

ADAPTING THE FORCHHEIMER EQUATION FOR THE FLOW OF AIR THROUGH WHEAT STRAW BEDS

by

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The paper presents the results of an experimental investigation of air pressure drop while flowing through wheat straw beds. According to Darcy's law, the smaller the porosity of the bed, the bigger the pressure drop. The investigation was conducted using three different porosities (or three bed densities), and for two different air flow rates. After determining porosity (which is directly measurable), the permeability of straw could be found. For high flow velocities, such as the velocity of air flowing through a straw bale, the Forchheimer equation becomes more relevant as a correction of Darcy's law with inertial effects included. Otherwise, the permeability tensor depends only on the geometry of the porous medium. When permeability is established, the Forchheimer equation coefficients can be easily estimated. These results may be important for the future development of efficient biomass combustion facilities. The measurement methods and facility characteristics are described in more detail.

Key words: *biomass combustion, air pressure drop, porosity, Forchheimer equation, wheat straw permeability*

Introduction

Due to the increasing environmental concerns related to energy production, as well as the continuously increasing prices of fossil fuels, biomass is receiving growing attention as an energy resource alternative to fossil fuels.

Conventional combustion equipment is not designed for burning agricultural residue. It can be combusted in existing facilities with certain adaptations, or in new facilities specially designed for such purposes. However, efficient devices for agricultural waste combustion are still in the development phase. That is why grate-fired boilers burning agricultural residue are often associated with high emission levels and relatively poor fuel burnout. For the further development of these types of energy systems, a fuller understanding of the factors which affect the efficiency of power production is particularly important [1].

Serbia has great potential for using biomass as a primary energy resource. Solid agricultural waste stands out as one of the most reliable resources, primarily because of the huge amounts and quality. Table 1 shows the amount of agricultural biomass gathered in the province of Vojvodina (northern autonomous province of Serbia, area: 21506 km²) during the period from 2003 to 2007 [2].

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Table 1. Biomass production in Vojvodina region 2003-2007 [2]

Type of straw	Yield [tones/year]	Growing area [ha]
Wheat	1,120,074	297,403
Barley	154,964	47,921
Oats	12,140	5,179
Rye	4,506	1,474
Soy	620,358	127,856

Although there have been many improvements in the furnace design, packed bed combustion is still the most common method of biomass combustion. After ignition, a reaction front propagates from the surface of the bed down to the grate. This method results in a better combustion process than the ones with surface combustion, with total facility efficiency increased, and smaller emission of CO during combustion [3, 4].

The aim of this paper, is to determine the total pressure drop, as well as the local pressure drops at different heights in the straw bed, in order to obtain a better insight into the internal air flows through porous beds and determine optimal combustion conditions, considering the flow rate of theoretical air needed for combustion.

Permeability is a characteristic of porous media which affects the flow pattern and influences heat and mass transfer. Hence, the permeability and thickness of the straw bed are sufficient to describe the condition of air in the combustion area and to provide valuable data for its analysis. Due to high flowing velocities, the inertial factor cannot be neglected, so Darcy's law in its classical form is not relevant in this case. The Forchheimer equation should be used instead, because it presents the corrected form of Darcy's law, which is applicable for the flows with higher values of Reynolds number, such as the air flow through straw bales. Several papers offer very comprehensive theoretical and mathematical basis for deriving the Forchheimer equation, *e. g.* [5]. There is no data in the literature on the permeability of wheat straw, nor the Forchheimer viscous and inertial coefficients data, which we have determined in this paper.

A similar research was conducted in [6], for soya residue, so we used the procedure established there for determining the characteristics of wheat straw residue.

Porous media flow brief review

Mathematical models that describe flow through porous beds have their role in predicting mass and energy transport through the media [5]. They are created from laws of conservation of mass, energy and momentum with the application of thermodynamic relations. Porosity of media is defined as:

$$\varepsilon = \frac{V_p}{V_t} = \frac{V_p}{V_p + V_s} \quad (1)$$

Porosity estimation is experimental. Porosity is non-dimensional [m^3/m^3] and has a value between 0 and 1. Formula (1) shows that porosity of media is defined as the ratio of the volume of porous space and the total volume of bed (which is a sum of the volumes of solid fractions and porous space). The relation between porosity and pressure gradient can be expressed by Darcy's law. Darcy's law is the simplest (linear) relation between the superficial velocity and the pressure drop in the porous medium [7], and it states that:

It can be concluded that wheat straw is the most important resource because of its high annual yield and availability, so this experiment was conducted for this kind of agricultural residue. Due to the above mentioned reasons, combustion science and industry in Serbia should be focused on developing and designing wheat straw furnaces of high efficiency.

A moving grate furnace is a typical way to burn solid biomass in many combustion plants.

$$w = -\frac{K}{\mu} \frac{dp}{dx} \quad (2)$$

Permeability K is a property of porous media whose effect is taken into account by the pressure gradient term ∇p in the momentum equation. For very high velocities in porous media, inertial effects can also become significant. An inertial term added to the Darcy's equation is known as Forchheimer term.

When the Reynolds number is small enough ($Re < 1$) [7, 8], permeability is a linear function of velocity. Darcy's law is reliable for such cases where linear dependence may persist. Forchheimer term is able to account for the non-linear behavior of the pressure difference vs. velocity data. The pressure gradient ∇p is usually modeled by the Forchheimer equation [8-10]:

$$\frac{\partial p}{\partial x} = -\frac{\mu}{K_{1,ij}} w_j - \frac{\rho}{K_{2,ij}} w_{ij}^2 \quad (3)$$

where $K_{1,ij}$ and $K_{2,ij}$ are second-order tensors representing permeability of the viscous and inertial term in the Forchheimer equation, respectively.

Biomass bales analysis can include assumption of isotropic case, so a simplified form of eq. (3) is used, modified to be representative of a single flow direction:

$$\frac{\Delta p}{\Delta x} = \frac{\mu}{K_1} w + \frac{\rho}{K_2} w^2 \quad (4)$$

Experimental investigation

The investigation was conducted in an experimental facility located at the Faculty of Technical Sciences, University of Novi Sad, Novi Sad, Serbia.

Figure 1(a) presents the working part of the investigation facility with the pressure measuring ducts. The working part consists of one vertical cylinder with three pressure measuring ducts. On the top of the cylinder, there is a perforated lid, which allows the release of air and holds the straw inside the cylinder. Figure 1(b) shows the cylinder lid.

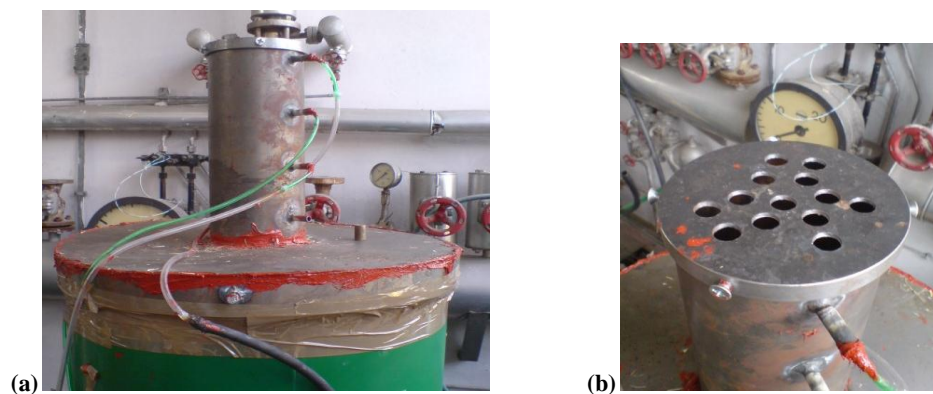


Figure 1. (a) Pressure measuring ducts, (b) Perforated lid

Measuring method and facility description

The average porosity was estimated by the method of *pouring* where two vessels were used – one with the straw placed inside and one without it. If both vessels are filled up

with liquid to the same height, the volume (or mass) of the filling liquid will be different and we can calculate porosity simply by using the ratio of these volumes. The calculated porosities are 0.91, 0.82, and 0.79, for three different densities that were chosen according to the dimensions of the measuring vessels in the laboratory.

The gauge pressure was measured at three different heights in the bed. For this purpose, the classical method of U-tubes was used, so the static pressure inside the bed was calculated from the U-tube pressure difference read-offs.

Figure 2 shows the facility setup plan. Point 1 presents the air inlet. At point 2 there is a flow measuring aperture (where the gauge pressure is also measured with U-tubes), and at point 3 there is a flow rate regulating valve. Point 4 presents an electrical centrifugal fan, and 5 is an air duct. Cylinder 6 served as a flow equalization cylinder. Point 7 is the inlet air pressure measuring point, and 8, 9, and 10 are other pressure measuring points. After point 10 the air comes to the outlet. Point 11 presents the measuring part of the facility which is filled with straw.

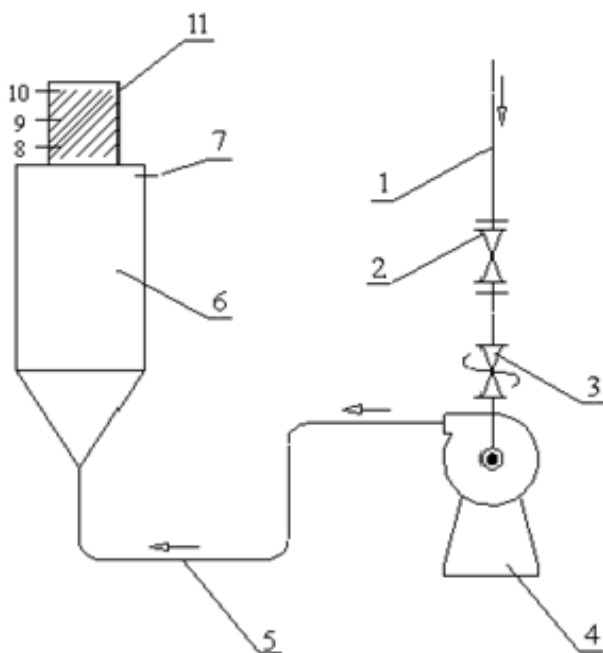


Figure 2. Facility set-up:

1 – air inlet, 2 – flow measuring aperture, 3 – valve, 4 – electrical fan, 5 – air duct, 6 – flow equalization cylinder, 7, 8, 9, 10 – static pressure measuring ducts, 11 – measuring cylinder

The cylinder is filled with the straw of previously measured porosity. After air flow is established, the pressure under the straw bed starts to increase, and pressure gradients occur. After the pressure reaches the necessary value, air will be conducted through the bed. Logically, the smaller the porosity, the larger the pressure gradients necessary for establishing the flow, so the flow rate must be increased.

The flow rate was measured using the already mentioned aperture at point 5, by measuring the pressure differences before and after the appearance of aperture disturbance, and calculating the flow from the relevant relations.

The investigation was conducted for three different porosities, and for two air flow rates for each porosity - full and

reduced flow rate (opened and half-closed valve). The calculated flow rates are 370 m³/h of air for fully open valve and 279 m³/h for half-closed valve. The ambient temperature was 20°C, so the density and viscosity of the gas were $\rho = 1.205 \text{ kg/m}^3$ and $\mu = 18.1 \cdot 10^{-6} \text{ Pa}\cdot\text{s}$.

Results analysis

The experimental results are presented in the foregoing diagrams. Figure 3 contains all the measured data that served for developing all other dependences and relations. More

precisely, fig. 3 shows the pressure drops vs. the straw layer height lines. Each of the six lines presents individual pressure drops for one value of flow rate and one porosity value. The lines with *square*, *x* and *circle* markers present the total pressure drop for the porosities of 0.91, 0.82, and 0.79, respectively, (without the release point which is at atmospheric pressure). It can be noticed that the air flows through the straw of larger porosities produced higher pressures at some points, mainly the mid-height and exhaust ones, regardless of the fact that the pressure of air intake was smaller. This observation could be explained by the fact that the straw loads with smaller porosity are liable to lodgment near the exhaust lid, which affects the increase of local pressure values. This phenomenon is restricted to laboratory conditions and should not occur in real cases with compact straw bales. Bigger loads of straw possess the structure of more expressed solid-shape characteristics, and are not liable to this kind of shape deformation. In any case, the overall pressure drop (from air inlet to release point which is at atmospheric pressure) will be equal to the gauge pressure at the inlet, *i.e.* total and atmospheric pressure difference.

By dividing the volume flow rates of air by the surface area of the straw bed (which was $A = d^2\pi/4 = 0.0201 \text{ m}^2$), cross sectional velocity (superficial velocity) w is obtained. The pressure gradient is calculated by dividing the pressure drop by the thickness of the straw bed. The dependence of superficial velocity on the pressure gradient is shown in fig 4.

The coefficients K_1 and K_2 from eq. (4) may be determined from fig. 4, by fitting a second-order polynomial function to the experimentally determined correlations (for the known density and viscosity), applying a last-squares fit. [6, 11]

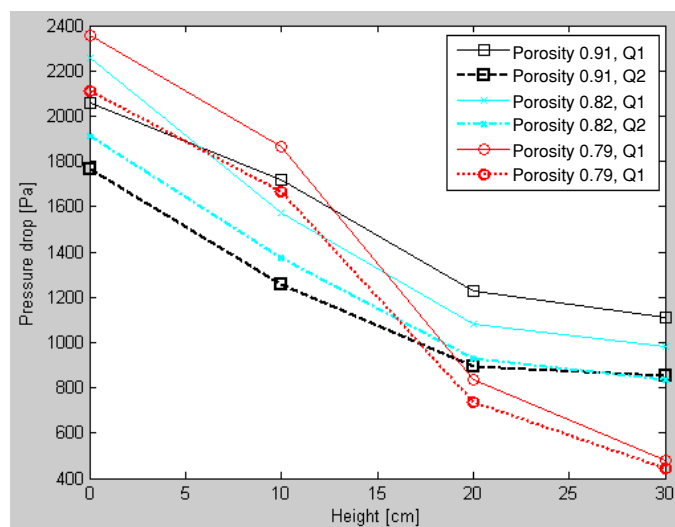


Figure 3. Pressure drop measurement results for different bed heights

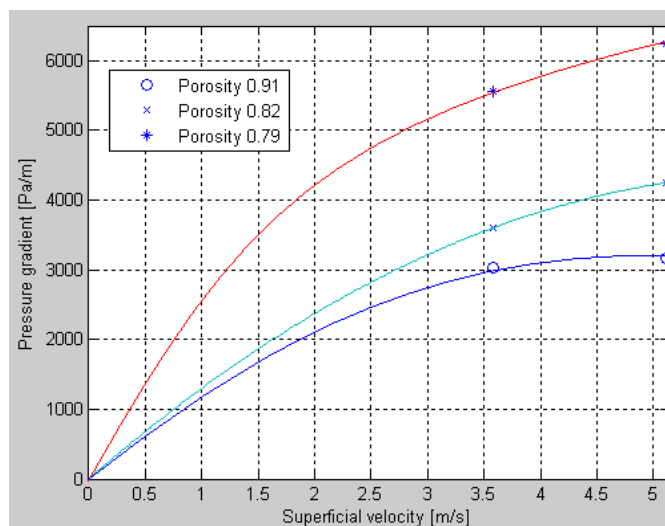


Figure 4. Pressure gradient values in relation to superficial velocity

By implementing this procedure, the proportionality coefficients $A = \mu/K_1$ and $B = \rho/K_2$ of eq. (4) were obtained, and so the coefficients K_1 and K_2 . Table 2 shows these values.

Table 2. Viscous and inertial permeability values

Porosity $E[-]$	Proportionality coefficients Viscous, Inertial		Viscous permeability, $K_1 \cdot 10^9 [m^2]$	Inertial, permeability $K_2 \cdot 10^3 [m]$
	A	B		
0.79	2541.8	-239.8	7.12	5.02
0.82	2147.6	-173.5	8.43	6.94
0.91	2009.5	-167.8	9.01	7.19

Using a similar procedure (the method of polynomial fitting), the dependence of permeability on porosity can be obtained. These dependences are shown in figs. 5 and 6.

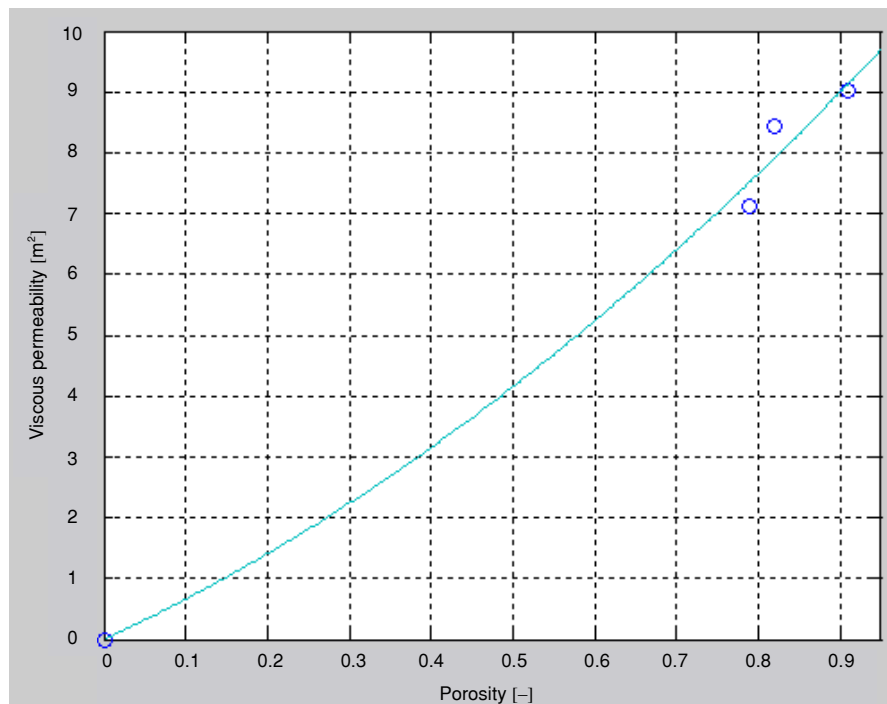


Figure 5. Variation of viscous permeability with porosity

By applying the above-mentioned method, the dependence of viscous and inertial permeability K_1 and K_2 on the porosity variations may be expressed:

$$K_1 = 4.22 \cdot \varepsilon^2 + 6.19 \cdot \varepsilon \quad (5)$$

$$K_2 = 7.64 \cdot \varepsilon^2 + 1.16 \cdot \varepsilon \quad (6)$$

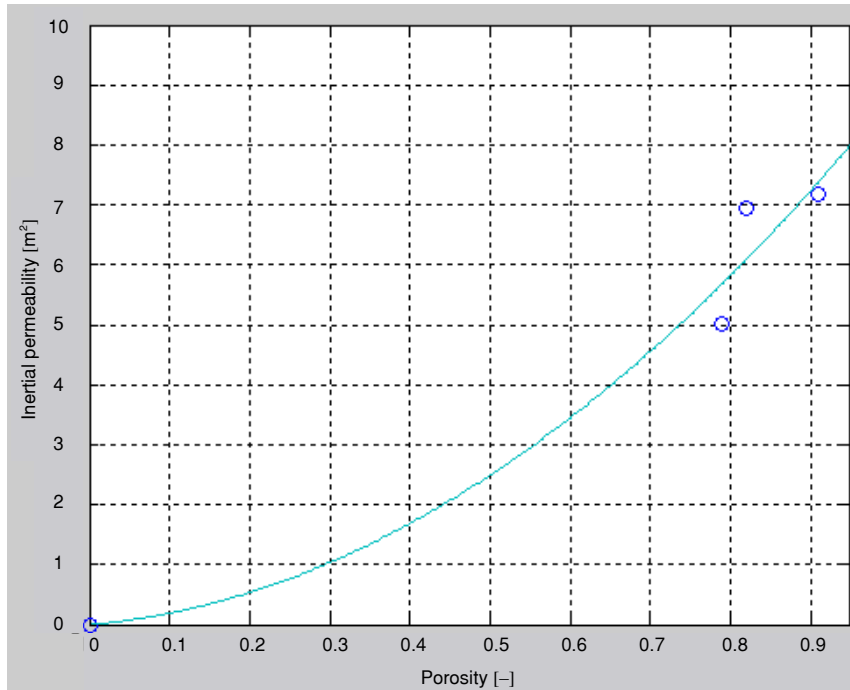


Figure 6. Variation of inertial permeability with porosity

Hence, the Forchheimer equation developed for wheat straw has the following form:

$$\frac{\Delta p}{\Delta x} = \frac{\mu}{4.22 \cdot \varepsilon^2 + 6.19 \cdot \varepsilon} w + \frac{\rho}{7.64 \cdot \varepsilon^2 + 1.16 \cdot \varepsilon} w^2 \quad (7)$$

Coefficients K_1 and K_2 could also be expressed in the form of a rational function, if it is considered more appropriate, according to Ergun's relation [12] for porous materials.

Conclusions

Based on the literature survey, we believe that due to different straw particles structure, practically every kind of agricultural waste will have different air flow characteristics, so the conclusions about the Forchheimer coefficients for different sorts of mediums must be strictly distinguished. In addition, every type of straw should be investigated by several authors, in very different conditions, to gain even better insight into the flow through the medium. Otherwise, models could be poor and insufficient for the purposes of designing useful facilities.

This paper outlines a new formula, and considering all these facts, it might be a contribution to this area of combustion science.

By analyzing the obtained results, some significant conclusions important for combustion technology could be made. These results could serve as a theoretical basis for combustion kinetics considerations. Real agricultural waste bales are gathered and compressed in many different densities and dimensions, so by varying the thickness of straw bed (*i.e.* straw layer height) and its porosity, very different pressure gradients may appear. Combustion parameters, such as theoretical air amount and air pressure, could be largely influenced by these

variations. With models like the one presented in this paper, these conditions should be determined with ease.

It can be noticed that higher points have smaller pressure drops (or smaller pressure gradients). This fact could be explained by the appearance of developed porous media flow, *i.e.* the main stream is split into many smaller streamlines that withstand friction and drag forces of less intensity.

In order to get a wider set of experimental data, an essentially different experimental set-up should be used. This should be the subject of future works.

The internal bale humidity is neglected. However, the humidity of the straw was very low, so this parameter should not have much influence on the results. The reason for neglecting bale humidity is the assumption that the straw fed in the combustion facility was already exposed to some kind of drying process, which is common in such facilities.

Nomenclature

A – surface area, [m²]
 d – diameter of the porous bed, [m]
 K_1 – viscous permeability, [m²]
 K_2 – inertial permeability, [m]
 p – pressure, [Pa]
 V_p – volume of porous space, [m³]
 V_s – volume of solid fractions, [m³]
 V_t – total volume of bed, [m³]
 w – superficial velocity, [ms⁻¹]

x – coordinate, [m]

Greek symbols

ε – porosity, [-]
 μ – dynamic viscosity, [Pa·s]
 ρ – density, [kgm⁻³]

Subscripts

i, j – direction

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