THE EXACT EFFECTS OF RADIATION AND JOULE HEATING ON MAGNETOHYDRODYNAMIC MARANGONI CONVECTION OVER A FLAT SURFACE

by

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In this paper, we re-investigate the problem describing effects of radiation, Joule heating, and viscous dissipation on magnetohydrodynamic Marangoni convection boundary layer over a flat surface with suction/injection. The analytical solution obtained for the reduced system of non-linear-coupled differential equations governing the problem. Laplace transform successfully implemented to get the exact expression for the temperature profile. Furthermore, comparing the current exact results with approximate numerical results obtained using Runge-Kutta-Fehlberg method is introduced. These comparisons declare that the published numerical results agree with the current exact results. In addition, the effects of various parameters on the temperature profile are discussed graphically.

Key words: *radiation, Joule heating, Marangoni convection, flat surface, exact solution*

Introduction

The MHD of an electrically conducting fluid has important bearings in geophysics, astrophysics, aeronautics, engineering applications, and many other areas, [1]. Because of this reason, many researchers tend to apply the MHD flow into their problems. For instance, Amkadni and Azzouzi [2] studied the similarity solution of MHD boundary layer flow over a moving vertical cylinder. Meanwhile, Rajeswari *et al.* [3] analyzed the influence of chemical reaction parameter, magnetic parameter, buoyancy parameter, and suction parameter on non-linear MHD boundary layer flow through a vertical porous surface. Recently, Bhattacharyya and Layek [4] investigated the effects of chemical reaction in MHD boundary layer flow over a permeable stretching sheet subject to suction or injection. They discovered that the rate of solute transfer could be enhanced by increasing the values of the magnetic parameter and suction parameter. Moreover, Al-Mudhaf and Chamkha [5] investigated the MHD thermosolutal Marangoni convection boundary layer. They have studied the effects of heat generation/absorption, Hartmann number, the chemical reaction parameter and the suction/injection parameter on the flow.

Sulochana and Sandeep [6] investigated the MHD forced convective flow of a nanofluid over a slendering stretching sheet in porous medium in presence of thermal radia-

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tion and slip effects. Veerasuneela *et al.* [7] studied the heat and mass transfer flow of a viscous electrically conducting fluid in a vertical wavy channel under the influence of an inclined magnetic field with heat generating sources.

The study of thermal radiation has received considerable attention in recent years as its effect in high operating temperature process. For instance, Pathak and Maheshwari [8] have analyzed the influence of radiation on an unsteady free convection flow bounded by an oscillating plate with variable wall temperature. Sulochana *et al.* [9] studied the thermal radiation and viscous dissipation on 3-D MHD Casson fluid over a stretching surface in presence of chemical reaction. Sandeep *et al.* [10] analyzed the MHD, radiation, and chemical reaction effects on unsteady flow, heat and mass transfer characteristics in a viscous, incompressible, and electrically conducting fluid over a semi-infinite vertical porous plate through porous media. The effects of thermal radiation, buoyancy and suction/blowing on natural convection heat and mass transfer over a semi-infinite stretching surface has been studied by Shateyi [11]. Suneetha *et al.* [12] have investigated the radiation effects on the MHD free convection flow past an impulsively started vertical plate with variable surface temperature and concentration. The effects of Joule heating and viscous dissipation are usually can be characterized by the Eckert number and magnetic parameter. Both have a very important rule in geophysical flows and in nuclear engineering, Alim *et al.* [13]. Not only that, the effects of suction or injection on boundary layer flow also have a huge influence over the engineering application and have been widely investigated by numerous researchers. With this understanding, many researchers studied the effects of Joule heating and viscous dissipation along with the effects of suction or injection in various geometries. For example, Borisevish and Patanin [14] for heat transfer near a rotating disk, Duwairi [15] and Chen [16] for MHD convection flow in the presence of radiation and Turkyilmazoglu [17] for viscous incompressible Newtonian and electrically conducting fluid flow over a porous rotating disk. In addition, Zhang *et al.* [18] studied the steady, laminar, thermal Marangoni convection flow of non-Newtonian power law fluid along a horizontal surface with variable surface temperature.

Hamid *et al.* [19] considered the combined effects of radiation, Joule heating, and viscous dissipation on MHD Marangoni convection flow in the presence of suction or injection. The objectives of their paper was to investigate the effects of radiation parameter, magnetic parameter, Eckert number, as well as suction or injection parameter on the surface velocity, surface temperature gradient, as well as the velocity and temperature profiles. Their numerical results obtained using the shooting method along with the Runge-Kutta-Fehlberg method. However, a number of researchers has recently proved that the approximate numerical results obtained by some semi-analytical methods were inaccurate enough to be presented to the scientific community [20-23]. Therefore, the current paper attempt to reinvestigate the work done by Hamid *et al.* [19].

The main objectives of the current work, therefore, can be summarized as follows.

– The first task is to obtain the exact solutions for the reduced system of non-linear coupled differential equations governing the flow.
– The obtained exact solutions used to introduce various predictions for the exact temperature and the exact fluid velocity.
– The final major task is to study the effects of various parameters on the included physical phenomena at the same values of parameters chosen by Hamid *et al.* [19] and hence checking the accuracy of their approximate numerical solutions.

Formulation of the physical problem

Hamid *et al.* [19] studied the problem of steady two dimensional, laminar boundary layer flow of an electrically conducting fluid over a flat surface through a uniformly distributed transverse magnetic field of strength B_0. With the usual boundary layer approximations, the governing equations written in the following form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\Delta\frac{B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{\sigma B_0^2}{\rho c_p}u^2+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

The boundary conditions are:

$$v(x,0)=-v_w,\quad T(x,0)=T_\infty+Ax^2,\quad \mu\frac{\partial u}{\partial y}=-\frac{\mathrm{d}\sigma}{\mathrm{d}T}\frac{\partial T}{\partial x}\quad \text{at}\quad y=0$$

$$u(x,\infty)=0,\quad T(x,\infty)=T_\infty\quad \text{as}\quad y\to\infty \tag{4}$$

where u and v are the velocity components in the x- and y-directions, ν – the kinematic viscocity, Δ – the electric conductivity, B_0 – the uniform magnetic field strength, ρ – the density of the fluid, T – the fluid temperature, k – the thermal conductivity of the fluid, c_p – the specific heat flux, v_w – the constant suction ($v_w > 0$) or injection ($v_w < 0$) velocity, and μ – the dynamic viscosity.

The radiative heat flux, q_r, under Rosseland approximation has the form [24]:

$$q_\mathrm{r}=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ^* is the Stefan-Boltzmann constant, and k^* is the mean absorption coefficient. The temperature difference within the flow are assumed to be sufficiently small such that T^4 may be expressed as linear function of temperature. Expanding T^4 using Taylor series and neglecting higher order terms yields [19]:

$$T^4\cong 4T_\infty^3 T-3T_\infty^4 \tag{6}$$

To convert the governing equations (1)-(4) into set of non-linear differential equations Hamid *et al.* [19] used the standard definition of the stream function $\psi(x, y)$, $u = \partial\psi/\partial x$, $\upsilon = \partial\psi/\partial x$, and the similarity transformation of [5]:

$$\psi(x,y)=\frac{1}{C_2}xf(\eta),\quad \theta(\eta)=\frac{[T(x,y)-T_\infty]x^{-2}}{A},\quad \eta=C_1 y \tag{7}$$

where

$$C_1 = \sqrt[3]{\frac{\rho A \dfrac{\mathrm{d}\sigma}{\mathrm{d}T}}{\mu^2}}, \quad C_2 = \sqrt[3]{\frac{\rho^2}{\mu A \dfrac{\mathrm{d}\sigma}{\mathrm{d}T}}} \tag{8}$$

Using equations (5) and (6) and (7) and (8), equations (1)-(3) can be reduced to the following form:

$$f'''(\eta) + f(\eta)f''(\eta) - [f'(\eta)]^2 - M^2 f'(\eta) = 0 \tag{9}$$

$$\left(\frac{1+Nr}{\Pr}\right)\theta''(\eta) - 2f'(\eta)\theta(\eta) + f(\eta)\theta'(\eta) + \mathrm{Ec}\{M^2[f''(\eta)]^2 + [f''(\eta)]^2\} = 0 \tag{10}$$

The transformed boundary conditions are:

$$f(0) = f_w, \quad f''(0) = -2, \quad f'(\infty) = 0 \tag{11}$$

$$\theta(0) = 1, \quad \theta(\infty) = 0 \tag{12}$$

where the prime (′) denotes differentiation with respect to η, $Nr = 16\sigma^* T_\infty^3 / 3kk^*$ is the radiation parameter, $M^2 = \Delta B_0^2 C_2 / \rho C_1$ – the magnetic field parameter, Pr – the Prandtl number, $\mathrm{Ec} = C_1^2 / A C_p C_2^2$ – the Eckert number, f_w (> 0) – the constant suction parameter, and f_w (< 0) – the constant injection parameter. It should be noticed that the effects of viscous dissipation is characterized by the Eckert number while Joule heating effects is represented by the product of Ec and M. The system (9)-(12) was numerically solved in [19] by using the Runge-Kutta-Fehlberg method. This approximate numerical solution used to discuss various physical phenomena. Hence, the subject of the next section is to obtain the exact solutions of the above system of non-linear differential equations (9)-(12).

The exact solutions

Before launching into the main objective of this section, it may reasonable first to describe the proposed approach for obtaining the exact solutions of the present system of non-linear differential equations given by eqs. (9)-(12) by following [25]. It should be noted that eq. (9) with BC (11) has an exact solution. Such exact solution can be assumed to take the following form:

$$f(\eta) = a + b\mathrm{e}^{-\beta\eta} \tag{13}$$

where the constants a and b are determined by applying the first two conditions in eq. (11) and therefore, their values are obtained:

$$a = f_w + \frac{2}{\beta^2}, \quad b = -\frac{2}{\beta^2} \tag{14}$$

In view of the assumed stream function $f(\eta)$ described by eq. (13), it can be easily seen that the boundary condition at infinity represented by eq. (11) is trivially satisfied for β > 0. Moreover, in order to find the governing equation of this unknown β, we substitute from eq. (13) into eq. (9) and this directly leads to following formula for β:

$$\beta^3 - c\beta^2 - M^2\beta - 2 = 0 \tag{15}$$

where β is a positive root for the previous equation.

The exact solution for the f, eq. (9) expressed in eqs. (13)-(15) can be verified by direct substitution into the governing non-linear eq. (9) and the corresponding BC (11). Similar algebraic equations have been obtained in recent studies [26, 27]. The main characteristic of this cubic algebraic eq. (15) that it has only one positive real root, while the two other roots are either complex or negative, see [28]. Now, insert eq. (13) into eq. (10), we obtain a second-order linear ODE with exponential function coefficients. In order to simplify the resulting ODE in the unknown θ, we follow the approach proposed by Ebaid *et al.* [25] by introducing a new independent variable t which takes the form, $t = \mathrm{e}^{-\beta\eta}$ Consequently, the transformed θ-equation is given as: ϑ

$$t\theta''(t) + (n - mt)\theta'(t) + 2m\theta(t) = -\lambda t \tag{16}$$

where

$$n = 1 - \frac{a}{\Omega\beta}, \quad m = \frac{b}{\Omega\beta}, \quad \lambda = \frac{\mathrm{Ec}\,\beta^2(\beta^2 + M^2)}{\Omega}, \quad \Omega = \frac{1 + Nr}{\mathrm{Pr}} \tag{17}$$

The transformed boundary conditions are:

$$\theta(0) = 0, \quad \theta(1) = 1 \tag{18}$$

Applying Laplace transform to eq. (16), we have:

$$(ms - s^2)\Theta'(s) + [(n-2)s + 3m]\Theta(s) = -\frac{\lambda}{s^2} \tag{19}$$

where $\Theta(s)$ is the Laplace transform of $\theta(t)$. Integrating eq. (19) and applying the Laplace transform to the resulting equation, we obtain:

$$\begin{aligned}\theta(t) &= -\frac{\lambda t^2}{2(n+1)} + \frac{c}{2\Gamma(-n-1)}\int_0^t (t-\tau)^2 \tau^{-(n+2)} \mathrm{e}^{m\tau} \mathrm{d}\tau = \\ &= -\frac{\lambda t^2}{2(n+1)} + \frac{c}{2\Gamma(-n-1)} t^{1-n} \int_0^1 (1-\mu)^2 \mu^{-(n+2)} \mathrm{e}^{(mt)\mu} \mathrm{d}\mu = \\ &= -\frac{\lambda t^2}{2(n+1)} + \frac{c}{\Gamma(2-n)} t^{1-n}\, F_{1,1}\left[-1-n, 2-n, mt\right], \quad n < 1\end{aligned} \tag{20}$$

where $F_{1,1}$ is the Kummer confluent hypergeometric function, and c – the constant of integration. It is clear from this equation that the boundary condition $\theta(0) = 0$ is automatically satisfied. Besides, the other boundary condition $\theta(1) = 1$ gives c by:

$$c = \frac{\left[1 + \dfrac{\lambda}{2(n+1)}\right]\Gamma(2-\mathrm{n})}{F_{1,1}\left[-1-n, 2-n, mt\right]} \tag{21}$$

Therefore, $\theta(t)$ is given in a closed form as:

$$\theta(t) = -\frac{\lambda t^2}{2(n+1)} + \left[1 + \frac{\Gamma}{2(n+1)}\right] \frac{t^{1-n}\, F_{1,1}\left[-1-n, 2-n, mt\right]}{F_{1,1}\left[-1-n, 2-n, mt\right]} \tag{22}$$

Analysis and discussions

In the previous section, we have obtained the exact solutions for the fluid velocity and the temperature profile. These exact solutions will be investigated here to discuss the effect of various parameters on the velocity and the temperature of the fluid. Table 1 shows comparisons between the current exact numerical results and those obtained by Hamid *et al.* [19] for the effects of Prandtl number, Pr, magnetic parameter, M, radiation parameter, Nr, Joule heating, and viscous dissipation on the surface temperature gradient, $\theta'(0)$, when $f_w = 0$. It is observed that when Ec = 0 (no Joule heating and viscous dissipation), the increase in Prandtl number increases the surface temperature gradients, *i. e.*, the interface heat transfer. On the other hand, increasing the values of M and Nr reduces the surface temperature gradient. When viscous dissipation and Joule heating are considered, a reduction in the interface heat transfer is noticed when we increase the values of the parameter M, Ec, and Nr.

It has been found in some previous work that some of the approximate solutions used to solve some kind of systems of differential equations are not accurate enough [22, 23, 26, 28] or even wrong in some others [20, 21], if it compared with the exact solutions. Fortunately, enough, regarding tab. 1, it can easily noticed that the percentage error between the approximate numerical values obtained by [19] and the current exact calculations not exceed 0.007 % in most calculations. This is of course is an advantage Runge-Kutta-Fehlberg method [19] and indicates of course how much accuracy of the numerical technique used to solve the governing set of coupled non-linear differential equations (9)-(12) comparing with new exact solution obtained in the current work.

Figures 1-3 show the effects of Pr, Nr, and combined effects of Joule and viscous heating in the existence of suction and injection on the temperature profiles. Depreciation in the temperature profiles of the flow has been observed by increasing Prandtl number, it is an expected result which has been previously shown by several authors, among [6], where with such increasing impact of Prandtl number the thermal boundary layers become thinner. On the other hand, a raise in the temperature profiles is noticed with increasing the values of the radiation parameter Nr and Eckert number. For the former case, it is evident that increasing in the radiation parameter releases the heat energy to the flow which causes to develop the thermal boundary layer thickness. The same later conclusion can be also followed regarding the raise in the temperature profiles with increasing Eckert number due to the increase in thermal boundary layer thickness.

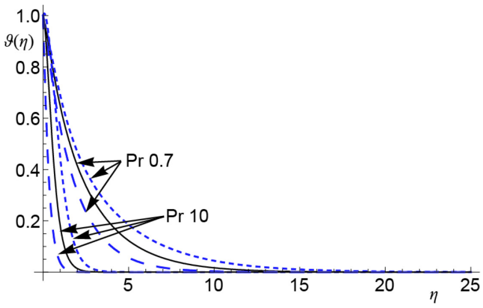

Figure 1. Effects of Pr on temperature profiles when Nr = 1, Ec = 1, M = 0.5 with values of f_w = –1 (dashed line), f_w = 0, (solid line), and f_w = 1 (long dashed line)

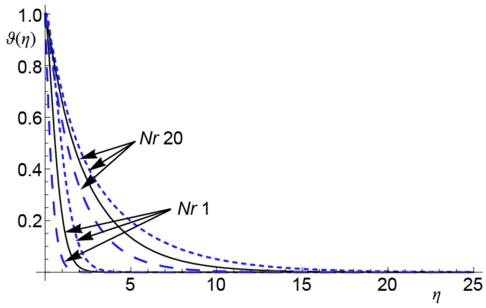

Figure 2. Effects of Nr on temperature profiles when Pr = 7, Ec = 1, M = 0.5 with values of f_w = –1 (dashed line), f_w = 0 (solid line), and f_w = 1 (long dashed line)

Table 1. Effects of Pr, *M, Nr*, and Ec on the heat transfer at the interface, $-\theta'(0)$

$\theta'(0)$							
Pr	M	Nr	Ec	No Joule and viscous heating (Ec = 0)		With Joule and viscous heating	
				Numeric [19]	Exact [current]	Numeric [19]	Exact [current]
0.7 (air)	0.1	1	0	0.846557	0.846535		
			1			0.397620	0.397592
			2			–0.051315	–0.051351
		2	0	0.629720	0.629635		
			1			0.313849	0.313827
			2			–0.001954	–0.0019811
	0.5	1	0	0.800423	0.800417		
			1			0.305089	0.305080
			2			–0.190245	–0.190257
		2	0	0.590971	0.59096		
			1			0.243832	0.243819
			2			–0.103307	–0.103323
7 (water)	0.1	1	0	3.437454	3.437450		
			1			0.709948	0.709949
			2			–2.017556	–2.017560
		2	0	2.745009	2.745010		
			1			0.712576	0.712576
			2			–0.1319857	–0.131986
	0.5	1	0	3.329354	3.329350		
			1			0.256583	0.256584
			2			–2.816186	–2.816190
		2	0	2.653193	2.653190		
			1			0.371772	0.371773
			2			–1.909647	–1.909650
100 (eng. oil)	0.1	1	0	13.890674	13.89070		
			1			–1.154713	–1.154710
			2			–16.20010	–16.20010
		2	0	11.283932	11.28390		
			1			–0.559073	–0.559074
			2			–12.402077	–12.40210
	0.5	1	0	13.530921	13.53090		
			1			–3.751520	–3.751520
			2			–21.033964	–21.03400
		2	0	10.987189	10.98720		
			1			–2.586174	–2.586170
			2			–16.159538	–16.15950

In addition, we present in fig. 4 the effects of magnetic parameter M on the temperature profiles which teach us that the temperature profiles increase as the parameter M increases. Regarding the effect of the suction/injection parameter f_w, it can be concluded from the results in figs. 1-4 that the suction parameter decreases the temperature profiles while injection parameter increases the temperature profiles.

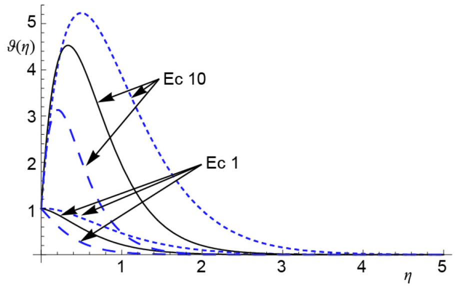

Figure 3. Combined effects of Joule heating and viscous dissipation on temperature profiles when Pr = 7, *Nr* = 1, *M* = 0.5 with values of f_w = –1 (dashed line), f_w = 0 (solid line), and f_w = 1 (long dashed line)

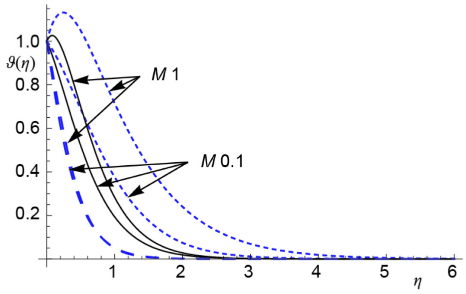

Figure 4. Effects of *M* on temperature profiles when Pr = 7, *Nr* = 1, Ec = 1with values of f_w = –1 (dashed line), f_w = 0 (solid line), and f_w = 1 (long dashed line)

Conclusion

In this paper, we have obtained the exact analytical solution for the system of non--linear coupled differential equations describing effects of radiation, Joule heating and viscous dissipation on MHD Marangoni convection boundary layer over a flat surface with suction/injection. Laplace transform has been successfully applied to get the exact expression for the temperature profile. Comparisons with published numerical results obtained by using Runge-Kutta-Fehlberg method [19] have been introduced. These comparisons] declare that the approximate numerical values obtained by [19] not exceed 0.007% in most calculations when compared with the current exact calculations. Moreover, the effects of various parameters on the temperature profile have been analyzed.

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Nomenclature

B_0 – uniform magnetic field strength
c – constant of integration
c_p – specific heat flux, [J/kgK]
Ec – Eckert number, [–]
$F_{1,1}$ – Kummer confluent hypergeometric function
f_w – constant suction or injection parameter, [–]
k – thermal conductivity of the fluid
k^* – mean absorption coefficient, [W/m^2]
M – magnetic field parameter, [–]
Nr – radiation parameter, [–]
Pr – Prandtl number, [–]
q_r – radiative heat flux, [W/m^2]
T – fluid temperature, [K]
u, v – velocity components, [m/s]
v_w – constant suction velocity, [m/s]

Greek symbols

Δ – electric conductivity
μ – dynamic viscosity, [kg/ms]
ν – kinematic viscocity, [m^2/s]
ρ – density of the fluid, [kg/m^3]
σ^* – Stefan-Boltzmann constant, [W/m^{2}K^4]
ψ – stream function, [m^2/s]

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