

EFFECT OF THERMAL RADIATION ON NATURAL CONVECTION IN A SQUARE POROUS CAVITY FILLED WITH A FLUID OF TEMPERATURE-DEPENDENT VISCOSITY

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A numerical study of the natural convection combined with thermal radiation inside a square porous cavity filled with a fluid of temperature-dependent viscosity is carried out. The side horizontal walls are assumed to be adiabatic while both the left and right vertical walls are kept at constant but different temperatures. The Rosseland diffusion approximation is used to describe the radiative heat flux in the energy equation. The governing equations formulated in dimensionless stream function, vorticity and temperature variables are solved using finite difference method. A parametric analysis illustrating the effects of the radiation parameter ($0 \leq R_d \leq 10$), Darcy number ($10^{-5} \leq Da \leq 10^{-2}$) and viscosity variation parameter ($0 \leq C \leq 6$) on fluid flow and heat transfer is implemented. The results show an essential intensification of convective flow with an increase in the radiation parameter.

Key words: Thermal radiation, Natural convection, Temperature-dependent viscosity, Porous medium, Square cavity, Numerical method.

1. Introduction

An analysis of natural convection in porous cavities has essential fundamental and practical values because of its relevance to the thermal systems such as solar collectors, electronic devices, cooling of nuclear reactors, crystal growth and building insulations [1–4]. Radiative heat transfer inside porous enclosures is neglected in many papers. However, it is very important mechanism even for small temperature differences [5–10]. For example, Badruddin et al. [5] have numerically analyzed natural convection with thermal radiation inside a Darcy porous cavity using local thermal non-equilibrium model. It has been found that the average Nusselt number for solid increases with an increase in the radiation parameter while the average Nusselt number for fluid decreases with an increase in the radiation parameter. The effect of the radiation parameter on the total Nusselt number reduces for high values of conductivity ratio. Badruddin et al. [6] have studied numerically the combined effect of thermal radiation and viscous dissipation on natural convection in a Darcy porous vertical annular cylinder. The authors have shown that an increase in the radiation parameter leads to the heat transfer enhancement while the viscous dissipation parameter leads to both an increase in the average Nusselt number at cold surface and a decrease in the average Nusselt number at hot surface. Ahmed et al. [7]

have studied MHD natural convection in a square differentially heated porous cavity taking into account the influence of viscous dissipation and radiation. They have shown that an increase in the radiation parameter leads to an intensification of conductive heat transfer. The average Nusselt number has been found to be an increasing function of the radiation parameter. Ahmed et al. [8] on the basis of the finite volume method have investigated natural convection and thermal radiation in an inclined porous cavity with corner heater. It has been ascertained that heat transfer is increased with the radiation parameter and decreased with decreasing of the Darcy number. Mahapatra et al. [9] have analyzed the combined effects of thermal radiation and heat generation on free convection in a differentially heated square porous cavity. The authors have shown that the vertical velocity and average Nusselt number is an increasing function of the radiation parameter. Abdou [10] has studied the effect of thermal radiation on boundary layer flow with temperature-dependent viscosity and thermal conductivity past a stretching sheet inside a porous medium. It has been found that the average Nusselt number increases with the radiation and thermal conductivity parameters.

In many practical situations the real fluid inside porous enclosures has a variable viscosity. Therefore, it is necessary to analyze the effect of variable viscosity on fluid flow and heat transfer in order to control and intensify the technological processes. Thus, Makinde et al. [11] using the similarity method have numerically investigated the steady state mixed convective flow and heat transfer of a chemically reacting variable viscosity fluid past a vertical porous permeable plate inside a porous medium. It has been revealed that temperature dependent viscosity reduces skin friction and increases Nusselt number, while the dimensionless temperature increases with radiative parameter. Astanina et al. [12] have numerically examined transient natural convection with temperature-dependent viscosity in a non-Darcy porous cavity. It has been found that an increase in the viscosity variation parameter leads to an intensification of convective flow and heat transfer and a formation of a single-core convective cell for the porous cavity. More detailed literature review for an effect of the variable viscosity on natural convection in clear and porous enclosures can be found in [12].

However these above-mentioned papers did not take into account simultaneously the influence of thermal radiation and temperature-dependent viscosity on fluid flow and heat transfer inside porous cavities using the Brinkman–extended Darcy model. The main purpose of the present paper is a numerical analysis of thermal radiation effect on natural convection in a differentially heated porous cavity saturated with a fluid of temperature-dependent viscosity. It should be noted that the present work is an extension to the thermal radiation effect of previous published paper by Astanina et al. [12].

2. Basic equations

The physical model is schematically shown in Fig. 1. It is assumed that the vertical walls of the enclosure are isothermal with constant temperatures T_h and $T_c < T_h$ while horizontal walls are adiabatic. The cavity boundaries are rigid, impermeable with no-slip boundary conditions for velocity vector. Viscosity of the fluid is varied with temperature [12–14], and the flow is laminar. The fluid is assumed to be heat-conducting and Newtonian. Boussinesq approximation is accepted. It is supposed that the temperature of the fluid phase is equal to the temperature of the solid structure everywhere in the porous region, and local thermal equilibrium model is applicable in the present investigation. The properties of the fluid and those of the porous medium are homogeneous and isotropic. Radiation heat flux inside the fluid-saturated porous cavity is modeled on the basis of the Rosseland approximation

[15, 16]. It should be noted that taking into account the Rosseland approach the porous medium behaves as an optically thick gray body or we have a large effective optical depth in a medium.

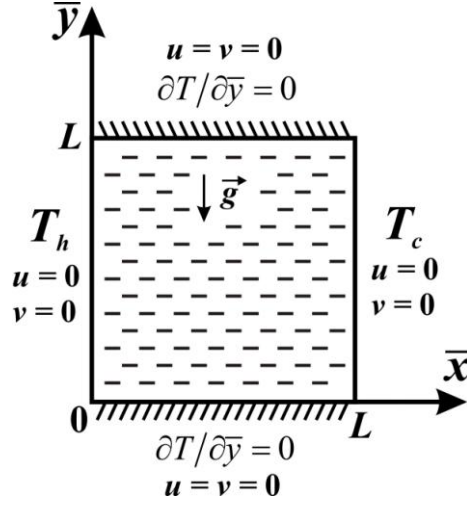


Fig. 1. Physical model

Under above-mentioned assumptions and using the Brinkman–extended Darcy model [12, 17] the governing equations for laminar two-dimensional natural convection flow in the porous cavity with the effect of thermal radiation can be written in dimensionless variables as follows:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \quad (1)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} = \text{Pr} \left(\frac{\partial^2 (\mu \omega)}{\partial x^2} + \frac{\partial^2 (\mu \omega)}{\partial y^2} - \frac{\mu \omega}{\text{Da}} \right) + \text{Ra} \cdot \text{Pr} \frac{\partial \theta}{\partial x} +$$

$$+ 2 \text{Pr} \left[\frac{1}{2 \text{Da}} \frac{\partial \psi}{\partial y} \frac{\partial \mu}{\partial y} + \frac{1}{2 \text{Da}} \frac{\partial \psi}{\partial x} \frac{\partial \mu}{\partial x} + \frac{\partial^2 \mu}{\partial x^2} \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \mu}{\partial y^2} \frac{\partial^2 \psi}{\partial x^2} - 2 \frac{\partial^2 \mu}{\partial x \partial y} \frac{\partial^2 \psi}{\partial x \partial y} \right] \quad (2)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} = \left(1 + \frac{4R_d}{3} \right) \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \quad (3)$$

Here we have the following governing parameters: Rayleigh number (Ra), Prandtl number (Pr), Darcy number (Da), temperature-dependent dimensionless dynamic viscosity (μ) and radiation parameter (R_d) which are defined as

$$\text{Ra} = \frac{\rho g \beta (T_h - T_c) L^3}{\alpha \mu_0}, \quad \text{Pr} = \frac{\mu_0}{\rho \alpha}, \quad \text{Da} = \frac{K}{L^2}, \quad \mu = \exp(-C\theta), \quad R_d = \frac{4\sigma T_c^3}{k\beta_r} \quad (4)$$

Invoking Rosseland approximation [15, 16] for radiation we have $q_{rx} = -\frac{4\sigma}{3\beta_r} \frac{\partial T^4}{\partial x}$, $q_{ry} = -\frac{4\sigma}{3\beta_r} \frac{\partial T^4}{\partial y}$. In the case of Boussinesq approach [18] it is considered that the temperature differences inside the cavity are small. Therefore, T^4 may be expressed as a linear function of the

temperature, utilizing the Taylor series for T^4 about T_c and after neglecting higher order terms we have $T^4 \approx 4TT_c^3 - 3T_c^4$. In the case of the Rosseland approximation, it is possible to assume that radiative energy transport for the medium is of the same nature as a heat conduction with an “effective heat conductivity” = $16\sigma T_c^3/3\beta_r$.

The boundary conditions for the governing equations (1)–(3) are as follows

$$\begin{aligned} \psi = 0, \omega = -\frac{\partial^2 \psi}{\partial x^2}, \theta = 1 \quad \text{at } x = 0 \text{ and } 0 \leq y \leq 1 \\ \psi = 0, \omega = -\frac{\partial^2 \psi}{\partial x^2}, \theta = 0 \quad \text{at } x = 1 \text{ and } 0 \leq y \leq 1 \\ \psi = 0, \omega = -\frac{\partial^2 \psi}{\partial y^2}, \frac{\partial \theta}{\partial y} = 0 \quad \text{at } y = 0, 1 \text{ and } 0 < x < 1 \end{aligned} \quad (5)$$

Here we introduced the following dimensionless variables

$$\begin{aligned} x = \bar{x}/L, y = \bar{y}/L, \theta = (T - T_0)/(T_h - T_c), u = \bar{u}L/\alpha, \\ v = \bar{v}L/\alpha, \psi = \bar{\psi}/\alpha, \omega = \bar{\omega}L^2/\alpha, \mu = \bar{\mu}/\mu_0 \end{aligned} \quad (6)$$

The physical quantity of interest is the average Nusselt number defined as:

$$\text{Nu}_{\text{avg}} = \int_0^1 \left| \frac{\partial \theta}{\partial x} \right|_{x=0} dy \quad (7)$$

3. Numerical method

The partial differential equations (1)–(3) with corresponding boundary conditions (5) were solved by finite difference method using the uniform grid. The used numerical technique has been described in detail in [12, 15, 17, 19–21].

The accuracy of the numerical code developed by the authors was checked by preparing the benchmark solutions for natural convection with temperature-dependent viscosity in a square differentially heated cavity filled with a pure fluid [13]. This benchmark solution was obtained when $\text{Da} = \infty$ and $R_d = 0.0$ in the partial differential equations (1)–(3). Table 1 shows a good agreement between the obtained maximum values of the stream function for different values of the Rayleigh number and viscosity variation parameter and the results by [13].

Tab. 1. Comparison of maximum values of the stream function (ψ_{max}) between present results and data of Hyun and Lee [13].

Ra	C	Data [13]	Obtained results
$3.5 \cdot 10^4$	0	1.155	1.154
	1	1.165	1.165
	3	1.240	1.219

$3.5 \cdot 10^5$	0	2.135	2.186
	1	2.156	2.198
	3	2.256	2.243

The grid independent solution was performed by preparing the solution for steady-state free convection in a square porous cavity filled with a fluid of temperature-dependent viscosity at $Ra = 10^6$, $Pr = 7.0$, $Da = 10^{-3}$, $R_d = 5.0$, $C = 3.0$. Four cases of the uniform grid are tested: a grid of 50×50 points, a grid of 100×100 points, a grid of 150×150 points, and a much finer grid of 200×200 points. Table 2 shows an effect of the mesh on the average Nusselt number of the left vertical wall and maximum absolute value of the stream function inside the cavity.

Tab. 2. Variations of the average Nusselt number of the left vertical wall and maximum absolute value of the stream function inside the cavity with the uniform grid.

Uniform grids	Nu_{avg}	$\Delta_{Nu} = \frac{ Nu_{avg, i \times j} - Nu_{avg, 150 \times 150} }{Nu_{avg, i \times j}} \times 100\%$	$ \psi _{max}$	Δ_ψ
50×50	2.90424	2.16	41.4749	0.08
100×100	2.84831	0.24	41.4756	0.08
150×150	2.84145	–	41.5092	–
200×200	2.83981	0.06	41.5282	0.05

On the basis of the conducted verifications the uniform grid of 150×150 points has been selected for the following analysis.

4. Results and discussion

Numerical study has been conducted at the following values of the key parameters: Rayleigh number ($Ra = 10^6$), Darcy number ($10^{-5} \leq Da \leq 10^{-2}$), Prandtl number ($Pr = 7.0$), viscosity variation parameter ($0 \leq C \leq 6$) and radiation parameter ($0 \leq R_d \leq 10$). Particular efforts have been focused on the effects of the radiation parameter, viscosity variation parameter and Darcy number on the fluid flow and heat transfer. Streamlines, isotherms, average Nusselt number at the left hot wall and maximum absolute values of the stream function inside the cavity for different values of key parameters mentioned above are illustrated in Figs. 2–6.

Figure 2 shows streamlines and isotherms for different values of the radiation parameter. Regardless of the radiation parameter value a single convective cell forms inside the cavity illustrating ascending flows near the hot vertical wall and descending flows near the right cold wall. It is worth noting that a presence of the fluid with temperature-dependent viscosity leads to non-uniform distributions of streamlines and isotherms. In the case of $C = 3$ we have the fluid where the dynamic viscosity decreases with temperature and as a result for high temperature this fluid is more movable. Therefore one can find near the left hot wall more intensive circulation of the fluid. Moreover the convective cell core appears near this hot wall with an intensive penetration of the high temperature

along the upper wall inside the cavity. While the low temperature wave penetrates inside the cavity along the bottom wall less intensive. An increase in the radiation parameter leads to both an intensification of convective flow taking into account the maximum absolute values of the stream function and a reduction of the convective cell core size. Such changes occur owing to less intensive heating of the upper part of the enclosure (see the variation of the position of isotherm $\theta = 0.4$). Also an increase in the radiation parameter leads to an attenuation of convective heat transfer mechanism and an intensification of heat conduction as it has been mentioned by Ahmed et al. [6]. One can find that distribution of isotherms reflects both a straightening of the isotherms in the central part of the cavity and a decrease in the thermal boundary layers close to the vertical isothermal walls. It should be noted that observable fluid structures and temperature fields are defined with the radiation parameter as well Prandtl number (in the present work $Pr = 7.0$). The attenuation of convective heat transfer and the circulation intensification with the radiation parameter are presented in Figs. 5 and 6.

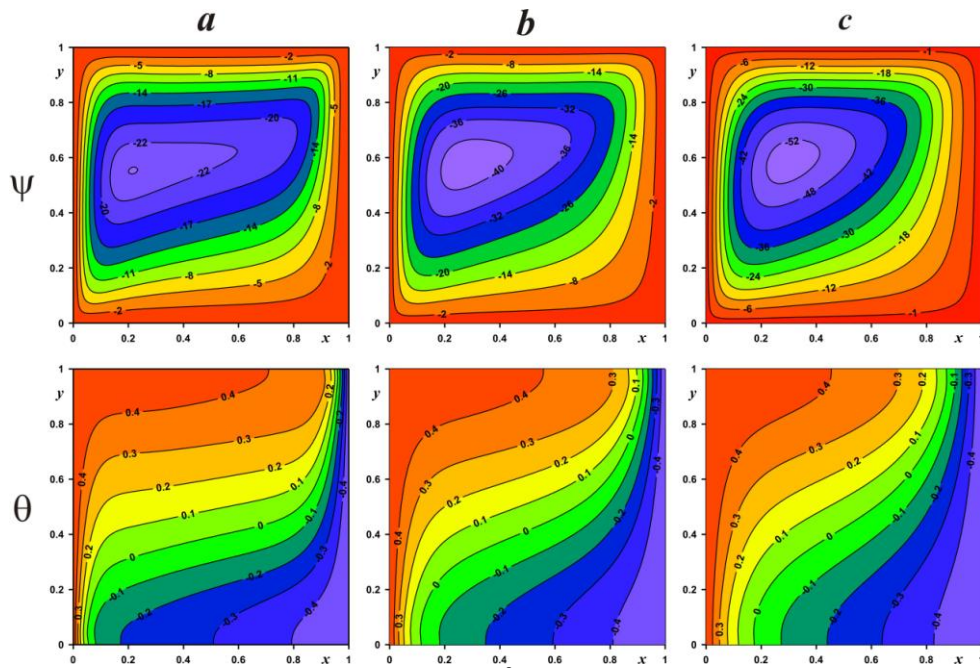


Fig. 2. Streamlines ψ and isotherms θ at $Da = 10^{-3}$, $C = 3$: $R_d = 1 - a$, $R_d = 5 - b$, $R_d = 10 - c$

Figures 2b and 3 illustrate the effect of the Darcy number for $R_d = 5$, $C = 3$. In the case of low value of the Darcy number ($Da = 10^{-5}$) one can find weak fluid circulation inside the cavity with dominance of heat conduction. It should be noted that also in this case when heat convection is the weakest heat transfer mechanism the convective cell core forms close to the left hot wall due to the influence of the temperature-dependent viscosity as has been mentioned above. An increase in the Darcy number leads to an intensification of convective flow and heat transfer where the main convective core displaces to the left upper corner. One can find a formation of minor convective cell core close to the right cold vertical wall. Therefore it is possible to conclude that an increase in the permeability leads to a transport from a single-core convective cell regime to double-core convective cell mode. At the same time the central part of the cavity is characterized by thermal stratification for high values of Da . Figure 5 confirms that an increase in the Darcy number leads to an increase in the heat transfer and flow rates. Moreover for high values of Da one can find more intensive increase in

$|\psi|_{\max}$ with the radiation parameter while for low values of Da ($Da = 10^{-5}$) we have dominance of conductive heat transfer.

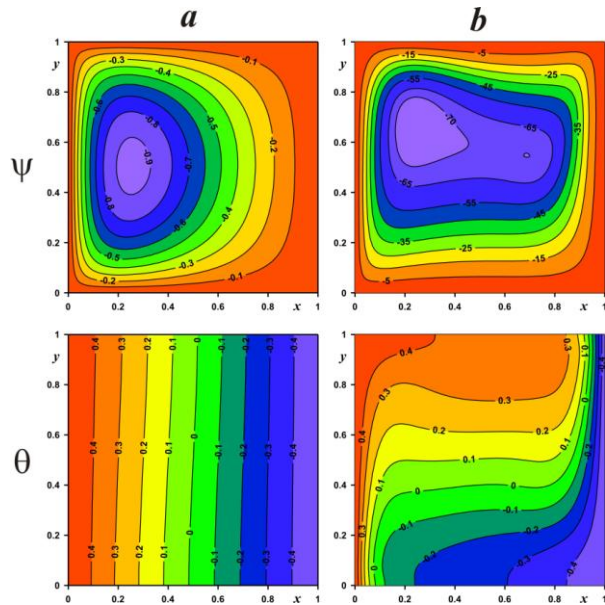


Fig. 3. Streamlines ψ and isotherms θ at $R_d = 5$, $C = 3$: $Da = 10^{-5} - a$, $Da = 10^{-2} - b$

The effect of viscosity variation parameter on streamlines and isotherms is presented in Figs. 2b and 4 for $Da = 10^{-3}$, $R_d = 5$. An increase in C leads an intensification of convective flow inside the cavity taking into account the maximum absolute value of the stream function. Also one can find more intensive penetration of high temperature wave from the left hot wall to the cavity along the upper adiabatic wall while the penetration intensity of the low temperature wave from the right cold wall decreases. An increment of C leads to a decrease in the convective core sizes and this core displaces close to the left upper corner. Such changes are due to more intensive motion of the fluid with low viscosity in high temperature zones.

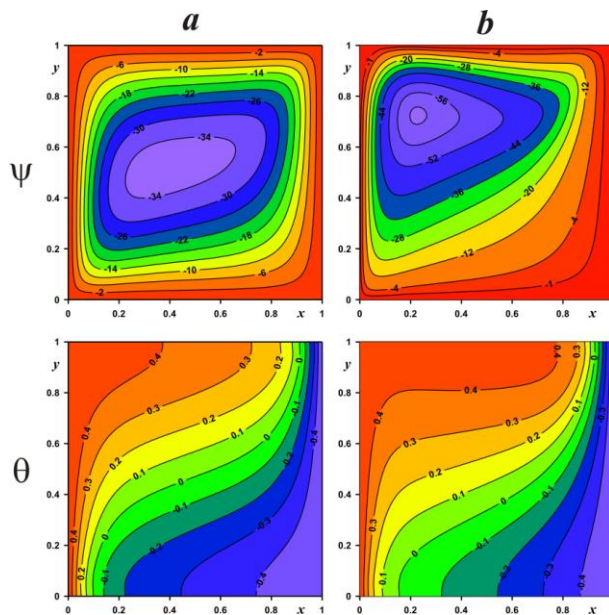


Fig. 4. Streamlines ψ and isotherms θ at $Da = 10^{-3}$, $R_d = 5$: $C = 1 - a$, $C = 6 - b$

Figure 6 shows the effect of the viscosity variation parameter and radiation parameter on the average Nusselt number at the hot vertical wall and maximum absolute value of the stream function for $Da = 10^{-3}$. As has been mentioned above an increase in R_d leads to a decrease in the heat transfer rate and an increase in the flow rate. While an increase in C leads to an intensification of heat transfer and convective flow inside the cavity.

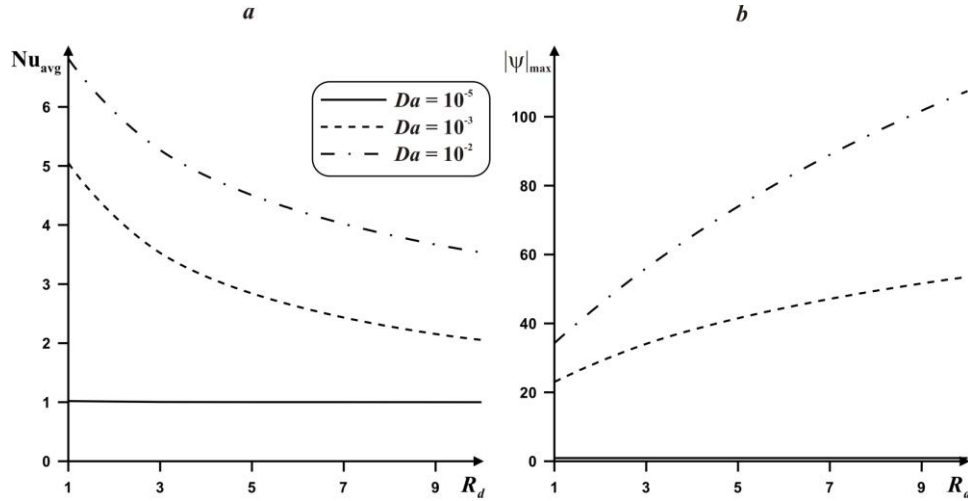


Fig. 5. Variation of the average Nusselt number (a) and maximum absolute value of the stream function (b) with radiation parameter and Darcy number for $C = 3$

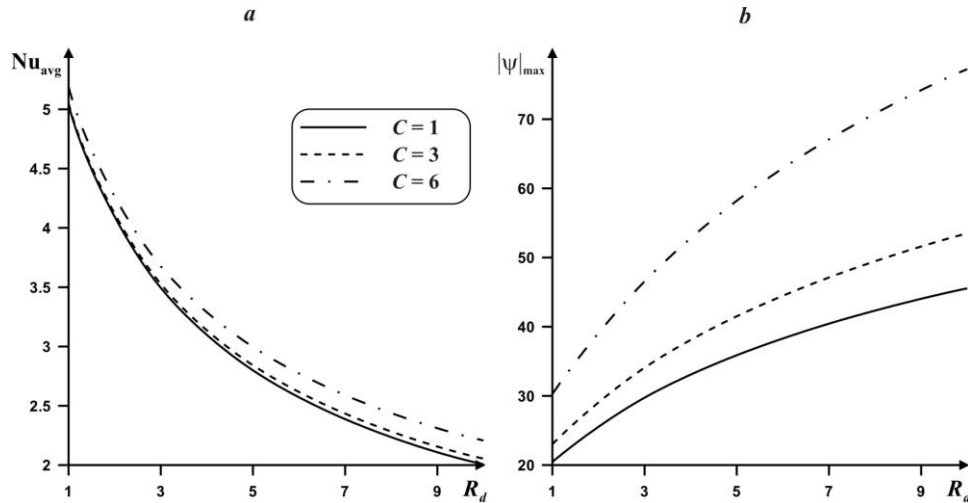


Fig. 6. Variation of the average Nusselt number (a) and maximum absolute value of the stream function (b) with radiation parameter and viscosity variation parameter for $Da = 10^{-3}$

5. Conclusions

A laminar two-dimensional natural convection combined with thermal radiation in a square differentially heated porous cavity filled with a fluid of temperature-dependent viscosity has been studied. The governing equations formulated in dimensionless stream function, vorticity and temperature taking into account the Brinkman–extended Darcy model, Boussinesq and Rosseland diffusion approximation have been solved by finite difference method. Based on the findings in this study, we conclude the following:

(1) An increase in the radiation parameter leads to an intensification of convective flow rate, attenuation of the heat transfer rate, a decrease in the thermal boundary layers thickness and a decrease in the convective core sizes. For high values of thermal radiation parameter the heat conduction is a dominating heat transfer mechanism.

(2) An increase in the Darcy number leads to an intensification of convective flow and heat transfer and a formation of a double-core convective cell for high values of Da. Also an increase in Da leads to a displacement of the major convective core to the left upper corner. For high values of the Darcy number one can find the thermal stratification in the central part of the cavity.

(3) An increase in the viscosity variation parameter leads to a reduction of the convective core sizes and a displacement of this core close to the left upper corner. The average Nusselt number and convective flow rate are the increasing functions of the viscosity variation parameter.

Acknowledgement

This work of Marina S. Astanina and Mikhail A. Sheremet was conducted as a government task of the Ministry of Education and Science of the Russian Federation, Project Number 13.1919.2014/K.

Nomenclature

C	viscosity variation parameter, [-]
C_p	heat capacity, [$\text{Jkg}^{-1}\text{K}^{-1}$]
Da	Darcy number ($= K/L^2$), [-]
g	gravitational acceleration, [m/s^2]
K	permeability of the porous medium, [m^2]
k	thermal conductivity of the porous medium, [$\text{Wm}^{-1}\text{K}^{-1}$]
L	size of the cavity, [m]
Nu_{avg}	average Nusselt number, [-]
p	pressure, [Pa]
Pr	Prandtl number ($= \mu_0/(\rho\alpha)$), [-]
Ra	Rayleigh number ($= \rho g \beta (T_h - T_c) L^3 / (\alpha \mu_0)$), [-]
R_d	thermal radiation parameter ($= 4\sigma T_c^3 / (k\beta_r)$), [-]
T	dimensional fluid temperature, [K]
T_0	mean dimensional temperature of the cavity ($= 0.5(T_h + T_c)$), [K]
T_c	dimensional temperature at right vertical wall, [K]
T_h	dimensional temperature at left vertical wall, [K]
\bar{u}, \bar{v}	dimensional velocity components along horizontal and vertical directions, [ms^{-1}]
u, v	dimensionless velocity components along horizontal and vertical directions, [-]
\bar{x}	dimensional Cartesian coordinate measured along the bottom wall of the cavity, [m]
\bar{y}	dimensional Cartesian coordinate measured along the vertical wall of the cavity, [m]

x, y dimensionless Cartesian coordinates, [-]

Greek symbols

α	thermal diffusivity of the porous medium, [m^2s^{-1}]
β	coefficient of thermal expansion, [K^{-1}]
β_r	extinction coefficient, [m^{-1}]
θ	dimensionless temperature, [-]
μ	dimensionless dynamic viscosity ($= \exp(-C\theta)$), [-]
μ_0	dimensional dynamic viscosity at mean temperature, [$\text{Pa}\cdot\text{s}$]
$\bar{\mu}$	dimensional dynamic viscosity $\left(= \mu_0 \cdot \exp\left(-C \frac{T - T_0}{T_h - T_c}\right) \right)$, [$\text{Pa}\cdot\text{s}$]
ρ	density, [kg/m^3]
ρC_p	volumetric heat capacity of the fluid, [$\text{Jm}^{-3}\text{K}^{-1}$]
σ	Stephan–Boltzmann constant, [$\text{Wm}^{-2}\text{K}^{-4}$]
ψ	dimensionless stream function, [-]
$\bar{\psi}$	dimensional stream function, [m^2s^{-1}]
ω	dimensionless vorticity, [-]
$\bar{\omega}$	dimensional vorticity, [s^{-1}]

Subscripts

c	cold
h	hot
0	reference value

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