An Analytical Model on Thermal Performance Evaluation of Counter Flow Wet Cooling Tower

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This paper proposes an analytical model for simultaneous heat and mass transfer processes in a counter flow wet cooling tower, with the assumption that the enthalpy of the saturated air is a linear function of the water surface temperature. The performance of the proposed analytical model is validated in some typical cases. The validation reveals that, when cooling range is in a certain interval, the proposed model is not only comparable with the accurate model, but also can reduce computational complexity. In addition, with the proposed analytical model, the thermal performance of the counter flow wet cooling towers in power plants is calculated. The results show that the proposed analytical model can be applied to evaluate and predict the thermal performance of counter flow wet cooling towers.

Key words: *heat and mass transfer, counter flow, wet cooling tower, analytical model, thermal performance, power plants*

1. Introduction

Wet cooling towers are widely applied in many industrial fields, such as power plants, air-conditioning and petroleum industries, to withdraw the waste heat to the environment. The heat is discharged in the processes of the simultaneous heat and mass transfer when the water is in direct contacted with the ambient air in the wet cooling tower. To depict the processes of the heat and mass transfer, the Merkel method [1] was proposed in 1925 with some simplifying assumptions, which are: 1) the water flow rate is constant in energy balance by neglecting the water loss by evaporation, 2) the outlet air is saturated and it is only function with its enthalpy, 3) the Lewis factor which can be expressed as $k_c/(k_D.c_{p,a})$ is unity. Many researchers' experiments were based on Merkel method [2-7], which is widely used to rate and design the wet cooling tower.

Jaber and Webb [8] presented an *e-NTU* method which is particularly useful in the cross flow cooling tower. The similar assumptions used in Merkel method were also made in the *e-NTU* method. Based on NTU method, Saravanan et al [9] analysis the performance of cooling tower through the perspectives of energy and exergy. The inlet air wet bulb temperature is the most important parameter through the analysis. Kloppers and Rögener [10-11] gave a detailed derivation of the differential equations which were based on the mass and energy balance. And Merkel numbers employing Merkel method, *e-NTU* method and Poppe method were deduced. The performance of the cooling tower was evaluated by

employing Merkel, *e-NTU* and Poppe methods, respectively. Kloppers and Rögener also found that the outlet water temperature evaluated by the Merkel, *e-NTU* and Poppe approaches were identical.

To rate the performance of the cooling tower, the numerical solutions of the ordinary differential equations were available in the literatures [12-13]. Their calculated results were acceptable in comparison with their experimental results. Khan et al [14] evaluated the performance of the counter flow wet cooling towers with a constant slope of tie line and the slope[15] was given by $E=-h_{cw}/h_D$. It was found that the evaporation caused about 62.5%-90% of the total rate of heat transfer from bottom to the top of the tower. The Merkel method was revisited by Picardo and Variyar [16], they developed a novel method to compute the packed height of a counter flow cooling tower. Asvapoositkul et al [17] presented a simplified method to evaluate the thermal performance capacity of the cooling tower. However, it was applied only when the flow rate and the inlet temperature were near the design conditions. Ren [18] developed an accurate and quick approach to analyze the tower performance. In his model, the humidity ratio of air at the water surface was assumed to be linearized with respect to the water surface temperature. And then, he obtains an analytical solution for evaluating the counter flow wet cooling tower performance. However, a set of ordinary differential equations should be solved in his model, which can lead to more computational complexity. A general non-dimensional model for cooling tower was presented by Halasz [19-20], which could be applied in the counter flow, parallel flow and cross flow towers. This method could yield an analytical solution for the counter flow and parallel towers while a numerical solution could be obtained for cross flow towers. Makkinejad [21] proposed a new method which could obtain an acceptable solution for cooling tower. In his method, the absolute gas humidity was assumed to be a linear function of the liquid temperature. Hasan [22] proposed a modified *e-NTU* method to achieve sub-wet bulb temperature. His model was based on the assumption that the enthalpy of the saturated air is a linear function of the temperature of the saturated air.

The aim of this paper is to develop a model, which is easily to be solved to obtain the analytical solution of outlet water temperature, in order to evaluate the performance of the counter flow wet cooling tower. Firstly, based on the assumption that the enthalpy of air at the water surface is a linear function of the water surface temperature, Merkel number will be integrable and an analytical solution for outlet water temperature will be deduced. Secondly, results of the analytical model will be in comparison with the accurate model in some typical cases. Finally, the proposed model will be used to calculate the thermal performance of the counter flow wet cooling towers in power plants.

2. Mathematical model for counter flow wet cooling tower

The simultaneous heat and mass transfer in the counter flow wet cooling tower has been shown in Fig. 1. The analytical model is mainly based on the following assumptions:
1) The water loss in the energy balance is neglected.
2) The Lewis factor is equal to unity.
3) The enthalpy of air at the water surface is a linear function of water surface temperature.
4) The enthalpy of bulk air is a linear function of water surface temperature.

5) The specific heats of air, water vapor and water are constant.

6) The effect of water film heat transfer resistance is negligible.

2.1 Governing equations for heat and mass transfer

The energy balance equation between water and air is given by:

$$m_w dh_w + h_w dm_w = m_{da} dh_a \tag{1}$$

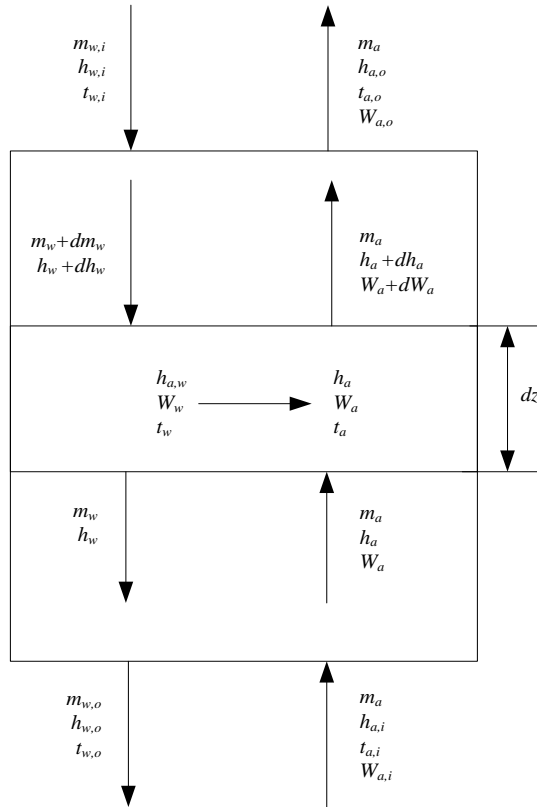The mass balance equation between water and air is given by:

$$dm_w = m_{da} dW_a \tag{2}$$

The convective heat transfer from water to air yields:

$$dQ_c = k_c (t_w - t_a) dA \tag{3}$$

Where dA is given by:

$$dA = a_{fi} A_{fr} dz \tag{4}$$

Fig.1. Heat and mass transfer in counter flow wet cooling tower

The mass transfer from water to air yields:

$$dm_w = k_D (W_{s,w} - W_a) dA \tag{5}$$

The evaporation heat transfer from water to air is expressed as:

$$dQ_v = h_{v,s} dm_w = h_{v,s} k_D (W_{s,w} - W_a) dA \tag{6}$$

Where $h_{v,s}$ is given by:

$$h_{v,s} = h_{ba} + c_{p,v} t_w \tag{7}$$

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The total heat transfer of convective and evaporation heat transfer is expressed as:

$$dQ = dQ_v + dQ_c = k_c(t_w - t_a)dA + h_{v,s}k_D(W_{s,w} - W_a)dA \qquad (8)$$

The energy balance at the water surface is given by:

$$dQ = m_{da}dh_a = m_w dh_w + h_w dm_w \qquad (9)$$

The enthalpy of air at the water surface is expressed by:

$$h_{a,s} = c_{p,da}t_w + W_{s,w}(h_{ba} + c_{p,v}t_w) \qquad (10)$$

The enthalpy of bulk air is expressed by:

$$h_a = c_{p,da}t_a + W_a(h_{ba} + c_{p,v}t_a) \qquad (11)$$

By subtracting Eq.(11) from Eq.(10), the resultant equation is expressed by:

$$t_w - t_a = (h_{a,s} - h_a - (W_{s,w} - W_a)(h_{ba} + c_{p,v}t_w))/c_{p,a} \qquad (12)$$

Where $c_{p,a}$ is given by Eq.(13)

$$c_{p,a} = c_{p,da} + W_a c_{p,v} \qquad (13)$$

Substituting Eq.(12) into Eq.(8), we obtain:

$$dQ = h_c(h_{a,s} - h_a - (W_{s,w} - W_a)(h_{ba} + c_{p,v}t_w))/c_{p,a}dA + h_{v,s}k_D(W_{s,w} - W_a)dA \qquad (14)$$

Substituting Eq.(14) into Eq.(9), we obtain:

$$m_w dh_w + h_w dm_w = h_c(h_{a,s} - h_a - (W_{s,w} - W_a)(h_{ba} + c_{p,v}t_w))/c_{p,a}dA + h_{v,s}k_D(W_{s,w} - W_a)dA \qquad (15)$$

2.2 Analytical model

According to the assumption 1) and 2), the second term at the left hand of Eq.(15) is ignored and the Lewis factor is equal to 1 ($Le = k_c/(c_{p,a}k_D)=1$). Then, Eq.(15) can be reduced as:

$$m_w dh_w = h_D(h_{a,s} - h_a - (W_{s,w} - W_a)(h_{ba} + c_{p,v}t_w))dA + h_{v,s}k_D(W_{s,w} - W_a)dA \qquad (16)$$

Substituting Eq.(7) into Eq.(16), we obtain:

$$Me = k_D dA/m_w = dh_w/(h_{a,s} - h_a) \qquad (17)$$

According to the literature [23], the enthalpy of the bulk air is expressed as:

$$h_a = k_1(t_w - t_{w,o}) + h_{a,i} \qquad (18)$$

Where k_1 is given by Eq(19):

$$k_1 = m_w c_{p,w}/m_{da} \qquad (19)$$

According to the assumption 3), the enthalpy of saturated air at the water surface temperature, t_w, is expressed as:

$$h_{a,s} = k_2(t_w - t_{wb,i}) + h_{wb,i} \qquad (20)$$

Where $h_{wb,i}$ is the enthalpy of saturated air at the temperature, $t_{wb,i}$.

The relationship between an assumed straight air saturation line and a real saturation line is shown in Fig. 2. To get a reasonable k_2, the midpoint, $t_{w,m}$, between inlet water temperature, $t_{w,i}$, and outlet water temperature , $t_{w,o}$, should be found. The midpoint, $t_{w,m}$, can be expressed as:

$$t_{w,m} = \left(t_{w,i} + t_{w,o}\right)/2 \tag{21}$$

Then, k_2, the slope of the assumed straight saturation line shown in Fig.2, can be yielded as:

$$k_2 = (h_{a,m} - h_{wb,i})/(t_{w,m} - t_{wb,i}) \tag{22}$$

Where $h_{a,m}$ is the enthalpy of saturated air at the temperature, $t_{w,m}$.

Here, we assume that the enthalpy of inlet air, $h_{a,i}$, is equal to the enthalpy of saturated air, $h_{wb,i}$, at the temperature, $t_{wb,i}$. Substituting Eq.(18) and Eq.(19) into Eq.(17) gives:

$$Me = c_{p,w}/(k_2 - k_1)\ln(k_2(t_{w,i} - t_{wb,i}) - k_1(t_{w,i} - t_{w,o})/k_2(t_{w,o} - t_{wb,i})) \tag{23}$$
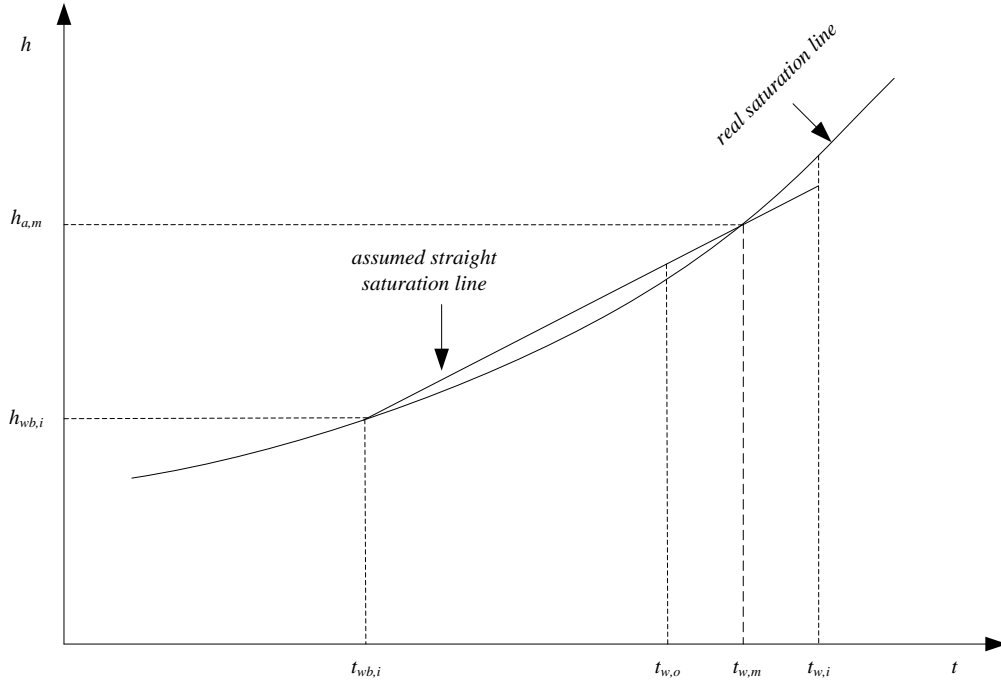
Fig.2. An illustration of the assumed straight and real saturation lines

Rearrange Eq.(24), we obtain the analytical expression of $t_{w,o}$ as follows:

$$t_{w,o} = (k_2 - k_1)(t_{w,i} - t_{wb,i})/(k_2 B - k_1) + t_{wb,i} \tag{24}$$

Where B can be expressed as:

$$B = exp\left(Me(k_2 - k_1)/c_{p,w}\right) \tag{25}$$

The cooling efficiency of the counter flow wet cooling tower is usually defined as:

$$\varepsilon = (t_{w,i} - t_{w,o})/(t_{w,i} - t_{wb,i}) \tag{26}$$

Substituting Eq.(25) into Eq.(26) gives:

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$$\varepsilon = 1 - (k_2 - k_1)/(k_2 B - k_1) \tag{27}$$

3. Results validation

To validate this analytical model, Merkel number evaluated by this model are compared with the Merkel number calculated by accurate model[24], Merkel model, Boris's Non-dimensional model [19], and the outlet water temperature evaluated by this model are compared with the outlet water temperature calculated by accurate model[18] and Merkel model.

3.1 Results validation for the Merkel number Me

The relationship between N_G used in [19, 24] and Me is expressed as:

$$N_G = h_D A / m_{da} = (m_w / m_{da}) h_D A / m_w = (m_w / m_{da}) Me \tag{28}$$

Me^A, Me^M, Me^B and Me^P represents the Merkel number calculated by Poppe's accurate model[24], Merkel model, Boris's Non-dimensional model[19] and the proposed model, respectively. The relative errors of Merkel model, Boris's Non-dimensional model and the proposed model are expressed as e^M, e^B, e^P, respectively.($e^X = (Me^X - Me^A)/Me^A \times 100\%$, here X represents M or B or P)

Tab.1 The Comparison of the results for Merkel number

No.	$t_{w,i}$	$t_{w,o}$	$t_{a,i}$	$t_{a,wb}$	m_a/m_w	Me^A	Me^M	e^M	Me^B	e^B	Me^P	e^P
01	30	26	8	4	0.25	0.530	0.475	-10.4	0.527	-0.7	0.511	-3.6
02	30	26	8	4	0.30	0.419	0.385	-8.1	0.407	-2.8	0.400	-4.5
03	30	26	8	8	0.30	0.533	0.485	-9.0	0.524	-1.7	0.507	-4.9
04	34	30	16	12	0.20	0.941	0.684	-27.3	-	-	0.948	0.7
05	34	30	24	20	0.30	0.874	0.745	-14.8	0.921	5.4	0.881	0.8
06	34	30	24	20	0.35	0.655	0.588	-10.2	0.656	0.2	0.640	-2.3
07	34	30	24	20	0.40	0.568	0.518	-8.8	0.562	-1.1	0.551	-3.0
08	34	24	16	12	0.50	3.577	2.723	-23.9	-	-	-	-
09	34	24	16	12	0.80	1.251	1.165	-6.9	1.235	-1.3	1.227	-1.9
10	34	24	16	12	1.00	1.086	1.020	-6.1	1.054	-2.9	1.048	-3.5
11	34	24	16	16	1.00	1.497	1.397	-6.7	1.444	-3.5	1.438	-3.9
12	34	24	24	20	1.00	2.603	2.404	-7.6	2.534	-2.7	2.707	4.0
13	34	24	24	20	1.50	1.926	1.817	-5.7	1.835	-4.8	1.915	-0.6
14	34	24	24	20	2.00	1.722	1.634	-5.1	1.634	-5.1	1.697	-1.5
15	40	20	16	12	1.50	2.340	2.234	-4.5	2.099	-10.3	2.227	-4.8
16	40	20	16	12	2.00	2.062	1.976	-4.2	1.836	-11.0	1.931	-6.4
17	40	20	16	12	3.00	1.851	1.779	-3.9	1.644	-11.2	1.718	-7.2
18	40	20	16	16	3.00	2.625	2.517	-4.1	2.235	-14.9	2.363	-10.0
19	40	20	22	18	3.00	3.486	3.381	-3.0	2.934	-15.8	3.123	-10.4
20	40	20	22	18	5.00	3.115	3.030	-2.7	2.650	-14.9	2.801	-10.1
21	40	20	22	18	8.00	2.944	2.864	-2.7	2.512	-14.7	2.651	-10.0
22	54	24	16	12	1.50	1.725	1.662	-3.7	1.395	-19.1	1.580	-8.4
23	54	24	16	12	2.00	1.584	1.528	-3.5	1.280	-19.2	1.432	-9.6
24	54	24	16	16	2.00	1.922	1.852	-3.6	1.498	-22.1	1.693	-11.9

Tab. 1 lists the input data and the results calculated by four models mentioned above. The results of the proposed model are more accurate than Merkel model for case 1 to 7 and case 9 to 14. We can observe that the temperature difference between inlet and outlet water are less than 10℃ in all these cases. The results of the proposed model almost have the same accuracy as Boris's Non-dimensional model since the two models have the same average absolute relative error (2.7%) for the cases mentioned above, except case 4 where Boris's Non-dimensional model yields no results.

According to tab. 1, the proposed model yields no results for case 8 while Boris's Non-dimensional model yields no results for the case 4 and 8. Both of the two cases are in extreme conditions where the enthalpy of outlet air, $h_{a,o}$, is almost equal to the enthalpy of the saturated air, $h_{a,wi}$, at the inlet water temperature, $t_{w,i}$. For case 8, the enthalpy of the saturated air, $h_{a,wi}$, evaluated by Eq.(20) is less than the enthalpy of outlet air, $h_{a,o}$, and the antilogarithm at the right hand of Eq.(23) will be negative. Therefore, Eq.(23) is unable to get reasonable results. For case 4, the enthalpy of the saturated air, $h_{a,wi}$, evaluated by Eq.(20) is larger than the enthalpy of outlet air, $h_{a,o}$, so the results can be yielded. For the proposed model, large temperature difference between inlet and outlet water may yield no results in extreme conditions, such as case 8. The process lines of water cooling in cooling tower for the two extreme cases are illustrated in Fig.3. (The enthalpy of inlet air and the temperature of inlet water are the same for case 4 and 8 according to tab. 1.)
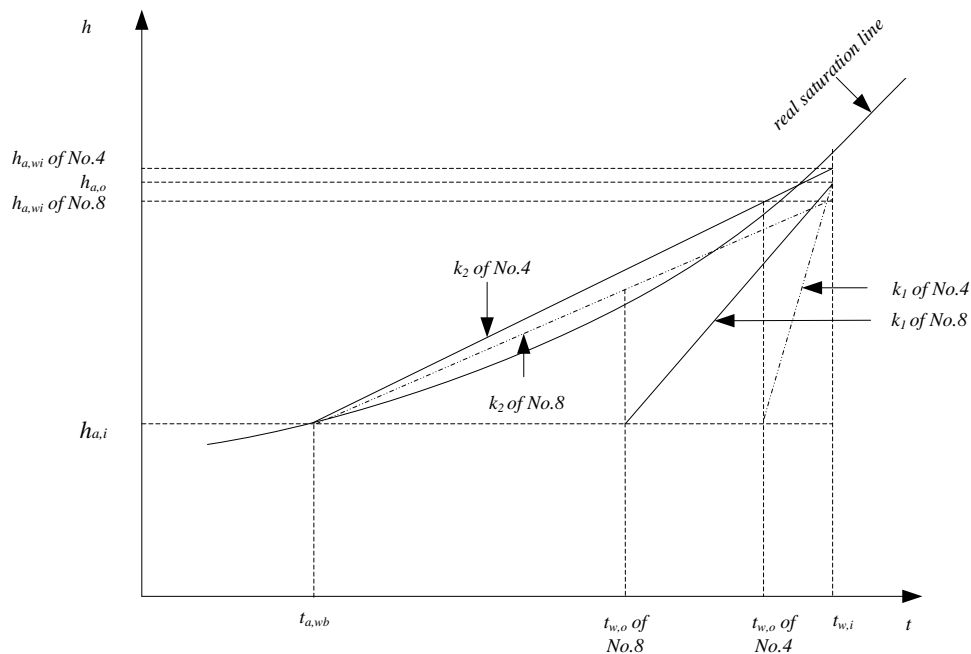
Fig.3. The process lines for water cooling in the cooling tower for Case.4 and 8

For cases 15 to 24, the results of the proposed model are less accurate than Merkel model. These large errors are caused by the linearization assumption, which introduces large errors under large temperature difference between inlet and outlet water (larger than 20℃). However, the results of the proposed model are more accurate than Boris's Non-dimensional model for cases 15 to 24.

3.2 Results validation for the outlet water temperature

The outlet water temperature evaluated by the proposed model is compared with an accurate model under some typical operating conditions investigated by Ren [18]. For the accurate model, Equations (29), (30) and (31) are solved by the finite difference method. If the air becomes supersaturated, Equations (29), (30) and (31) should be replaced with Equations (32), (33) and (34).

$$dw_a = (W_{s,w} - W_a)dNTU \tag{29}$$

$$dt_a = (Le + (c_{p,v}/c_{p,a})(W_{s,w} - W_a))dNTU \tag{30}$$

$$dt_w = (m_{da}c_{p,a}/m_w c_{p,w})(Le(t_w - t_a) + (h_{ba} + (c_{p,v} - c_{p,w})t_w)(W_{s,w} - W_a)/c_{p,a})dNTU \tag{31}$$

$$dw_a = (W_{s,w} - W_{s,a})dNTU \tag{32}$$

Where $W_{s,a}$ is humidity ratio of saturated moist air at dry bulb temperature.

$$dt_a = (Lec_{p,a}(t_w - t_a) + (h_{ba} + c_{p,v}t_w - c_{p,w}t_a)(W_{s,w} - W_{s,a}))/(c_{p,a} + (W_a - W_{s,a})c_{p,w})dNTU$$

$$-(h_{ba} + (c_{p,v} - c_{p,w})t_a)/(c_{p,a} + (W_a - W_{s,a})c_{p,w})dW_{s,a} \tag{33}$$

$$dt_w = (m_{da}c_{p,a}/m_w c_{p,w})(Le(t_w - t_a) + (h_{ba} + (c_{p,v} - c_{p,w})t_w)(W_{s,w} - W_{s,a})/c_{p,a})dNTU \tag{34}$$

The slope of the assumed straight saturation line, k_2, in Eq.(24), will depend on the outlet water temperature, therefore, some steps in iteration will be needed for the solutions. $t^A_{w,o}$, $t^M_{w,o}$, and $t^P_{w,o}$ represents the outlet water temperature calculated by accurate model, Merkel model and the proposed model. The relative errors of Merkel model and the proposed model are expressed as e^M_t and e^P_t, respectively. ($e^X_t = (t_{w,o}^X - t_{w,o}^A)/t_{w,o}^A \times 100\%$, here X represents M or P)

Tab. 2 The comparison of outlet water temperature with different models

No.	$t_{w,i}$	$t_{a,i}$	$t_{wb,i}$	m_a/m_w	NTU	$t^A_{w,o}$	$t^M_{w,o}$	e^M_t	$t^P_{w,o}$	e^P_t
1	60	35	20	1	3	29.68	25.38	-14.5	24.47	-17.6
2	30	35	20	1	3	23.50	22.65	-3.6	23.03	-2.0
3	40	35	30	1	3	32.05	31.03	-3.2	30.99	-3.3
4	40	25	20	1	3	26.03	24.25	-6.8	24.46	-6.0
5	40	35	20	1	3	26.03	24.19	-7.1	24.46	-6.0
6	40	35	20	0.5	3	31.35	29.68	-5.3	30.18	-3.7
7	40	35	20	2	3	21.94	20.56	-6.3	20.50	-6.6
8	40	35	20	1	0.5	34.46	32.30	-6.3	32.39	-6.0
9	40	35	20	1	6	23.59	22.28	-5.6	22.55	-4.4

The results calculated by three models are listed in tab. 2. The absolute relative errors of the proposed model are below 5% except case 1. We can observe that the range of temperature between inlet water and outlet water is larger than 30℃ in the case, so the linearization for enthalpy of saturated air produces large error. As shown in tab. 2, the absolute relative errors of the proposed model increases as the temperature difference between

inlet and outlet water increases.

To compare the accuracy between accurate model and the proposed model in a relative narrow range, the regression analysis is conducted between $t^A_{w,o}$ and $t^P_{w,o}$ for cases 2-10. The regression analysis between $t^A_{w,o}$ and $t^P_{w,o}$ for cases 2-10 and a $\pm 5\%$ error band are also shown in Fig.4. According to Fig.4, the regression analysis yields an R^2 of 0.9980 and an RMSE of 0.29℃ for the proposed model. The results show the proposed model and accurate model show good agreement. The regression analysis also yields an R^2 of 0.9987 and an RMSE of 0.33℃ for Merkel model. The results also show good agreement between Merkel model and accurate model. However, as shown in the tab. 3, the steps in iteration for the proposed model are much less than that for Merkel model. S^P and S^M represent the steps in iteration for the proposed model and Merkel model, respectively. (The initial value of the outlet water, $t_{w,o}$, is set to $t_{w,i}$-5).

Tab. 3. The comparison of steps in iteration with the proposed model and Merkel model

No.	1	2	3	4	5	6	7	8	9	10
S^P	6	3	7	3	5	3	4	5	4	5
S^M	3002	637	1932	398	1081	1411	537	1086	271	1281
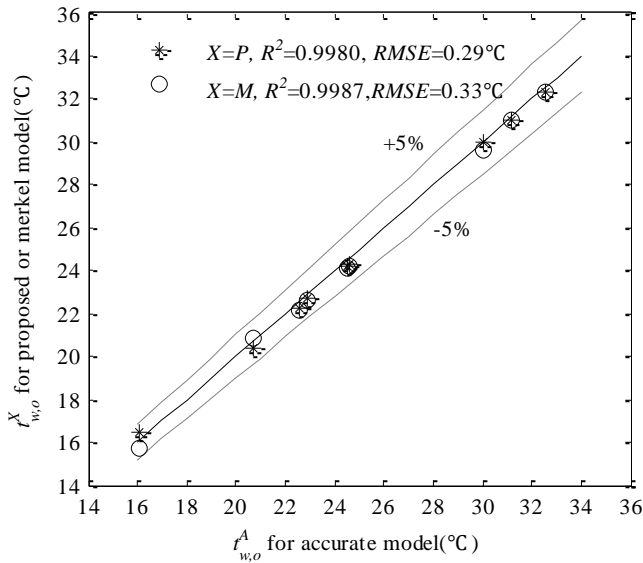
Fig.4. Comparison of outlet water temperature between accurate model and proposed (or Merkel) model

4. Application in thermal power plants

In this section, the proposed model is applied to calculate the performance of cooling tower used to discharge the waste heat to the environment in the power plant. Two cooling towers are chosen here. One is served for a 300 MW power plant while another is served for a 600 MW power plant. Tab. 4 shows the specification of the two towers. e^{Ms} in tab. 4 is the absolute value of the difference value between outlet water temperature ($t^{Ms}_{w,o}$) measured and water outlet temperature($t^P_{w,o}$) calculated by the proposed model($e^{Ms} = |t^{Ms}_{w,o} - t^P_{w,o}|$).

Tab. 4. The specification of the cooling towers

	Total height	Tip diameter	Zero meter[1]	Water drenching area	Sprinkle density
unit	m	m	m	m^2	t/(m^2.h)
300MW	128	61.7	101.4	6000	6.48
600MW	150	71.4	118.9	9000	8.46

Tab. 5. Validation of the model result

	m_w	$t_{w,i}$	m_a	$t_{a,i}$	$t_{a,wb}$	$t^{Ms}_{w,o}$	$t^{P}_{w,o}$	e^{Ms}
unit	t/h	℃	t/h	℃	℃	℃	℃	℃
300MW	38880	38.10	31025	30.75	22.90	30.06	**29.91**	**0.15**
	38880	41.40	30495	34.10	28.80	32.90	**32.64**	**0.26**
600MW	77642	41.93	41528	35.70	26.90	34.35	**34.01**	**0.34**
	67322	40.94	45779	32.60	27.40	32.69	**32.64**	**0.05**

For the 300MW case, the two working conditions are both full load. For the 600 MW case, one working condition is full load, and the other one is partial load. As shown in tab. 5, the errors of two cases for 300MW are 0.15 ℃ and 0.26 ℃, respectively. And the errors of two cases for 600 MW are 0.34 ℃ and 0.05 ℃, respectively. Therefore, the results calculated by the proposed model are absolutely acceptable since the errors are less than 0.34℃。The good results are due to that the errors caused by the linearization assumption are small since the temperature difference between the inlet and outlet water is less than 10 ℃ in our experiments.

5. Conclusions

A simple and analytical model based on the assumption that enthalpy-temperature relation of saturated air is linear is proposed to evaluate the thermal performance of the counter flow wet cooling tower. In this model, the analytical expression of the outlet water temperature is available. The proposed analytical model is validated with Poppe's accurate model [24] for Merkel number and Ren's accurate model [18] for outlet water temperature. The results show that the proposed analytical model can yield acceptable accuracy within a relative narrow cooling range which is temperature difference between inlet and outlet water. Furthermore, when the proposed model is applied to calculate the performance of cooling towers used in power plants, its results are quite acceptable. Therefore, it can be applied to online calculation since it is easily to be solved.

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[1] The diameter at the height of zero meter

Nomenclature

A	area, (m^2)	t	temperature, (℃)
A_{fr}	front area, (m^2)	W_a	humidity ratio of moist air , (kgkg^{-1})
a_{fi}	area per unit volume, (m^2/m^3)		
c_p	specific heat at constant pressure (kJkg^{-1}K^{-1})	$W_{s,w}$	humidity ratio of saturated moist air at water temperature, (kgkg^{-1})
e	relative error		
h	specific enthalpy, (kJkg^{-1})	z	coordinate, (m)
$h_{a,s}$	Enthalpy of saturated moist air at water temperature, (kJkg^{-1})	**Greek letters**	
		ε	cooling efficiency
h_{ba}	latent heat of evaporation at 0℃, (kJkg^{-1})	**Subscripts**	
h_w	specific enthalpy of water, (kJkg^{-1})	a	air
$h_{v,s}$	specific enthalpy of saturated water vapor at water temperature, (kJkg^{-1})	da	dry air
		i	inlet
k_D	mass transfer coefficient, (kgm^{-2}s^{-1})	o	outlet
k_c	heat transfer coefficient, (Wm^{-2}K^{-1})	v	vapor
Le	Lewis factor	w	water
m_{da}	dry air mass flow rate, (kgs^{-1})	wb	wet bulb
Me	Merkel number, $h_D A/m_w$	**Superscripts**	
NTU	Number of transfer coefficient, ($=h_D A/m_a$)	A	accurate model
		B	Boris's Non-dimensional model
Q	total heat transfer rate, (W)		
Q_c	convective heat transfer rate, (W)	M	Merkel model
Q_v	evaporation heat transfer rate, (W)	Ms	Measurement
R^2	absolute fraction of variance	P	proposed model
$RMSE$	root mean square error	S	steps in iteration

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