AN ANALYTICAL MODEL ON THERMAL PERFORMANCE EVALUATION OF COUNTER FLOW WET COOLING TOWER

by

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This paper proposes an analytical model for simultaneous heat and mass transfer processes in a counter flow wet cooling tower, with the assumption that the enthalpy of the saturated air is a linear function of the water surface temperature. The performance of the proposed analytical model is validated in some typical cases. The validation reveals that, when cooling range is in a certain interval, the proposed model is not only comparable with the accurate model, but also can reduce computational complexity. In addition, with the proposed analytical model, the thermal performance of the counter flow wet cooling towers in power plants is calculated. The results show that the proposed analytical model can be applied to evaluate and predict the thermal performance of counter flow wet cooling towers.

Key words: *heat and mass transfer, counter flow, wet cooling tower, analytical model, thermal performance, power plants*

Introduction

Wet cooling towers are widely applied in many industrial fields, such as power plants, air-conditioning, and petroleum industries, to withdraw the waste heat to the environment. The heat is discharged in the processes of the simultaneous heat and mass transfer when the water is in direct contacted with the ambient air in the wet cooling tower. To depict the processes of the heat and mass transfer, the Merkel method [1] was proposed in 1925 with some simplifying assumptions, which are: the water flow rate is constant in energy balance by neglecting the water loss by evaporation, the outlet air is saturated and it is only function with its enthalpy, and the Lewis factor which can be expressed as $k_c/(k_D{\cdot}c_{p,\mathrm{a}})$ is unity. Many researchers experiments were based on Merkel method [2-7], which is widely used to rate and design the wet cooling tower.

Jaber and Webb [8] presented an *e-NTU* method which is particularly useful in the cross-flow cooling tower. The similar assumptions used in Merkel method were also made in the *e-NTU* method. Based on NTU method, Saravanan *et al.* [9] analysis the performance of cooling tower through the perspectives of energy and exergy. The inlet air wet bulb temperature is the most important parameter through the analysis. Kloppers and Kroger [10, 11] gave a detailed derivation of the differential equations which were based on the mass and energy balance. Merkel numbers employing Merkel method, *e-NTU* method, and Poppe method were deduced. The performance of the cooling tower was evaluated by employing Merkel, *e-NTU,* and Poppe

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methods, respectively. Kloppers and Kroger [10, 11] also found that the outlet water temperature evaluated by the Merkel, *e-NTU*, and Poppe approaches were identical.

To rate the performance of the cooling tower, the numerical solutions of ODE were available in the literatures [12, 13]. Their calculated results were acceptable in comparison with their experimental results. Khan *et al.* [14] evaluated the performance of the counter flow wet cooling towers with a constant slope of tie line and the slope [15] was given by $E = -k_c/k_D$. It was found that the evaporation caused about 62.5-90% of the total rate of heat transfer from bottom to the top of the tower. The Merkel method was revisited by Picardo and Variyar [16], they developed a novel method to compute the packed height of a counter flow cooling tower. Asvapoositkul and Treeutok [17] presented a simplified method to evaluate the thermal performance capacity of the cooling tower. However, it was applied only when the flow rate and the inlet temperature were near the design conditions. Ren [18] developed an accurate and quick approach to analyze the tower performance. In his model, the humidity ratio of air at the water surface was assumed to be linearized with respect to the water surface temperature. Then, he obtains an analytical solution for evaluating the counter flow wet cooling tower performance. However, a set of ODE should be solved in his model, which can lead to more computational complexity. A general non-dimensional model for cooling tower was presented by Halasz [19, 20], which could be applied in the counter flow, parallel flow, and cross flow towers. This method could yield an analytical solution for the counter flow and parallel towers while a numerical solution could be obtained for cross flow towers. Makkinejad [21] proposed a new method which could obtain an acceptable solution for cooling tower. In his method, the absolute gas humidity was assumed to be a linear function of the liquid temperature. Hasan [22] proposed a modified *e-NTU* method to achieve sub-wet bulb temperature. His model was based on the assumption that the enthalpy of the saturated air is a linear function of the temperature of the saturated air.

The aim of this paper is to develop a model, which is easily to be solved to obtain the analytical solution of outlet water temperature, in order to evaluate the performance of the counter flow wet cooling tower. Firstly, based on the assumption that the enthalpy of air at the water surface is a linear function of the water surface temperature, Merkel number will be integrable and an analytical solution for outlet water temperature will be deduced. Secondly, results of the analytical model will be in comparison with the accurate model in some typical cases. Finally, the proposed model will be used to calculate the thermal performance of the counter flow wet cooling towers in power plants.

Mathematical model for counter flow wet cooling tower

The simultaneous heat and mass transfer in the counter flow wet cooling tower has been shown in fig. 1. The analytical model is mainly based on the following assumptions.

- The water loss in the energy balance is neglected.
- The Lewis factor is equal to unity.
- The enthalpy of air at the water surface is a linear function of water surface temperature.
- The enthalpy of bulk air is a linear function of water surface temperature.
- The specific heats of air, water vapor, and water are constant.
- The effect of water film heat transfer resistance is negligible.

Governing equations for heat and mass transfer

The energy balance equation between water and air is given by:

$$m_{\mathrm{w}} \mathrm{d}h_{\mathrm{w}} + h_{\mathrm{w}} \mathrm{d}m_{\mathrm{w}} = m_{\mathrm{da}} \mathrm{d}h_{\mathrm{a}} \tag{1}$$

The mass balance equation between water and air is given by:

$$\mathrm{d}m_\mathrm{w} = m_\mathrm{da}\mathrm{d}W_\mathrm{a} \tag{2}$$

The convective heat transfer from water to air yields:

$$\mathrm{d}Q_c = k_c(t_\mathrm{w} - t_\mathrm{a})\mathrm{d}A \tag{3}$$

where dA is given by:

$$\mathrm{d}A = a_{fi}A_{fr}\,\mathrm{d}z \tag{4}$$

The mass transfer from water to air yields:

$$\mathrm{d}m_\mathrm{w} = k_D(W_{s,\mathrm{w}} - W_\mathrm{a})\mathrm{d}A \tag{5}$$

The evaporation heat transfer from water to air is expressed as:

$$\mathrm{d}Q_v = h_{v,s}\mathrm{d}m_\mathrm{w} = h_{v,s}k_D(W_{s,w} - W_\mathrm{a})\mathrm{d}A \tag{6}$$

where $h_{v,s}$ is given by:

$$h_{v,s} = h_{ba} + c_{p,v}t_\mathrm{w} \tag{7}$$

The total heat transfer of convective and evaporation heat transfer is expressed:

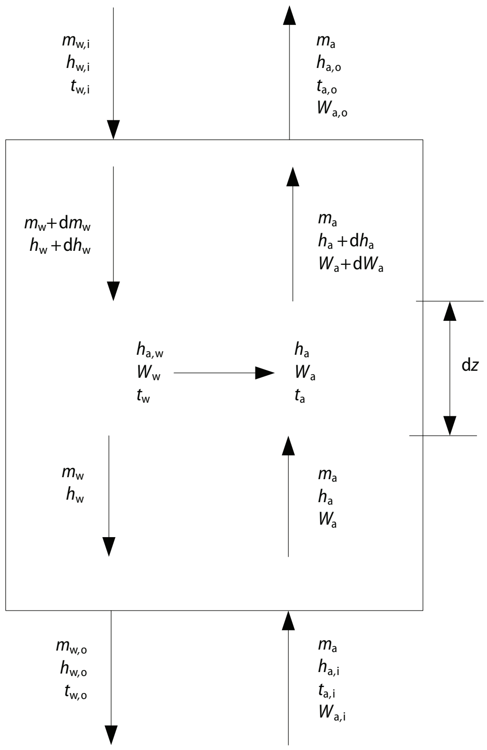

Figure 1. Heat and mass transfer in counter flow wet cooling tower

$$\mathrm{d}Q = \mathrm{d}Q_v + \mathrm{d}Q_c = k_c(t_\mathrm{w} - t_\mathrm{a})\mathrm{d}A + h_{v,s}k_D(W_{s,w} - W_\mathrm{a})\mathrm{d}A \tag{8}$$

The energy balance at the water surface is given by:

$$\mathrm{d}Q = m_\mathrm{da}\mathrm{d}h_\mathrm{a} = m_\mathrm{w}\mathrm{d}h_\mathrm{w} + h_\mathrm{w}\mathrm{d}m_\mathrm{w} \tag{9}$$

The enthalpy of air at the water surface is expressed by:

$$h_{a,s} = c_{p,\mathrm{da}}t_\mathrm{w} + W_{s,\mathrm{w}}(h_{ba} + c_{p,v}t_\mathrm{w}) \tag{10}$$

The enthalpy of bulk air is expressed by:

$$h_\mathrm{a} = c_{p,\mathrm{da}}t_\mathrm{a} + W_\mathrm{a}(h_{ba} + c_{p,v}t_\mathrm{a}) \tag{11}$$

By subtracting eq. (11) from eq. (10), the resultant equation is expressed by:

$$t_\mathrm{w} - t_\mathrm{a} = \frac{h_{a,s} - h_\mathrm{a} - (W_{s,\mathrm{w}} - W_\mathrm{a})(h_{ba} + c_{p,v}t_\mathrm{w})}{c_{p,a}} \tag{12}$$

where $c_{p,\mathrm{a}}$ is given by eq. (13):

$$c_{p,\mathrm{a}} = c_{p,\mathrm{da}} + W_\mathrm{a}c_{p,v} \tag{13}$$

Substituting eq. (12) into eq. (8), we obtain:

$$\mathrm{d}Q = h_\mathrm{c} \frac{h_{a,s} - h_\mathrm{a} - (W_{s,w} - W_\mathrm{a})(h_{ba} + c_{p,v} t_\mathrm{w})}{c_{p,\mathrm{a}}} \mathrm{d}A + h_{v,s} k_D (W_{s,w} - W_a) \mathrm{d}A \tag{14}$$

Substituting eq. (14) into eq. (9), we obtain:

$$m_\mathrm{w} \mathrm{d}h_\mathrm{w} + h_\mathrm{w} \mathrm{d}m_\mathrm{w} = k_\mathrm{c} \frac{h_{a,s} - h_\mathrm{a} - (W_{s,w} - W_\mathrm{a})(h_{ba} + c_{p,v} t_\mathrm{w})}{c_{p,\mathrm{a}}} \mathrm{d}A + h_{v,s} k_D (W_{s,w} - W_\mathrm{a}) \mathrm{d}A \tag{15}$$

Analytical model

According to the assumption (1) and (2), the second term at the left hand of eq. (15) is ignored and the Lewis factor is equal to 1 [Le = $k_c/(c_{p,\mathrm{a}} k_D) = 1$]. Then, eq. (15) can be reduced:

$$m_\mathrm{w} \mathrm{d}h_\mathrm{w} = k_D \left[h_{a,s} - h_\mathrm{a} - (W_{s,w} - W_\mathrm{a})(h_{ba} + c_{p,v} t_\mathrm{w}) \right] \mathrm{d}A + h_{v,s} k_D (W_{s,w} - W_\mathrm{a}) \mathrm{d}A \tag{16}$$

Substituting eq. (7) into eq. (16), we obtain:

$$\mathrm{Me} = \frac{k_D \mathrm{d}A}{m_\mathrm{w}} = \frac{\mathrm{d}h_\mathrm{w}}{h_{a,s} - h_\mathrm{a}} \tag{17}$$

According to [23], the enthalpy of the bulk air is expressed:

$$h_\mathrm{a} = k_1 (t_\mathrm{w} - t_\mathrm{w,o}) + h_\mathrm{a,i} \tag{18}$$

where k_1 is given:

$$k_1 = \frac{m_\mathrm{w} c_{p,\mathrm{w}}}{m_\mathrm{da}} \tag{19}$$

According to the assumption (3), the enthalpy of saturated air at the water surface temperature, t_w, is expressed:

$$h_{a,s} = k_2 (t_\mathrm{w} - t_\mathrm{wb,i}) + h_\mathrm{wb,i} \tag{20}$$

where $h_\mathrm{wb,i}$ is the enthalpy of saturated air at the temperature, $t_\mathrm{wb,i}$.

The relationship between an assumed straight air saturation line and a real saturation line is shown in fig. 2. To get a reasonable k_2, the midpoint, $t_\mathrm{w,m}$, between inlet water temperature, $t_\mathrm{w,i}$, and outlet water temperature, $t_\mathrm{w,o}$, should be found. The midpoint, $t_\mathrm{w,m}$, can be expressed:

$$t_\mathrm{w,m} = \frac{t_\mathrm{w,i} + t_\mathrm{w,o}}{2} \tag{21}$$

Then, k_2, the slope of the assumed straight saturation line shown in fig. 2, can be yielded:

$$k_2 = \frac{h_\mathrm{a,m} - h_\mathrm{wb,i}}{t_\mathrm{w,m} - t_\mathrm{wb,i}} \tag{22}$$

where $h_\mathrm{a,m}$ is the enthalpy of saturated air at the temperature, $t_\mathrm{w,m}$.

Here, we assume that the enthalpy of inlet air, $h_\mathrm{a,i}$, is equal to the enthalpy of saturated air, $h_\mathrm{wb,i}$, at the temperature, $t_\mathrm{wb,i}$. Substituting eqs. (18) and (19) into eq. (17) gives:

$$\mathrm{Me} = \frac{c_{p,\mathrm{w}}}{k_2 - k_1} \ln \left[k_2 (t_\mathrm{w,i} - t_\mathrm{wb,i}) - \frac{k_1 (t_\mathrm{w,i} - t_\mathrm{w,o})}{k_2 (t_\mathrm{w,o} - t_\mathrm{wb,i})} \right] \tag{23}$$

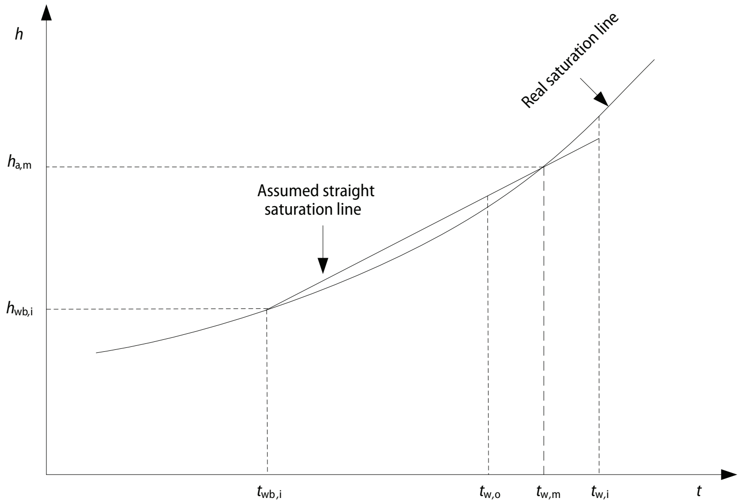

Figure 2. An illustration of the assumed straight and real saturation lines

Rearrange eq. (24), we obtain the analytical expression of $t_{\mathrm{w,o}}$:

$$t_{\mathrm{w,o}} = \frac{(k_2 - k_1)(t_{\mathrm{w,i}} - t_{\mathrm{wb,i}})}{k_2 B - k_1} + t_{\mathrm{wb,i}} \tag{24}$$

where B can be expressed:

$$B = \exp\left[\frac{\mathrm{Me}\left(k_2 - k_1\right)}{c_{p,\mathrm{w}}}\right] \tag{25}$$

The cooling efficiency of the counter flow wet cooling tower is usually defined:

$$\varepsilon = \frac{t_{\mathrm{w,i}} - t_{\mathrm{w,o}}}{t_{\mathrm{w,i}} - t_{\mathrm{wb,i}}} \tag{26}$$

Substituting eq. (25) into eq. (26) gives:

$$\varepsilon = 1 - \frac{k_2 - k_1}{k_2 B - k_1} \tag{27}$$

Results validation

To validate this analytical model, Merkel number evaluated by this model are compared with the Merkel number calculated by accurate model [24], Merkel model, Halasz [19] non-dimensional model, and the outlet water temperature evaluated by this model are compared with the outlet water temperature calculated by accurate model [18] and Merkel model.

Results validation for the Merkel number

The relationship between N_G used in [19, 24] and Merkel model is expressed:

$$N_G = \frac{k_D A}{m_{\mathrm{da}}} = \frac{\dfrac{m_{\mathrm{w}}}{m_{\mathrm{da}}} k_D A}{m_{\mathrm{w}}} = \frac{m_{\mathrm{w}}}{m_{\mathrm{da}}}\mathrm{Me} \tag{28}$$

The Me^A, Me^M, Me^B, and Me^P represents the Merkel number calculated by Poppes [24] accurate model, Merkel model, Halasz [19] non-dimensional model and the proposed model, respectively. The relative errors of Merkel model, Halasz non-dimensional model, and the proposed model are expressed as e^M, e^B, e^P, respectively, $\{e^X = [(Me^X – Me^A)/ Me^A]\cdot 100\%$, here X represents M, B, or $P\}$

Table 1 lists the input data and the results calculated by four models aforementioned. The results of the proposed model are more accurate than Merkel model for case 1 to 7 and case 9 to 14. We can observe that the temperature difference between inlet and outlet water are less than 10 °C in all these cases. The results of the proposed model almost have the same accuracy as Halasz non-dimensional model since the two models have the same average absolute relative error (2.7%) for the cases aforementioned, except case 4 where Halasz non-dimensional model yields no results.

According to tab. 1, the proposed model yields no results for case 8 while Halasz non-dimensional model yields no results for the case 4 and 8. Both of the two cases are in extreme conditions where the enthalpy of outlet air, $h_{a,o}$, is almost equal to the enthalpy of the saturated air, $h_{a,wi}$, at the inlet water temperature, $t_{w,i}$. For case 8, the enthalpy of the saturated air, $h_{a,wi}$, evaluated by eq. (20) is less than the enthalpy of outlet air, $h_{a,o}$, and the antilogarithm at the right hand of eq. (23) will be negative. Therefore, eq. (23) is unable to get reasonable results. For case 4, the enthalpy of the saturated air, $h_{a,wi}$, evaluated by eq. (20) is larger than the enthalpy of outlet air, $h_{a,o}$, so the results can be yielded. For the proposed model, large tem-

Table 1. The comparison of the results for Merkel number

	$t_{w,i}$	$t_{w,o}$	ta,i	$t_{a,wb}$	m_a/m_w	Me^A	Me^M	e^M	Me^B	e^B	Me^P	e^P
1	30	26	8	4	0.25	0.530	0.475	–10.4	0.527	–0.7	0.511	–3.6
2	30	26	8	4	0.30	0.419	0.385	–8.1	0.407	–2.8	0.400	–4.5
3	30	26	8	8	0.30	0.533	0.485	–9.0	0.524	–1.7	0.507	–4.9
4	34	30	16	12	0.20	0.941	0.684	–27.3	–	–	0.948	0.7
5	34	30	24	20	0.30	0.874	0.745	–14.8	0.921	5.4	0.881	0.8
6	34	30	24	20	0.35	0.655	0.588	–10.2	0.656	0.2	0.640	–2.3
7	34	30	24	20	0.40	0.568	0.518	–8.8	0.562	–1.1	0.551	–3.0
8	34	24	16	12	0.50	3.577	2.723	–23.9	–	–	–	–
9	34	24	16	12	0.80	1.251	1.165	–6.9	1.235	–1.3	1.227	–1.9
10	34	24	16	12	1.00	1.086	1.020	–6.1	1.054	–2.9	1.048	–3.5
11	34	24	16	16	1.00	1.497	1.397	–6.7	1.444	–3.5	1.438	–3.9
12	34	24	24	20	1.00	2.603	2.404	–7.6	2.534	–2.7	2.707	4.0
13	34	24	24	20	1.50	1.926	1.817	–5.7	1.835	–4.8	1.915	–0.6
14	34	24	24	20	2.00	1.722	1.634	–5.1	1.634	–5.1	1.697	–1.5
15	40	20	16	12	1.50	2.340	2.234	–4.5	2.099	–10.3	2.227	–4.8
16	40	20	16	12	2.00	2.062	1.976	–4.2	1.836	–11.0	1.931	–6.4
17	40	20	16	12	3.00	1.851	1.779	–3.9	1.644	–11.2	1.718	–7.2
18	40	20	16	16	3.00	2.625	2.517	–4.1	2.235	–14.9	2.363	–10.0
19	40	20	22	18	3.00	3.486	3.381	–3.0	2.934	–15.8	3.123	–10.4
20	40	20	22	18	5.00	3.115	3.030	–2.7	2.650	–14.9	2.801	–10.1
21	40	20	22	18	8.00	2.944	2.864	–2.7	2.512	–14.7	2.651	–10.0
22	54	24	16	12	1.50	1.725	1.662	–3.7	1.395	–19.1	1.580	–8.4
23	54	24	16	12	2.00	1.584	1.528	–3.5	1.280	–19.2	1.432	–9.6
24	54	24	16	16	2.00	1.922	1.852	–3.6	1.498	–22.1	1.693	–11.9

perature difference between inlet and outlet water may yield no results in extreme conditions, such as case 8. The process lines of water cooling in cooling tower for the two extreme cases are illustrated in fig. 3. (The enthalpy of inlet air and the temperature of inlet water are the same for case 4 and 8 according to tab. 1.)

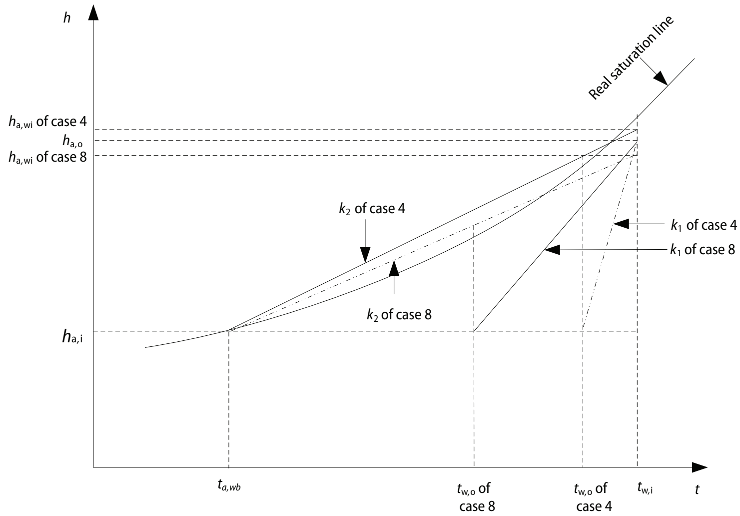

Figure 3. The process lines for water cooling in the cooling tower for case 4 and 8

For cases 15 to 24, the results of the proposed model are less accurate than Merkel model. These large errors are caused by the linearization assumption, which introduces large errors under large temperature difference between inlet and outlet water (larger than 20 °C). However, the results of the proposed model are more accurate than Halasz non-dimensional model for cases 15 to 24.

Results validation for the outlet water temperature

The outlet water temperature evaluated by the proposed model is compared with an accurate model under some typical operating conditions investigated by Ren [18]. For the accurate model, eqs. (29)-(31) are solved by the finite difference method. If the air becomes supersaturated, eqs. (29)-(31) should be replaced with eqs. (32)-(34).

$$\mathrm{d}w_\mathrm{a} = (W_{s,w} - W_\mathrm{a})\mathrm{d}NTU \tag{29}$$

$$\mathrm{d}t_\mathrm{a} = \left[\mathrm{Le} + \frac{c_{p,v}}{c_{p,\mathrm{a}}}(W_{s,w} - W_\mathrm{a})\right]\mathrm{d}NTU \tag{30}$$

$$\mathrm{d}t_\mathrm{w} = \frac{m_\mathrm{da} c_{p,\mathrm{a}}}{m_\mathrm{w} c_{p,\mathrm{w}}}\left\{\mathrm{Le}(t_\mathrm{w} - t_\mathrm{a}) + \left[h_{ba} + (c_{p,v} - c_{p,\mathrm{w}})t_\mathrm{w}\right]\frac{W_{s,w} - W_\mathrm{a}}{c_{p,\mathrm{a}}}\right\}\mathrm{d}NTU \tag{31}$$

$$\mathrm{d}w_\mathrm{a} = (W_{s,w} - W_{s,\mathrm{a}})\mathrm{d}NTU \tag{32}$$

where $W_{s,\mathrm{a}}$ is humidity ratio of saturated moist air at dry bulb temperature.

$$\mathrm{d}t_\mathrm{a} = \frac{\mathrm{Le}\, c_{p,\mathrm{a}}(t_\mathrm{w} - t_\mathrm{a}) + (h_{ba} + c_{p,v}t_\mathrm{w} - c_{p,\mathrm{w}}t_\mathrm{a})(W_{s,w} - W_{s,\mathrm{a}})}{c_{p,a} + (W_a - W_{s,a})c_{p,w}}\mathrm{d}NTU - \\ - \frac{h_{ba} + (c_{p,v} - c_{p,\mathrm{w}})t_\mathrm{a}}{c_{p,\mathrm{a}} + (W_\mathrm{a} - W_{s,a})c_{p,\mathrm{w}}}\mathrm{d}W_{s,a} \tag{33}$$

$$\mathrm{d}t_\mathrm{w} = \frac{m_\mathrm{da}c_{p,\mathrm{a}}}{m_\mathrm{w}c_{p,\mathrm{w}}}\left\{\mathrm{Le}(t_\mathrm{w} - t_\mathrm{a}) + \frac{\left[h_{ba} + (c_{p,v} - c_{p,\mathrm{w}})t_\mathrm{w}\right](W_{s,w} - W_{s,\mathrm{a}})}{c_{p,\mathrm{a}}}\right\}\mathrm{d}NTU \tag{34}$$

The slope of the assumed straight saturation line, k_2, in eq. (24), will depend on the outlet water temperature, therefore, some steps in iteration will be needed for the solutions. The $t^\mathrm{A}_\mathrm{w,o}$, $t^\mathrm{M}_\mathrm{w,o}$, and $t^\mathrm{P}_\mathrm{w,o}$ represents the outlet water temperature calculated by accurate model, Merkel model, and the proposed model. The relative errors of Merkel model and the proposed model are expressed as e^M_t and e^P_t, respectively. $\{e^\mathrm{X}_t = [(t^\mathrm{X}_\mathrm{w,o} - t^A_{w,o})/t^\mathrm{A}_\mathrm{w,o}]\cdot 100\%$, here X represents M or $P\}$

The results calculated by three models are listed in tab. 2. The absolute relative errors of the proposed model are below 5% except case 1. We can observe that the range of temperature between inlet water and outlet water is larger than 30 °C in the case, so the linearization for enthalpy of saturated air produces large error. As shown in tab. 2, the absolute relative errors of the proposed model increases as the temperature difference between inlet and outlet water increases.

Table 2. The comparison of outlet water temperature with different models

Case	$t_\mathrm{w,i}$	$t_\mathrm{a,i}$	$t_\mathrm{wb,i}$	$m_\mathrm{a}/m_\mathrm{w}$	NTU	$t^\mathrm{A}_\mathrm{w,o}$	$t^\mathrm{M}_\mathrm{w,o}$	e^M_t	$t^\mathrm{P}_\mathrm{w,o}$	e^P_t
1	60	35	20	1	3	25.93	25.02	–3.51	24.36	–6.05
2	30	35	20	1	3	22.86	22.64	–0.96	22.70	–0.70
3	40	7	–0.68	1	3	16.10	15.72	–2.36	16.62	3.23
4	40	35	30	1	3	31.19	31.03	–0.51	30.98	–0.67
5	40	25	20	1	3	24.56	24.22	–1.38	24.28	–1.14
6	40	35	20	1	3	20.72	20.88	0.77	20.41	–1.50
7	40	35	20	0.5	3	30.04	29.62	–1.40	30.02	–0.07
8	40	35	20	2	3	24.53	24.12	–1.67	24.24	–1.18
9	40	35	20	1	0.5	32.55	32.34	–0.65	32.28	–0.83
10	40	35	20	1	6	22.59	22.17	–1.86	22.29	–1.33

To compare the accuracy between accurate model and the proposed model in a relative narrow range, the regression analysis is conducted between $t^\mathrm{A}_\mathrm{w,o}$ and $t^\mathrm{P}_\mathrm{w,o}$ for cases 2-10. The regression analysis between $t^\mathrm{A}_\mathrm{w,o}$ and $t^\mathrm{P}_\mathrm{w,o}$ for cases 2-10 and a ±5% error band are also shown in fig. 4. According to fig. 4, the regression analysis yields an R^2 of 0.9980 and an RMSE of 0.29 °C for the proposed model. The results show the proposed model and accurate model show good agreement. The regression analysis also yields an R^2 of 0.9987 and an RMSE of 0.33 °C for Merkel model. The results also show good agreement between Merkel model and accurate model. However, as shown in the tab. 3, the steps in iteration for the proposed model are much less than that for Merkel model. The S^P and S^M represent the steps in iteration for the proposed model and Merkel model, respectively. (The initial value of the outlet water, $t_\mathrm{w,o}$, is set to $t_\mathrm{w,i} - 5$).

Application in thermal power plants

In this chapter, the proposed model is applied to calculate the performance of cooling tower used to discharge the waste heat to the environment in the power plant. Two cooling towers are chosen here. One is served for a 300 MW power plant while another is served for a 600 MW power plant. Table 4 shows the specification of the two towers. The e^{Ms} in tab. 4 is the absolute value of the difference value between outlet water temperature, $t_{w,o}^{Ms}$, measured and water outlet temperature, $t_{w,o}^{P}$, calculated by the proposed model, $e^{Ms} = |t_{w,o}^{Ms} - t_{w,o}^{P}|$.

For the 300 MW, the two working conditions are both full load. For the 600 MW, one working condition is full load, and the other one is partial load. As shown in the tab. 5, the errors of two cases for 300 MW are 0.15 °C and 0.26 °C, respectively. And the errors of two cases for 600 MW are 0.34 °C and 0.05 °C, respectively. Therefore, the results calculated by the proposed model are absolutely acceptable since the errors are less than 0.34 °C. The good results are due to that the errors caused by the linearization assumption are small since the temperature difference between the inlet and outlet water is less than 10 °C in our experiments.

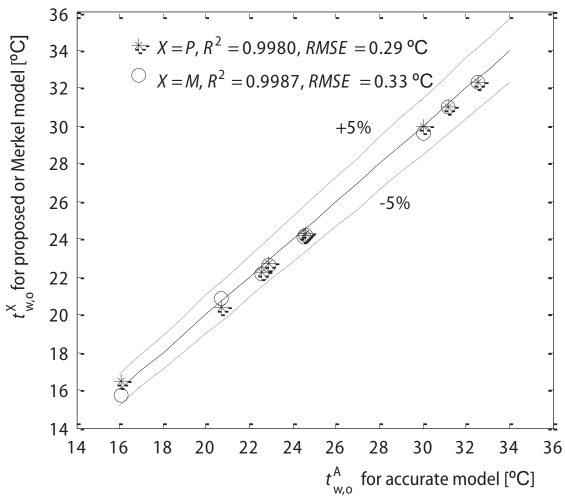

Figure 4. Comparison of outlet water temperature between accurate model and proposed (or Merkel) model

Table 3. The comparison of steps in iteration with the proposed model and Merkel model

Steps	1	2	3	4	5	6	7	8	9	10
S^P	6	3	7	3	5	3	4	5	4	5
S^M	3002	637	1932	398	1081	1411	537	1086	271	1281

Table 4. The specification of the cooling towers

	Total height	Tip diameter	Zero meter[1]	Water drenching area	Sprinkle density
Unit	m	m	m	m^2	$tm^{-2}h^{-1}$
300 MW	128	61.7	101.4	6000	6.48
600 MW	150	71.4	118.9	9000	8.46

[1] The diameter at the height of zero meter

Table 5. Validation of the model result

	m_w	$t_{w,i}$	m_a	$t_{a,i}$	$t_{a,wb}$	$t_{w,o}^{Ms}$	$t_{w,o}^{P}$	e^{Ms}
Unit	th^{-1}	°C	th^{-1}	°C	°C	°C	°C	°C
300 MW	38880	38.10	31025	30.75	22.90	30.06	29.91	0.15
	38880	41.40	30495	34.10	28.80	32.90	32.64	0.26
600 MW	77642	41.93	41528	35.70	26.90	34.35	34.01	0.34
	67322	40.94	45779	32.60	27.40	32.69	32.64	0.05

Conclusion

A simple and analytical model based on the assumption that enthalpy-temperature relation of saturated air is linear is proposed to evaluate the thermal performance of the counter flow wet cooling tower. In this model, the analytical expression of the outlet water temperature is available. The proposed analytical model is validated with Poppe [24] accurate model for Merkel number and Ren [18] accurate model for outlet water temperature. The results show that the proposed analytical model can yield acceptable accuracy within a relative narrow cooling range which is temperature difference between inlet and outlet water. Furthermore, when the proposed model is applied to calculate the performance of cooling towers used in power plants, its results are quite acceptable. Therefore, it can be applied to online calculation since it is easily to be solved.

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Nomenclature

A – area, [m^2]
A_{fr} – front area, [m^2]
a_{fi} – area per unit volume, [m^{2}m^{-3}]
c_p – specific heat at constant pressure, [kJkg^{-1}K^{-1}]
E – slope of tie line, [–]
e – relative error, [–]
h – specific enthalpy, [kJkg^{-1}]
$h_{a,s}$ – enthalpy of saturated moist air at water temperature, [kJkg^{-1}]
h_{ba} – latent heat of evaporation at 0 °C, [kJkg^{-1}]
h_w – specific enthalpy of water, [kJkg^{-1}]
$h_{v,s}$ – specific enthalpy of saturated water vapor at water temperature, [kJkg^{-1}]
k_D – mass transfer coefficient, [kgm^{-2}s^{-1}]
k_c – heat transfer coefficient, [Wm^{-2}K^{-1}]
Le – Lewis factor, [–]
m_da – dry air mass flow rate, [kgs^{-1}]
Me – Merkel number, (= $h_D A/m_\mathrm{w}$), [–]
NTU – number of transfer coefficient, (= $h_D A/m_\mathrm{a}$)
N_G – number of transfer units, [–]
Q – total heat transfer rate, [W]
Q_c – convective heat transfer rate, [W]
Q_v – evaporation heat transfer rate, [W]
R^2 – absolute fraction of variance
$RMSE$ – root mean square error
S – steps in iteration, [–]
t – temperature, [°C]
W_a – humidity ratio of moist air, [kgkg^{-1}]
$W_{s,w}$ – humidity ratio of saturated moist air at water temperature, [kgkg^{-1}]

Greek symbol

ε – cooling efficiency

Subscripts

a – air
da – dry air
i – inlet
o – outlet
v – vapor
w – water
wb – wet bulb

Superscripts

A – accurate model
B – Halasz non-dimensional model
M – Merkel model
Ms – measurement
P – proposed model

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