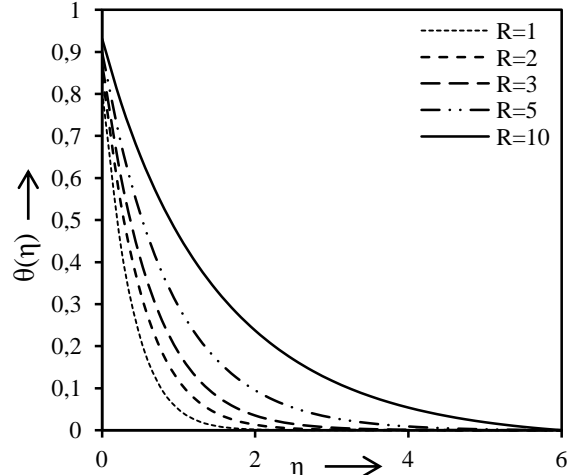


Figure 9. Influence of R on temperature against η for $S=0.1$, $\lambda=0.1$, $M=0.1$, $\delta=0.1$, $Pr=10$ and $Ec=0.01$.

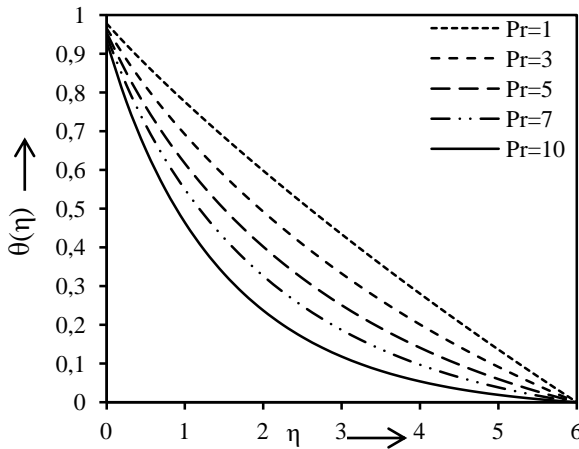


Figure 10. Influence of Pr on temperature against η for $S=0.1$, $\lambda=0.1$, $M=0.1$, $\delta=0.1$, $R=10$ and $Ec=0.01$.

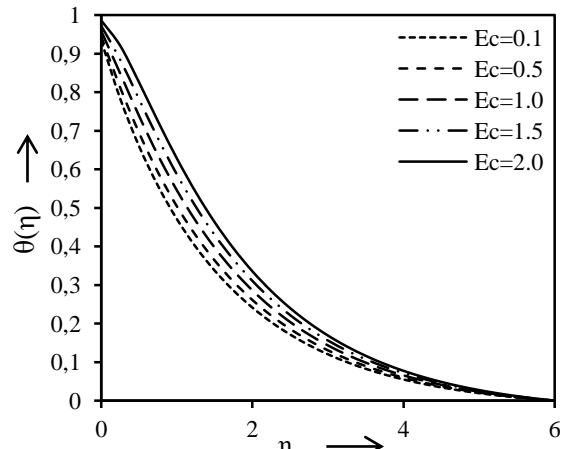


Figure 11. Influence of Ec on temperature against η for $S=0.1$, $\lambda=0.1$, $M=0.1$, $\delta=0.1$, $R=10$ and $Pr=10$.

friction coefficient $f''(0)$ and the local Nusselt number $\theta'(0)$ decrease with the increasing values of the suction or blowing parameter S but a reverse phenomenon occurs for the velocity slip parameter λ taking other parameters constant respectively. Further it may be seen that the skin friction coefficient $f''(0)$ decreases while the local Nusselt number $\theta'(0)$ increases with the increasing values of the magnetic parameter M . Moreover, it is found that the Nusselt number $\theta'(0)$ increases with the increasing values of the thermal slip parameter δ , the thermal radiation parameter R and the Eckert number Ec while an opposite effect occurs for the Prandtl number Pr , when other parameters kept constant respectively. This table also shows that the skin friction coefficient $f''(0)$ and the local Nusselt number $\theta'(0)$ are always negative for all the values of physical parameters considered. Physically, the negative sign of skin friction coefficient implies that the fluid exerts a drag force from the surface and negative local Nusselt number means there is a heat flow from the surface.

Table 2. Results of $f''(0)$ and $\theta'(0)$ for several values of S , λ , M , δ , R , Pr and Ec .

S	λ	M	δ	R	Pr	Ec	$-f''(0)$	$-\theta'(0)$
-0.5	0.1	0.1	0.1	10	10	0.01	0.9521657	0.529612
-0.3							1.0119855	0.575976
0.0							1.1112839	0.655864
0.3							1.2223470	0.748100
0.5							1.3027500	0.815985
0.1	0.1	0.1	0.1	10	10	0.01	1.1470008	0.685269
	0.3						0.8788629	0.633248
	0.5						0.7190887	0.596666
	0.7						0.6115220	0.568730
	0.9						0.5335684	0.546309
0.1	0.1	0.3	0.1	10	10	0.01	1.2057009	0.667330
		0.5					1.2604935	0.651047
		0.7					1.3119887	0.636170
		0.9					1.3606528	0.622508
0.1	0.1	0.1	0.3	10	10	0.01		0.602379
			0.5					0.537378
			0.7					0.485038
			0.9					0.441990
0.1	0.1	0.1	0.1	1	10	0.01		1.924327
				2				1.525495
				3				1.288670
				5				1.007521
0.1	0.1	0.1	0.1	10	1	0.01		0.223966
					3			0.339339
					5			0.447530
					7			0.548135
0.1	0.1	0.1	0.1	10	10	0.10		0.660506
						0.50		0.550453
						1.00		0.412886
						1.50		0.275318
						2.00		0.137750

8. Conclusions

The problem of hydromagnetic flow and heat transfer over an exponentially stretching surface is investigated with the radiation effects and partial slip boundary conditions. From the results of the problem, it can be concluded that the flow field, temperature profiles and the quantities of physical interest are significantly affected by these parameters:

- (i) The velocity boundary layer thickness decreases with the increasing values of the suction or blowing parameter, the velocity slip parameter and the magnetic parameter but a reverse behavior is noted being η greater than 2.7 in case of the velocity slip parameter.
- (ii) Thermal boundary layer thickness decreases with the increasing values of the suction or blowing parameter, the thermal slip parameter and the Prandtl number while it increases with the velocity slip parameter, the magnetic parameter, the thermal radiation parameter and the Eckert number.
- (iii) The wall shear stress decreases with the increasing values of the suction or blowing parameter and the magnetic parameter although an opposite phenomenon occurs for the velocity slip parameter.

- (iv) Finally the rate of heat transfer decreases with the increasing values of the suction or blowing parameter and the Prandtl number however it increases with an increment in the velocity slip parameter, the magnetic parameter, the thermal slip parameter, the thermal radiation parameter and the Eckert number.

Nomenclature

B_o	Uniform magnetic field, $[Kgs^{-2}A^{-1}]$	T	Temperature of the fluid, $[K]$
C_f	Local skin-friction coefficient $(= 2\tau_w / \rho U_w^2), [-]$	T_0	Reference temperature, $[K]$
C_p	Specific heat at constant pressure, $[JKg^{-1}K^{-1}]$	T_w	Surface temperature, $[K]$
D	Thermal slip factor, $[-]$	T_∞	Free stream temperature, $[K]$
D_1	Initial thermal slip factor, $[-]$	U_0	Reference velocity, $[ms^{-1}]$
Ec	Eckert number $(= U_w^2 / C_p T_0 e^{x/2L}), [-]$	U_w	Surface velocity, $[ms^{-1}]$
f	Dimensionless stream function, $[-]$	u	Velocity component in the x – direction, $[ms^{-1}]$
k^*	Absorption coefficient, $[m^{-1}]$	v	Velocity component in the y – direction, $[ms^{-1}]$
L	Reference length, $[m]$	V_0	Initial strength of suction and blowing, $[ms^{-1}]$
M	Magnetic parameter $(= 2L\sigma_e B_o^2 / \rho U_w), [-]$	V_w	Surface velocity of suction and blowing, $[ms^{-1}]$
N	Velocity slip factor, $[-]$	x	Along the exponentially stretching surface distance, $[m]$
N_1	Initial velocity slip factor, $[-]$	y	Normal distance, $[m]$
Nu_x	Local Nusselt number, $(= xq_w / \kappa(T_w - T_\infty)) [-]$	<i>Greek symbols</i>	
Pr	Prandtl number $(= \mu C_p / \kappa), [-]$	η	Similarity variable, $[-]$
q_r	Radiative heat flux, $[Kgm^{-2}]$	δ	Thermal slip parameter $(= D_1 \sqrt{U_0 / 2\nu L}), [-]$
q_w	Heat transfer from the sheet, $[Kgm^{-2}]$	θ	Dimensionless temperature, $[-]$
R	Thermal radiation parameter $(= 4\sigma^* T_\infty^3 / \kappa k^*), [-]$	κ	Thermal conductivity, $[Kgm^{-3}K^{-1}]$
Re_x	Local Reynolds number $(= U_w x / \nu), [-]$	λ	Velocity slip parameter $(= N_1 \sqrt{\nu U_0 / 2L}), [-]$
S	Suction or blowing parameter $(= V_0 \sqrt{2L / \nu U_0}), [-]$	μ	Coefficient of viscosity, $[Nsm^{-2}]$
		ν	Kinematic viscosity $(= \mu / \rho), [m^2s^{-1}]$
		ρ	Fluid density, $[Kgm^{-3}]$
		σ_e	Electrical conductivity, $[s^3A^2Kg^{-1}m^{-3}]$

σ^*	Stefan-Boltzmann constant, [$Kgm^{-2}K^{-4}$]	<i>Superscript</i> '	Differentiation with respect to η
τ_w	Wall shear stress, [$Kgm^{-1}s^{-2}$]	<i>Subscripts</i> w	Surface conditions
ψ	Stream function, [m^2s^{-1}]	∞	Conditions for away from the surface

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