

ADAPTIVE CONTINUOUSLY VARIABLE TRANSMISSION USED FOR MAINTAINING STATIONARY REGIME OF DRIVING MACHINE

by

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Continuously variable transmissions (CVT) contained in many complex systems of modern techniques can be designed as purely mechanical systems that due to their structure make self adjustments, meaning they can change their own characteristics in order to adapt to external parameters variations. In this way adaptive CVT can be used for automatic regulation of a system. A general approach to dynamical description of adaptive CVT behaviour as a non-holonomic, completely mechanical system has been developed. For dynamic description of mechanical non-holonomic system, Appell's differential equations are used. By numerically solving differential equations of motion, answers about the working stability as well as dynamic and kinematic behaviour of the observed CVT system are obtained thus proving CVT functional applicability as a regulator for a changeable working regime. Presented approach can be used in choosing optimal parameters for the synthesis of this type of transmissions.

Key words: *continuously variable transmissions, non-holonomic mechanical system, adaptive mechanical system*

Introduction

The general trend in modern mechanical design is aimed at creating automatic machines which function with as little involvement from human operators as possible. Due to their adaptability the parts of the machine that automatically control/regulate the process can provide optimal kinematic and dynamical working regime under any exploitation parameters to the working elements, and even more importantly from an energy efficiency stand point, to the motor as well. Thus, they become a very important part of machine, since they not only make sure that the given task is executed but can also improve the machines work characteristics. The working components in systems for automatic control/regulation are usually hydraulic or pneumatic in nature, while the sensors and control structures are electronic. The presence of a system with different medium (hydraulics/pneumatics) significantly complicates the structure and functioning of the machine while the use of electronics can be problematic because of the need for specialized software. Its alternative is presented here as the use of completely mechanical, adaptive systems that due to their structure do self adjustments, meaning they can change their own characteristics in accordance with external parameters. For exam-

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ple, they can change their geometry, which changes their kinematic and dynamical behaviour, and therefore the final performance of the machine.

Continuously variable transmissions

Continuously variable transmissions (CVT) are mechanical systems that can change their transmission ratio according to either a given program or signals received from the machine during work, for example the speed or force. They can be sorted by structure [1]:

- (1) CVT that transmit the load through friction
 - frictional CVT (half toroidal, full toroidal, *etc.*),
 - belt CVT (cone type, variable pulley type, *etc.*), and
- (2) CVT that transmits the load through teeth:
 - gear CVT,
 - chain type CVT.

They are traditionally used in all transmissions with variable working regimes: winding and unwinding machinery in the paper, textile, cable and metal industry, dozers, loaders, machinery in the processing industry, machinery for centrifugal moulding, *etc.*

Lately their use has skyrocketed in the automobile industry because it has become apparent that CVT systems not only greatly increase automobile performance and engine life expectancy but also reduce greenhouse gas emissions [2, 3]. All types of CVT can be found in contemporary automotive industry. Nearly all snowmobiles, utility vehicles, golf carts and motor scooters use CVT, typically the rubber belt/variable pulley variety Torotrak Ltd. implemented the first full toroidal friction system to be manufactured for outdoor power equipment [4]. In 1987 the Ford Fiesta and Fiat Uno became the first European cars to be equipped with steel-belted CVT [5]. Fiat in 2000 introduced a cone-type CVT as an option on its models Punto and Lancia. After researching pulley type CVT for some time, Honda also introduced their own version in the 1995 Honda Civic [6]. In the late 1990's, Nissan was the first car manufacturer to introduce a frictional CVT to the market in recent years. Their toroidal CVT, named the Extroid [7], was available on the Japanese market Nissan Gloria and Skyline. In 2006, Nissan announced that they will use CVT for all automatic versions of their vehicles in North America, thus establishing CVT as a mainstream transmission system. Audi has, since 2000, offered a chain type CVT (Multitronic) [8] as an option on A5 and A6 models. Most of the mentioned CVT systems use electronics as control component, and because of the magnitude of the forces involved, hydraulics for the working components [9, 10]. A large number of agricultural machines also have CVT [11] – the CVT allows forward motion speed of the combine to be adjusted independently of the engine speed thus enabling the operator to slow or accelerate as needed to accommodate variations in thickness of the crop.

The CVT are also a very important component in power plants – especially in wind generators [12]. A major problem regarding wind turbines is that wind intensity and force are time variables. To obtain an efficient connection between the wind turbine and the distributive network, variations in frequency and induced power must be within very narrow boundaries which imply implementation of CVT in the wind generator system. From [13, 14], it is plain to see that the use of CVT greatly increases the efficiency of wind turbines, but, still, automatic control is done by electronics.

The adaptive CVT should have a more complex structure to be able to do the actual self adjustments than typical basic CVT: there should be an additional mechanism with an element that will perform an additional motion – the control motion which will change the kinematic structure of the basic CVT as needed, and in addition, the main motion as well. Obviously, this introduces another degree of freedom into the system. Also, the additional mechanism has to have structural characteristics needed to appropriately and reliably correct the main motion. Since both the basic CVT and additional mechanism are mechanical systems, the behaviour of the whole system can be described by the laws of mechanics. In [15, 16] the development of dynamic model and friction stability analysis of CVT system which uses mechanical system to move the working components – screw pair, has been shown. An electronic system was used for automatic control. The parameter required to maintain is the prescribed value of the output angular velocity. The Rout-Hurwitz and Vyshnegradsky problem methods are used to determine the system parameters needed for the motion to be stable. A geared CVT, shown in [17, 18], is designed as a planetary-differential transmission with two degrees of freedom and one input, where the control degree of freedom (the motion) turns on according to the variation of the outside load. The paper [19] shows a completely mechanical system that maintains the output angular velocity during load and work condition variations, the additional mechanism being a lever mechanism with a Watt governor. During external load variations, the system steadily and in proper time maintains the output angular velocity.

A very significant type of regulation, especially in the fields like transportation and energy production, is maintaining the stationary mode of a working machine (motor) during load variation. That mode is most economic because the motor power is fully utilized. For this case, a self-adjusting CVT will be considered. A solution for the adaptive CVT (basic CVT + additional mechanism) similar to the one from [19] will be proposed; the model will be formed, behaviour simulation executed and stability of the system examined.

Method for the dynamic analysis of non-holonomic systems

According to [1], the basic CVT represents a non-holonomic mechanical system. The transmission ratio of CVT, *i.e.* the equation connecting the input and output angular velocity represents a non-holonomic connection. Due to the presence of non-holonomic constraints, it is impossible to use the second form of Lagrange's equation, so some other kinds of approach from analytical mechanics have to be used, most notably the Lagrange multiplier method and Appel's equations [20]. The equations of motion are also attainable through the direct use of general dynamical equations, as well as through the application of kinetostatic methods. The latter two methods call for decomposition of the system and the analysis of every single element, as well as taking into consideration the internal reactions of the system, making them less than ideal in this case. For complex systems, the Lagrange multiplier method is computationally demanding, so the procedure used for formation of CVT model will be using Appel's equations. The convenience of using Appel's equations is that already existing typical equations for the acceleration of a moving body can be used.

A general dynamical system can be described with n generalized coordinates: q_1, q_2, \dots, q_n . Let there be k non-holonomic connections in the system:

$$\sum_{j=1}^n A_{pj} \dot{q}_j + A_p = 0, \quad p = 1, 2, \dots, k \quad (1)$$

A_{pj} and A_p are coefficients in function of q_i and t only.

Appell's equations are given in the following form:

$$\frac{\partial S^*}{\partial \ddot{q}_v} = Q_v^* \quad (v = 1, 2, \dots, p), \quad p = n - k \quad (2)$$

Energy of acceleration of a system S^* is sum of energies of accelerations of its components S_i^* .

For bodies performing planar motion, the energy of acceleration can be calculated:

$$S_i^* = \frac{1}{2} (m_i a_{Ci}^2 + J_C \ddot{\phi}_i^2) \quad (3)$$

For bodies performing motion in space, the energy of acceleration can be calculated:

$$S_i^* = S_{i \text{ translation}}^* + S_{i \text{ rotation}}^* = \frac{1}{2} m_i a_{Ci}^2 + \frac{1}{2} (J_\xi \ddot{\phi}_\xi^2 + J_\eta \ddot{\phi}_\eta^2 + J_\zeta \ddot{\phi}_\zeta^2) + \\ + (J_\eta - J_\xi) \dot{\phi}_\xi \dot{\phi}_\eta \ddot{\phi}_\zeta + (J_\zeta - J_\eta) \dot{\phi}_\eta \dot{\phi}_\zeta \ddot{\phi}_\xi + (J_\xi - J_\zeta) \dot{\phi}_\zeta \dot{\phi}_\xi \ddot{\phi}_\eta \quad (4)$$

Generalized forces Q_v^* are determined using the principle of virtual work.

Dynamical behaviour of the adaptive CVT

The frictional adaptive CVT with half balls is shown in fig. 1. Power is transferred from the driving machine in this case an electro motor (*EM*) to CVT input link 1, then, through disc 3 to the output link 2 and, finally, to the working machine. Gearboxes GB1 and GB2 with constant transmission ratios can be integrated into the whole transmission system if needed. Variation of the output angular velocity happens due to a change in position of contact points A and B of disk 3 and links 1 and 2, respectively. Disk 3 is mounted on link 4 in such a way that it can rotate both around its axis and also (along with link 4) around the axis through O_4 . Thus, the distance between the contact points on input and output links and their respective axes of rotation have been changed giving the variable transmission ratio between links 1 and 2.

Spring controlled centrifugal governor of Wilson-Hartnell type (fig. 2) – GOV is integrated into the transmission system in order to ensure closed automatic regulation. Balls 1 with mass m_i are connected by two springs 2, with the spring constant c_1 , called the main springs. Levers 3 have two arms – vertical, with length ℓ_1 and horizontal with, length ℓ_2 . The horizontal arms have rollers 4 at their end which are placed in a groove on slider 5, with mass m_k and a moment of inertia J_k . The adjustable auxiliary spring 6 with a spring constant c_2 is connected to the slider by lever 7. Governor elements rotate around the governor axis while slider can perform an additional translational motion along the axis. Centrifugal force F_C and slider displacement x are defined:

$$F_{C0} - F_C(t) = \left(4c_1 + \frac{\ell_2}{\ell_1} \frac{\ell_4^2}{\ell_3^2} c_2 \right) (r_0 - r(t)) = \left(4c_1 + \frac{\ell_2}{\ell_1} \frac{\ell_4^2}{\ell_3^2} c_2 \right) \frac{\ell_1}{\ell_2} x \quad (5)$$

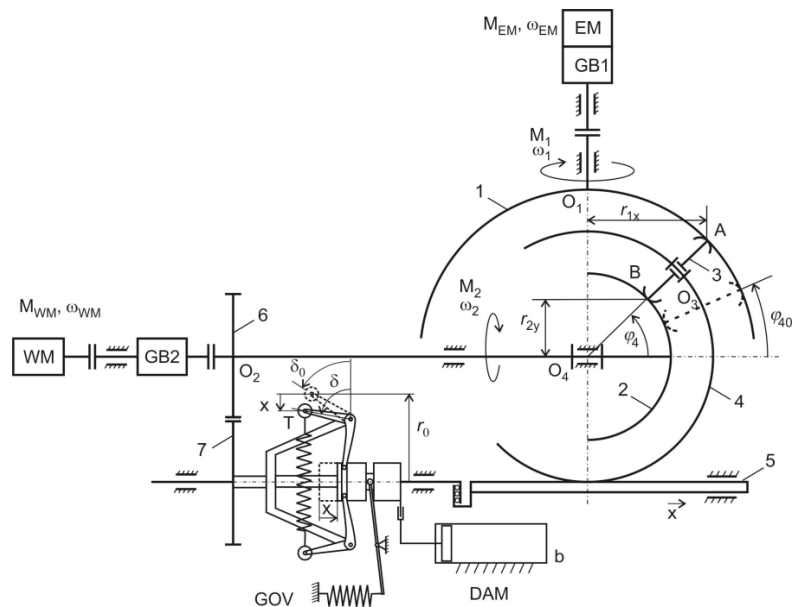


Figure 1. Continuously variable transmission with a frictional adaptive CVT as a closed mechanical regulating system

For a stationary working regime $\omega_2 = \text{const}$ i. e. $r(t) = \text{const}$ there is no change in slider position ($x = \text{const}$, $\dot{x} = 0$) so link 5 – the lathe and link 4 do not move. Rotation is transferred from the output shaft to the governor shaft via gears 6 and 7. If ω_2 changes, r and x will change as well, resulting, at the end, in the change of position of disc 3 and consequently in the change of transmission ratio of the basic CVT. Damper DAM with damping coefficient b is connected to the slider in order to stabilize motion.

Dynamical analysis will be done by using Appell's equations for non-holonomic systems.

Three generalized coordinates which describe the system are: rotation of the input shaft φ_1 , rotation of the output φ_2 and displacement of lathe x which indirectly defines the position of element 3. Rotation of output shaft 2 and displacement x are adopted as independent generalized accelerations, so Appell's equations for this system are given:

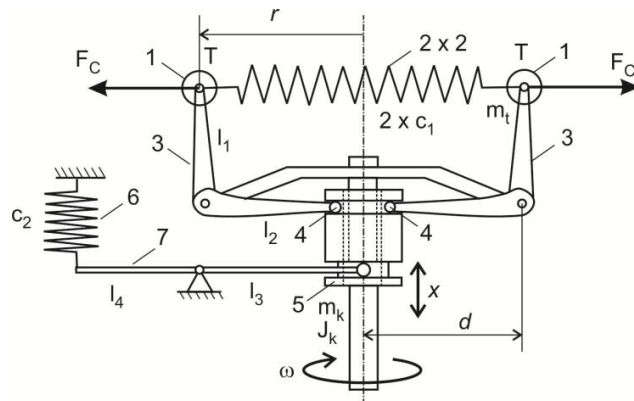


Figure 2. Spring controlled centrifugal governor

$$\frac{\partial S^*}{\partial \ddot{\varphi}_2} = Q^*_{\varphi_2}, \quad \frac{\partial S^*}{\partial \ddot{x}} = Q^*_x \quad (6)$$

The non-holonomic connection between generalized co-ordinates φ_1 and φ_2 is defined by the following differential equation:

$$\dot{\varphi}_1 - i_v \dot{\varphi}_2 = \dot{\varphi}_1 - \left(\frac{R_2}{R_1} \tan \varphi_4 \right) \dot{\varphi}_2 = 0 \quad (7)$$

The energy of acceleration for this system is:

$$S^* = \frac{1}{2} (J_1 + J_{EM} + J_{GB1}) \ddot{\varphi}_1^2 + \frac{1}{2} (J_2 + J_{GB2} + J_{WM} + J_6 + J_7 + J_k) \ddot{\varphi}_2^2 + \\ + \frac{1}{2} m_3 (R_2 + R_3)^2 \ddot{\varphi}_4^2 + \frac{1}{2} (J_{3\xi} \ddot{\varphi}_3^2 + J_{3\zeta} \ddot{\varphi}_4^2) + \frac{1}{2} J_4 \ddot{\varphi}_4^2 + \frac{1}{2} m_k \ddot{x}^2 + 2S_T^* \quad (8)$$

The connection between $\ddot{\varphi}_1$ and $\ddot{\varphi}_2$ is obtained by differentiating non-holonomic connection (7).

The equation connecting the tangential velocities of elements 2 and 3 in point B is as following:

$$R_3 \dot{\varphi}_3 = \dot{\varphi}_2 r_{2,y} \Rightarrow \dot{\varphi}_3 = \dot{\varphi}_2 \frac{R_2 \sin \varphi_4}{R_3} \quad (9)$$

Angular acceleration $\ddot{\varphi}_3$ is obtained by differentiation of eq. (9).

To express φ_4 through generalized co-ordinate x , the following geometric connection is used:

$$\varphi_4 - \varphi_{40} = \frac{x}{R_4} \quad (10)$$

Angular velocity $\dot{\varphi}_4$ and angular acceleration $\ddot{\varphi}_4$ are obtained by differentiation of eq. (10).

Energy of acceleration S_T of regulator balls is:

$$S_T^* = \frac{1}{2} a_T^2 m_T \quad (11)$$

Acceleration of point T – the centre of governor ball is:

$$\vec{a}_T = \vec{a}_{pn} + \vec{a}_{pt} + \vec{a}_{tm} + \vec{a}_{rt} + \vec{a}_k \quad (12)$$

Magnitudes of normal \vec{a}_{pn} and tangential \vec{a}_{pt} components of the transmission acceleration of point T are $a_{pn} = (r_0 - x) \ddot{\varphi}_2$ and $a_{pt} = (r_0 - x) \ddot{\varphi}_2$, respectively. Magnitudes of the normal \vec{a}_{rn} and tangential \vec{a}_{rt} components of relative acceleration of point T are $a_{rn} = \ell_1 \delta^2$ and $a_{rt} = \ell_1 \delta$, respectively. The magnitude of Coriolis acceleration \vec{a}_k is $a_k = 2 \ell_1 \dot{\delta} \ddot{\varphi}_2$.

Acceleration for point T is:

$$a_T^2 = (a_{pn} \cos \delta + a_{tm})^2 + (a_{rt} + a_{pn} \sin \delta)^2 + (a_{pt} + a_k)^2 \quad (13)$$

Angle δ can be expressed through generalized co-ordinate x :

$$\cos \delta = \frac{r_0 - d - x}{\ell_1} \quad (14)$$

First and second derivate of eq. (14) are:

$$\dot{\delta} = \frac{\dot{x}}{\ell_1 \sin \delta} = \frac{\dot{x}}{\left[\ell_1^2 - (r_0 - d - x)^2 \right]^{1/2}} \quad (15)$$

$$\ddot{\delta} = \frac{\ddot{x}}{\left[\ell_1^2 - (r_0 - d - x)^2 \right]^{1/2}} - \frac{\dot{x}^2 (r_0 - d - x)}{\left[\ell_1^2 - (r_0 - d - x)^2 \right]^{3/2}} \quad (16)$$

With eq. (11) through eq. (16) energy S_T^* is defined.

Generalized forces are defined using the principle of virtual work:

$$\delta A = Q_{\varphi_2}^* \delta \varphi_2 + Q_x^* \delta x = M_1 \frac{R_2}{R_1} \tan \varphi_4 \delta \varphi_2 - M_2 \delta \varphi_2 - 2 \frac{\partial \Pi_1}{\partial x} \delta x - \frac{\partial \Pi_2}{\partial x} \delta x - b \dot{x} \delta x \quad (17)$$

The torque on shaft 1 depends on the torque of the driving machine and equals to:

$$M_1 = i_{GB1} M_{EM} \quad (18)$$

In the case of asynchronous motor, for nominal working regime, the torque on the shaft and its angular velocity are connected via relationship $M_{EM} = A - B \omega_{EM}$.

Coefficients A and B depend on the power and type of motor.

The torque on the shaft of element 2 depends on the load on working machine and equals:

$$M_2 = i_{GB2} M_{WM}(t) \quad (19)$$

The load is generally a variable and depends on the particular type of the working machine. For example, the M_{WM} of a fan is a constant, while in machines for winding and unwinding, it equals $M_{WM} = C \pm D t$, etc.

The generalized force for φ_2 is:

$$Q_{\varphi_2}^* = i_{GB1} (A - B \omega_{EM}) - i_{GB2} M_{WM}(t) \quad (20)$$

The current deformation of spring 1 is $L_1 + 2\Delta L_1 - 2x\ell_1/\ell_2$, so the potential energy can be written in the following form:

$$\Pi_1 = \frac{1}{2} c_1 \left[L_1 + 2\Delta L_1 - 2 \frac{\ell_1}{\ell_2} x \right]^2 \quad (21)$$

The current deformation of spring 2 is $L_2 + 2\Delta L_2 - x\ell_3/\ell_4$, so potential energy can be written in the following form:

$$\Pi_2 = \frac{1}{2} c_2 \left[L_2 + \Delta L_2 - \frac{\ell_3}{\ell_4} x \right]^2 \quad (22)$$

The generalized force for coordinate x is:

$$Q_x^* = 2c_1 \left[L_1 + 2\Delta L_1 - \frac{\ell_1}{\ell_2} 2x \right] + c_2 \left[L_2 + \Delta L_2 - \frac{\ell_3}{\ell_4} x \right] - b\dot{x} \quad (23)$$

Finally, differential equations that describe the dynamical behaviour of the adaptive CVT are:

$$\begin{aligned} & \left[2(J_1 + J_{EM} + J_{GB1}) \frac{R_2}{R_1} \tan^2 \left(\frac{x}{R_4} + \varphi_{40} \right) + 2m_T (r_0 - x)^2 + (J_2 + J_{GB2} + J_{WM} + J_6 + J_7 + J_k) + \right. \\ & \left. + J_{3\xi} \frac{R_2}{R_3} \sin \left(\frac{x}{R_4} + \varphi_{40} \right) \right] \ddot{\varphi}_2 + \left\{ 2(J_1 + J_{EM} + J_{GB1}) \frac{R_2}{R_1} \tan \left(\frac{x}{R_4} + \varphi_{40} \right) \left[\cos^2 \left(\frac{x}{R_4} + \varphi_{40} \right) \right]^{-1} \frac{\dot{x}}{R_4} + \right. \\ & \left. + 4m_T (r_0 - x) \dot{x} \left[1 - \left(\frac{r_0 - d - x}{\ell_1} \right)^2 \right]^{-1/2} + Bi_{GB1} \frac{R_2}{R_1} \tan \left(\frac{x}{R_4} + \varphi_{40} \right) \right\} \dot{\varphi}_2 - \\ & - Ai_{GB1} i_{GB2} \frac{R_2}{R_1} \tan \left(\frac{x}{R_4} + \varphi_{40} \right) + C + Dt = 0 \end{aligned} \quad (24)$$

and:

$$\begin{aligned} & \left\{ m_3 \frac{(R_2 + R_3)^2}{R_4^2} + J_{3\xi} \frac{1}{R_4^2} + J_4 \frac{1}{R_4^2} + m_K + 2m_T \left[1 - \left(\frac{r_0 - d - x}{\ell_1} \right)^2 \right] \right\} \ddot{x} - \\ & - \frac{2m_T}{\ell_1} \left[1 - \left(\frac{r_0 - d - x}{\ell_1} \right)^2 \right]^{-1} \left(\frac{r_0 - d - x}{\ell_1} \right) \dot{x}^2 + \\ & + 2m_T (r_0 - x) \dot{\varphi}_2^2 + b\dot{x} - 2c_1 \left(L_1 + 2\Delta L_1 - \frac{\ell_1}{\ell_2} 2x \right) + c_2 \left(L_2 + \Delta L_2 - \frac{\ell_3}{\ell_4} x \right) = 0 \end{aligned} \quad (25)$$

Results

Using eqs. (24) and (25), behaviour simulation of adaptive CVT was ran in MATLAB, for a number of different load variations. The results are shown in figs. 3 and 4 in the form of time-histories of relevant variables: input and output angular velocities and control motion (motion of lathe). For the system depicted on fig.1, the parameter values are:

$J_1 = 2.6 \text{ kgm}^2$; $J_{EM} + J_{GB1} = 0.2 \text{ kgm}^2$; $J_2 = 0.5 \text{ kgm}^2$; $J_{3\xi} = 0.1 \text{ kgm}^2$; $J_{3\xi} = 0.05 \text{ kgm}^2$;

$J_4 = 0.65 \text{ kgm}^2$; $J_{GB2} = 0.1 \text{ kgm}^2$; $J_{WM} = 0.5 \text{ kgm}^2$; $J_6 = J_7 = 0.04 \text{ kgm}^2$; $J_k = 0.01 \text{ kgm}^2$;

$R_1 = 0.3 \text{ m}$; $R_2 = 0.15 \text{ m}$; $R_3 = 0.075 \text{ m}$; $R_4 = 0.225 \text{ m}$;

$m_3 = 2.8 \text{ kg}$; $m_T = 0.2 \text{ kg}$; $m_K = 0.2 \text{ kg}$; $\ell_1 = \ell_2 = \ell_3 = \ell_4 = 0.15 \text{ m}$; $L_1 + 2\Delta L_1 = 0.205 \text{ m}$;

$L_2 + \Delta L_2 = 0.105 \text{ m}$; $c_1 = 681 \text{ Nm}^{-1}$; $c_2 = 1363 \text{ Nm}^{-1}$; $i_{GB1} = 7$; $i_{GB2} = 1$; $b = 5000 \text{ [Nsm}^{-1}]$;

$M_{EM} = A - B \omega_{EM}$; $A = 1500 \text{ [Nm]}$; $B = 188.5 \text{ [Nms}^{-1}]$;

$t_0 = 0 \text{ s}$; $\alpha_0 = 26.5^\circ$; $\omega_{20} = 41.89 \text{ [s}^{-1}]$; $x_0 = 0 \text{ m}$;

Case 1. $M_{WM} = C + Dt$, $C = 175 \text{ Nm}$; $D = 14.6 \text{ Nms}^{-1}$;

$$\text{Case 2. } M_{\text{WM}} = \begin{cases} C, & t < 4\text{s} \\ C + D(t-4)/4, & 4\text{s} < t < 8\text{s} \\ D, & t > 8\text{s} \end{cases}, \quad C = 175 \text{ Nm}; \quad D = 14.6 \text{ Nms}^{-1};$$

The diagrams show that the proposed adaptive mechanism system fulfils the requested function – it maintains the output angular velocity of the driving machine. At the beginning of motion, as the load changes so does ω_1 . In case 1, since variation of M_2 is continuous, the change in ω_1 is negligible, while in case 2, since M_2 variation is uncontinuous, the change in ω_1 is more significant, but after a period of 10 seconds, a stationary working regime has been established.

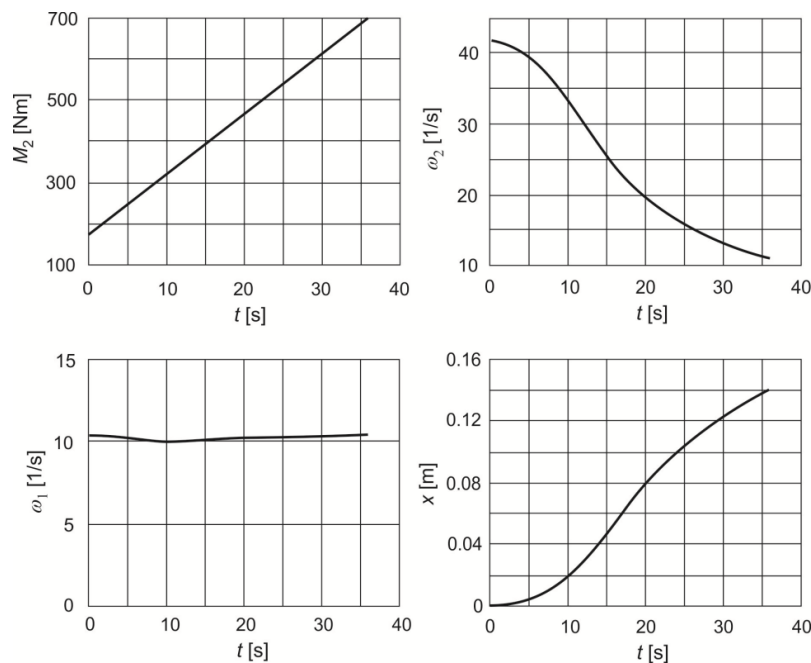


Figure 3. Time histories of output moment M_2 , output angular velocity ω_2 , input angular velocity ω_1 and displacement of the lathe x , case 1

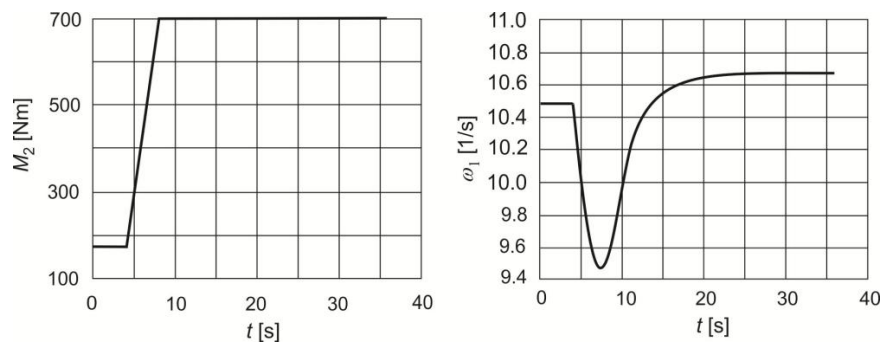


Figure 4. Time histories of output moment M_2 and input angular velocity ω_1 , case 2

Using the developed mechanism model, it is possible to analyze the influence of design parameters on CVT behaviour. In fig. 5. time histories of input angular velocity for different values of the governor balls mass and case 2 of working machine torque change are presented. Ratio c_1/c_2 (stiffness of main governor spring/stiffness of auxiliary governor spring) and geometrical parameters of governor are kept constant. As balls mass increases, magnitude of oscillation of ω_1 decreases and stationary regime is reached in less time. In fig. 6. time histories of input angular velocity for different values of the ratio c_1/c_2 and case 2. of working machine torque change are presented. Ball masses and geometrical parameters of the governor are kept constant. It can be seen that the change of c_1/c_2 causes small (practically negligible) changes of ω_1 . This is very significant information since it provides the designer more options in the process of calculating and adopting the springs.

Similar analysis can be performed for all design parameters of CVT and are going to be the topic of future research along with optimization of the system behaviour.

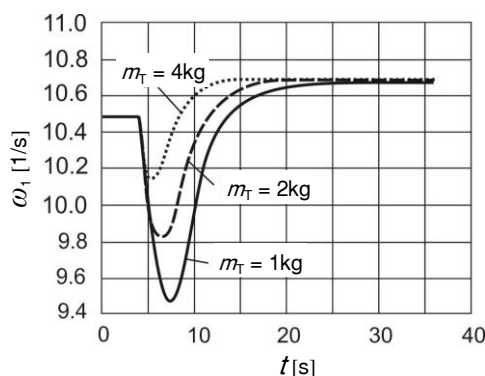


Figure 5. Time histories of input angular velocity ω_1 for different values of the governor balls mass, case 2, $c_1/c_2 = 1/2$

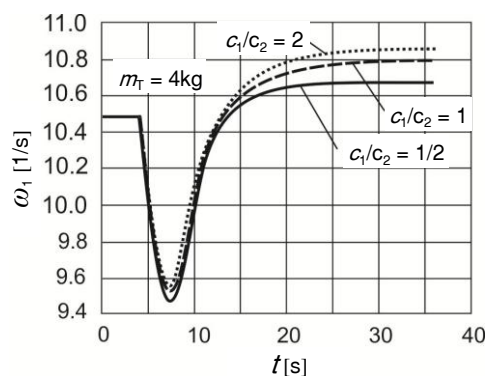


Figure 6. Time histories of input angular velocity ω_1 for different values of the ratio c_1/c_2 , $m_T = 1$ kg

Conclusions

Development of a purely mechanical CVT system that can self adjust to variations of the external load in order to maintain stationary regime of driving machine is presented in this paper.

It can be seen from literature that control of CVT in contemporary machinery is done by electronical and hydraulic systems. Presence of systems of different nature can significantly complicate structure and functioning of machine itself. Structurally simpler and more economic design is proposed – adaptive CVT which is completely mechanical and can adapt to variations of working regime by changing functional parameters due to its mechanical properties only. In this way adaptive CVT can be used for automatic regulation of a whole transmission system.

A general approach to the dynamical description of adaptive CVT behaviour is developed. Since CVT is a non-holonomic mechanical system, Appell's equations are used for dynamic modelling. By numerically solving differential equations of motion, dynamical and kinematic behaviour of CVT system was obtained. It immediately reacts to the external load perturbations and in reasonable time frame stabilizes the output angular velocity of the driving machine thus proving its functional applicability as regulator for changeable working regime.

In further work, an extensive study of the CVT design parameters influence on its characteristics and functioning has been intended. Based on the information that will be obtained and dynamical model developed in this paper, an optimization of CVT design will be performed as well.

Nomenclature

a_{C_i}	– acceleration of the centre of mass C_i of body i	J_{EM}	– moment of inertia of electro motor (reduced to shaft of input element 1)
c_1, c_2	– spring coefficient of main and auxiliary spring, respectively	J_{GB1}	– moment of inertia of gear box 1 (reduced to shaft of input element 1)
$\delta, \dot{\delta}, \ddot{\delta}$	– position angle, angular velocity and angular acceleration of governor bar	J_{GB2}	– moment of inertia of gear box 2 (reduced to shaft of input element 2)
$\Delta L_1, \Delta L_2$	– preload of main and auxiliary spring, respectively	J_k	– moment of inertia of governor's slider (reduced to shaft of input element 2)
$\dot{\phi}_i, \ddot{\phi}_i$	– angular and acceleration velocity of body i	J_{WM}	– moment of inertia of working machine (reduced to shaft of output element 2)
$\dot{\phi}_\xi, \ddot{\phi}_\xi, \dot{\phi}_\eta, \ddot{\phi}_\eta, \dot{\phi}_\zeta, \ddot{\phi}_\zeta, \dot{\phi}_\eta, \ddot{\phi}_\eta, \dot{\phi}_\zeta, \ddot{\phi}_\zeta$	– angular accelerations, angular velocities and angles about axes of coordinate system rigidly connected to the body, its origin coincides with C_i	L_1, L_2	– lengths of main and auxiliary spring, respectively
$\phi_1, \phi_2, \phi_3, \phi_4, \dot{\phi}_1, \dot{\phi}_2, \dot{\phi}_3, \dot{\phi}_4, \ddot{\phi}_1, \ddot{\phi}_2, \ddot{\phi}_3, \ddot{\phi}_4$	– angular position, angular velocity and angular accelerations of elements 1, 2, 3 and 4, respectively	m_i	– mass of body i
i_{GB1}, i_{GB2}	– transmission ratio of gearboxes 1 and 2, respectively	M_1, M_2	– moments on of input and output elements 1 and 2, respectively
J_1	– moment of inertia of input element 1 about its axis of rotation	Π_1, Π_2	– potential energy of the springs
J_2	– moment of inertia of output element 2 about its axis of rotation	q, \dot{q}, \ddot{q}	– generalized coordinate, velocity and acceleration
$J_{3\xi}$	– moment of inertia of element 3 about axis ξ (normal to disc and passing through O_3)	Q_v^*	– generalized force
$J_{3\zeta}$	– moment of inertia of element 3 about axis ζ (lying in the disc plane and passing through O_3)	R_1, R_2	– radii of input and output half ball elements 1 and 2, respectively
J_4	– moment of inertia of element 4 about its axis of rotation	R_4	– radius of element 4
J_6, J_7	– moments of inertia of elements 6 and 7, respectively (reduced to shaft of input element 2)	R_3	– radius of element 3
J_C	– moment of inertia for axis through C_i	S^*	– energy of acceleration of complete system
		S_{i}^{*}	– energy of acceleration of body i
		$S_{i \text{ translation}}^*$	– energy of acceleration of the center of the mass C_i of the body i
		$S_{i \text{ rotation}}^*$	– energy of the acceleration of the body i in spherical motion about point C_i
		t	– time
		x, \dot{x}, \ddot{x}	– speed, velocity and acceleration of the governor slider

References

- [1] Pronin, B. A., Revkov, G.A., *Besstupenchatie Klinoremennie i Frikcionnie Peredachi (Variatori) (Stepless V-belt and Frictional Continuously Variable Transmissions (Variators), in Russian)*, Mashinostroyeniye, Moscow, USSR, 1967
- [2] Brace, C., *et al.*, The Compromise in Reducing Exhaust Emissions and Fuel Consumption from a Diesel CVT Power Train over Typical Usage Cycles, *Proceedings, CVT'99*, Eindhoven, The Netherlands, 1999, pp. 27-33
- [3] Carbone, G., *et al.*, Fuel Consumption of a Mid Class Vehicle with Infinitely Variable Transmission, *SAE Transactions 2001 Journal of Engines*, 110 (2002), 3, pp. 2472-2483
- [4] Patterson, M., The Full Toroidal Variator in Theory and Practice, Torotrak Development Ltd., *Proceedings, CVT'96*, Yokohama, Japan, 1996, pp. 95-100
- [5] Poulton, M. L., *Fuel Efficient Car Technology*, WIT Press, Computational Mechanics Publications, United Kingdom, 1997

- [6] ***, <http://world.honda.com/automobile-technology/CVT/>
- [7] ***, http://www.nissan-global.com/PDF/tcvt_e.pdf
- [8] ***, <http://www.audi.co.uk/audi-innovation/our-technologies/transmissions.html>
- [9] Srivastava, N., Hakue, I., A Review on Belt and Chain Continuously Variable Transmissions: Dynamics and Control, *Mechanism and Machine Theory*, 44 (2009), 1, pp. 19-241
- [10] Fuchs, R., *et al.*, The Making of the Full Toroidal Variator, *JTEKT Engineering Journal*, 106E (2009), pp. 31-36
- [11] Renius, K., Resch, R., Continuously Variable Tractor Transmissions, *Proceedings*, Agricultural Equipment Technology Conference, Louisville, Ken., USA, 2005, pp. 1-37
- [12] Miltenović, V., *et al.*, Differential Planetary Transmission of Wind Turbine Continuously Variable Transmission, *Machine Design*, 2 (2010), May, pp. 123-128
- [13] Mangialardi, G., Automatically Regulated CVT in Wind Power Systems, *Renewable Energy*, 4 (1994), 3, pp. 299-310
- [14] Verdonschot, M. J., Modeling and Control of Wind Turbines Using CVT, M. Sc. thesis, University of Technology, Eindhoven, The Netherlands, 2009
- [15] Sergeev, V. I., The Dynamics of Frontal Frictional Mechanism (Speed Variator), *Journal of Machinery Manufacture and Reliability*, 43 (2014), 6, pp. 478-481
- [16] Sergeev, V. I., Kinematics of Mechanisms with Rolling Pairs, *Journal of Machinery Manufacture and Reliability*, 40 (2011), 4, pp. 313-318
- [17] Ivanov, K., Tultaev, B., *Toothed Continuously Variable Transmission – Industrial Realization*, New Trends in Mechanisms and Machine Science, Springer, Germany, 2013, pp. 329-335
- [18] Ivanov, K., Theory of Gear Adaptive Transmission, *American Journal of Mechanics and Application*, 2 (2014), 1, pp. 5-12
- [19] Zlokolica, M., *et al.*, General Approach to the Dynamical Behavior of Power Transmission as Non-holonomic System, *Proceedings*, International Conference on Motion and Power Transmissions MPT2009, Sendai, Japan, 2009, pp. 371-374
- [20] Dobronarov, V. V., *Osnovi Mehaniki Negolonomnih Sistem (Mechanics of Non-holonomic Systems, in Russian)*, Vissha Shkola, Moscow, USSR, 1970