SENSITIVE ANALYSIS OF HARMONICS ON THE ASSESSMENT OF THERMAL DIFFUSIVITY OF BUILDING MATERIALS OF A PERIODIC STATE

by

Geoffrey PROMIS*, Omar DOUZANE, Haykel BEN HAMED, and Thierry LANGLET

Laboratory of Innovative Technology, University of Picardie Jules Verne, Amiens, France

Original scientific paper
https://doi.org/10.2298/TSCI150702144P

Thermal efficiency of buildings requires a perfect knowledge of the thermal properties of materials which compose building envelope. To this end, the use of reliable testing methods for thermal diffusivity measurements of building materials is fundamental. Currently, periodic methods are based on the exploitation of fundamental harmonic, resulting from Fourier series decomposition. The objective of this paper is to demonstrate that fundamental is necessary and sufficient in order to characterize the thermal diffusivity. For those purposes, a sensitive analysis of harmonic contributions from Fourier series is supported by a substantial experimental campaign. Moreover, some dissymmetrical tests were carried out in order to highlight the preponderant influence of fundamental, compared to other harmonics. The analysis of contributions of harmonics shows that the evolution of thermal diffusivity is very slightly dependent on the number of harmonics. Then, the exploitation of fundamental is necessary and sufficient, and the characterization test protocol is validated by experimental results and by the comparison with commonly accepted values for these kinds of materials.

Key words: thermal metrology, thermal diffusivity, periodic method, sensitive analysis, dissymmetrical test, building materials

Introduction

The overall thermal performance of buildings is currently a main issue in the preservation of our renewable resources. Thermal efficiency requires a perfect knowledge of the thermal properties of building materials. To these ends, the evaluation of thermal conductivity and diffusivity is fundamental to control characteristics of building envelope [1-3].

Experimental methods for characterizing thermal properties (thermal conductivity, diffusivity, effusivity, and heat capacity) can be grouped into two main categories: steady-state measurements and dynamic state measurements. Nevertheless, all these methods consist in exciting a sample by absorption of a signal source and in quantifying the thermal response on the surface of the sample (variation of temperature through space and time). Signal source can be of different kinds:

− Periodic: conventional methods involve the heating of the sample with a sinusoidal signal [4-7]. Photoacoustic methods consist in the generation of a sinusoidal signal with a mi-
crophone [8-10]. The evolution of temperatures on the surface of the sample occurs at the same frequency that the signal source. Measurements of absorbed energy allow assessing the thermal properties of the material. Photopyroelectric methods focus on sinusoidal thermal photogenerated waves where the thermal response of sample is detected by a pyroelectric sensor [11-14].

Step: with these methods, the evolution of unidirectional flow, generated by a sudden change in temperature, is measured on both sides of the sample [15]. Flow analysis leads to the thermal properties of the material. Nowadays, hot wire methods are the most common. The hot wire, crossed by an electric current, is used as heat source. Measurements of temporal evolution of temperatures allow the assessment of thermal conductivity. Different probes are used in transient plane source method: hot disk sensor [16-19], needle probes for modified transient plane source [20, 21].

Impulse: firstly proposed by Parker et al. [22], flash method consists in the irradiation of the sample by a short heat pulse produced by a light source. The evolution of temperatures on both lighted up side and back side leads to the thermophysical properties of material [23]. Thermography protocol can also be included in this category [24]. These methods involve the heating of the sample with a laser or halogen lamps. Then, the thermal response can be detected by infrared camera [25-28], by the analysis of resonance frequency of protons and the nuclear spin phase shift on the surface of the sample [29].

Inverse methods estimate the thermal properties of material through indirect characterization. These protocols focus on the research of the boundary conditions of the test, with the knowledge of signal source and thermal properties [30-32].

All these methods are generally based on solving the heat conduction equation in finite or semi-infinite mediums, eq. (1), and considering a homogeneous material.

$$\div \varphi = -C_p \frac{\partial T(x,t)}{\partial t}$$

Moreover, many uncertainties related to experimental measurements exist such as radiation heat loss effect, finite pulse time effect... [33]. The development of new methods and the improvement of existing ones are necessary, regarding all these uncertainties from characterization methodologies [34, 35].

Previous papers [4, 36, 37] focus on a periodic method to assess the thermal diffusivity of building materials. The objective of this paper is to show that an experimental device, using the same elements as the means of the standardized guarded hot plate, is suitable for conventional building materials. With the generation of a periodic signal source, it is demonstrated that the use of fundamental is necessary and sufficient to assess the thermal diffusivity. The thermal model, consisting in solving heat conduction equation, is detailed in [37]. The experimental protocol is briefly presented here, as well as the limitation of the signal processing which is a main step that generated uncertainties. Indeed, a Fourier series decomposition of fundamental harmonic allows the evaluation of the thermal diffusivity. An analysis of the exploitation of fundamental is supported by a substantial experimental campaign carried out on four current building materials ($0.03 < \lambda < 2$ W/mK). Finally, an inverse method is presented in order to obtain the thermal response of a sample from the signal source and its thermophysical properties.
Formulation of the thermal model

A previous paper [37] presents in detail the procedure used to solve the thermal problem, in finite medium and in steady-state. Equations (2) govern the proposed thermal model, fig. 1.

\[
\begin{align*}
&\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T(x,t)}{\partial x^2} \\
&T(0,t) = \text{constant} \\
&T(2e) = T_e + |T| \sin(\omega t)
\end{align*}
\]

In steady-state, with \( A(\eta, x) \) and \( \Gamma(\eta, x) \) the damping and the phase shift, respectively, the solution is given by the following expression:

\[
T(x, t) = A(\eta, x) |T| \sin[\omega t + \Gamma(\eta, x)]
\]

At the middle of the sample (for \( x = e \)), the solution is then given by:

\[
T(e, t) = A(\eta) |T| |T(e, t)| \sin[\omega t + \Gamma(\eta)]
\]

Introducing \( \eta = (2e)^{1/2} \left( \frac{\omega}{2a} \right)^{1/2} \), it follows that:

\[
\begin{align*}
A(\eta) &= \sqrt{\frac{1}{2(\cosh \eta + \cos \eta)}} \\
\Gamma(\eta) &= \arctan \frac{\tan \eta}{\tanh \eta} - \arctan \frac{\tan \frac{\eta}{2}}{\tanh \frac{\eta}{2}}
\end{align*}
\]

In steady-state, a harmonic analysis of the signal source and the thermal response of the sample are performed through a Fourier series. Coefficients \( \alpha \) and \( \beta \) are the coefficients of the trigonometric functions of the fundamental harmonic (order 1). Following expressions are obtained for the damping and for the phase shift:

\[
\begin{align*}
&\left\{ \begin{array}{l}
A(\eta) = |\phi| \\
\Gamma(\eta) = \Gamma_\iota - \Gamma_\s
\end{array} \right. \\
&\text{where}
\end{align*}
\]

\[
\begin{align*}
&\phi_\iota = \sqrt{\alpha^2 + \beta^2} \\
&\Gamma_\iota = -\arctan \frac{\alpha}{\beta}
\end{align*}
\]

Then, the value of \( \eta \) can be calculated by damping and by phase shift, and the thermal diffusivity can be expressed by eq. (8), where \( P \) is the period:

\[
a = \frac{(2e)^2 \pi}{\eta^2 P}
\]
This thermal model of solving heat conduction equation requires a Fourier series decomposition of the signal source and thermal response at the middle of the sample, in order to assess trigonometric functions coefficients $\alpha$ and $\beta$ of the fundamental, eq. (7).

\[
\begin{align*}
\alpha_j &= \frac{2\Delta t}{3P} \left[ g_j(t_i) + 4 \sum_{l \text{ pair}} g_j(t_i) + 2 \sum_{l \text{ imp}} g_j(t_i) + g_j(t_n) \right] \\
\beta_j &= \frac{2\Delta t}{3P} \left[ h_j(t_i) + 4 \sum_{l \text{ pair}} h_j(t_i) + 2 \sum_{l \text{ imp}} h_j(t_i) + h_j(t_n) \right] \\
&\quad + \int g_j(t_i) \cos \left( \frac{2\pi j t}{P} \right) dt \\
&\quad + \int h_j(t_i) \sin \left( \frac{2\pi j t}{P} \right) dt 
\end{align*}
\]  

Figure 2 shows the algorithm used to measure the thermal diffusivity from characterization tests. Coefficients $\alpha$ and $\beta$ are obtained through an integral calculation using Simpson’s method. This method, easily programmable, consists in grouping three consecutive points of the curve $T_i$ and replacing the arc of the curve by a parabolic arc, eq. (9):

Finally, this algorithm allows a comparison of parameter $\eta$ obtained by damping and phase shift. In the case of a significant discrepancy, parameters of the test (period, thermal signals) must be modified, regarding the derivative of upper temperature evolution.

**Sensitive analysis of the thermal diffusivity**

**Physical properties and thermal conductivity of the studied materials**

Table 1 shows the main physical properties of the four studied materials. Experimental thermal diffusivity apparatus requires two samples of 500 × 500 mm for each material.
All tests were performed on dried material (oven at 70 °C to constant weight). The formulation of concrete corresponds to an usual one: 880 kg/m³ of sand, 950 kg/m³ of crushed stone, 380 kg/m³ of cement, and finally 190 kg/m³ of water.

The TAURUS TLP 500-X1 apparatus allows the assessment of the thermal conductivity of materials. Samples are subjected to various temperatures ranging from 10-40 °C, following European standard ISO 8302 [38]. The four materials show a weak linear evolution of thermal conductivity against mean temperature of sample. As expected, polyurethane foam presents a low thermal conductivity at 10 °C (\(\lambda_{10,\text{dry}}\)), followed by beechwood, concrete and finally marble.

Table 1. Physical properties of the materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness [mm]</th>
<th>Density [kg/m³]</th>
<th>(\lambda_{10,\text{dry}}) [W/mK]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polyurethane foam</td>
<td>60</td>
<td>0.03</td>
<td>0.034 ± 0.0007</td>
</tr>
<tr>
<td>Marble</td>
<td>50</td>
<td>2.96</td>
<td>1.908 ± 0.04</td>
</tr>
<tr>
<td>Beechwood</td>
<td>40</td>
<td>0.73</td>
<td>0.169 ± 0.003</td>
</tr>
<tr>
<td>Concrete</td>
<td>70</td>
<td>2.21</td>
<td>1.601 ± 0.03</td>
</tr>
</tbody>
</table>

**Characterization of the thermal diffusivity**

**Experimental set-up**

Figure 3 shows a schematic diagram of the experimental apparatus. This device is composed by two expanded polystyrene sheets – 1, two exchanging sheets – 2 maintained at constant temperature through thermostated baths – 5, two samples of the material (500 × 500 mm) – 3, and a hot plate provided including a thermopile – 4. Finally, a tightening device of 16 to 20 kN is applied to limit the contact resistance between the various components of the set-up.

The hot plate consists in a square central zone of 250 × 250 mm and an external guarded ring of 500 × 500 mm. The hot plate generates a periodic calorific flow and the guarded ring allows an unidirectional incoming heat flow, controlled by the thermopile. This one is composed by 20 thermocouples connected in series, in an alternative arrangement between the central zone and the guarded ring. Temperature
discrepancy measured by the thermopile should be close to zero in order to ensure an unidirectional heat flow.

The apparatus is instrumented with 12 T-type thermocouples – 6 to record both signal source and thermal response, and to validate thermal model assumptions. Indeed, a thermocouple is placed on the upper side of sample to ensure the stability of upper temperature, and a thermocouple is placed between the hot plate and the lower exchanging sheet to measure the heat loss. Last 10 thermocouples are located on both sides of the lower sample. Then, signal source and thermal response are obtained with the arithmetic average of these two sets of thermocouples.

**Test parameters**

Approximate values of thermal diffusivity $a$ can be found in literature. From this first approach, the period of the tests can be defined to obtain a value of $\eta$ between 2.0 to 3.5 (see next comment). From eq. (8), it follows:

\[ P = \frac{(2e)^2\pi}{\eta^2a} \]  

(10)

Periods used for thermal diffusivity assessment range between 20 to 240 minutes, depending on the studied material. Moreover, tests were carried out at mean temperatures varying between 5 °C and 30 °C with a step of 5 °C.

Transcendental eqs. (5) are solved by dichotomy; an optimum accuracy is reached for $2.0 < \eta < 3.5$, where the curves of damping, $A(\eta)$, and phase shift, $\Gamma(\eta)$, are quasi-linear. More information are given by [4].

**Results and discussion**

A first set of tests was performed with thermostated baths temperature regulated at 15 °C. These tests lead to the establishment of an optimal period of each material, regarding the assessment of parameter $\eta$ by damping, $\eta_d$, and phase shift, $\eta_f$ [4]. Results of these tests and the deviation obtained for $\eta$ are summarized in tab. 2.

<table>
<thead>
<tr>
<th>Period [minute]</th>
<th>Polyurethane foam</th>
<th>Beechwood</th>
<th>Marble</th>
<th>Concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>120</td>
<td>180</td>
<td>240</td>
<td>20</td>
</tr>
<tr>
<td>$\eta_d$</td>
<td>3.12</td>
<td>2.56</td>
<td>3.33</td>
<td>2.89</td>
</tr>
<tr>
<td>$\eta_f$</td>
<td>3.16</td>
<td>3.81</td>
<td>3.14</td>
<td>3.42</td>
</tr>
<tr>
<td>$\Delta\eta$</td>
<td>–1%</td>
<td>–49%</td>
<td>6%</td>
<td>–18%</td>
</tr>
</tbody>
</table>

The optimal period corresponds to a value of $\Delta\eta$ close to zero. For polyurethane foam, beechwood, and marble, chosen periods are 80, 180, and 60 minutes, respectively. Concrete shows a significant deviation $\Delta\eta$ for 80 and 120 minutes periods, but the sign of $\Delta\eta$ indicates that the optimal period lied between these two periods. This first step allows the characterization of thermal diffusivity against mean temperature of samples. This first experimental campaign highlights that the parameter $\eta$ must be close to 3.18±0.1 in order to obtain the weakest deviation between diffusivity assessed by damping and by phase shift (linear interpolation). A design optimal period $P_d^*$ can expressed by eq. (11), where $\eta_d = 3.18$: 

\[ P_d^* = \frac{(2e)^2\pi}{\eta_d^2a} \]
\[ P_d = \frac{(2e)^2 \pi}{\eta_d a} \quad (11) \]

Moreover, these results show that damping and phase shift cannot be considered as intrinsic parameters of materials. With the design optimal period, damping and phase shift are constant for all materials: \( A = 22.5 \pm 1.5\% \), and \( I = 93.3 \pm 3.8^\circ \).

This first step allows the assessment of thermal diffusivity against mean temperature of the sample, fig. 4. Hatched areas correspond to the value of common thermal diffusivity, estimated from literature. Weak deviations between experimental values and literature validate the proposed thermal model. These results are based on a Fourier series decomposition of the thermal signals resulting from magnitude and phase of fundamental from experimental signals.

Figure 4. Evolution of the thermal diffusivity of materials against mean temperature

**Influence of the number of harmonics on the establishment of the thermal diffusivity**

**Analysis of the thermal signals by Fourier series**

Figure 5 shows an example of superposition of thermal signals and Fourier series (test carried out on polyurethane foam, \( P = 80 \) minutes and \( T_{\text{mean}} = 18^\circ \text{C} \)). The first four harmonics seem to be sufficient to model experimental signals. Nevertheless, this comment needs to be confirm by the assessment of the contribution of each harmonic. The thermal response, very
close to a sinusoidal curve, seems to only need fundamental to be modelled. Even harmonics are superimposed with harmonics of rank \((n - 1)\), their contribution is negligible.

Harmonic contributions

To highlight the preponderant influence of fundamental on the thermal diffusivity, line spectra are drawn for each thermal signal and each material. These spectra allow the assessment of the \(n^{th}\) first harmonics contributions. The magnitude of harmonic is expressed by eq. (12):

\[
\theta_j = \sqrt{\alpha_j^2 + \beta_j^2}
\]  

Finally, the harmonic contribution with respect to fundamental is expressed by:

\[
C_j = \frac{\theta_j}{\theta_1}
\]  

For signal source and thermal response, fig. 6 shows a drop of contributions of two and more order harmonics, followed by a progressive decrease. As highlighted by fig. 5, thermal response is very close analysis can be brought for the signal source: a drop of contributions between fundamental and 3-order harmonic is found. The 3-order harmonics represent contributions which lie between 10% to 20%, while upper harmonic contributions are less than 9% for beechwood and approximatively 5% for polyurethane foam, marble, and concrete.

Contributions of even harmonics are negligible compared to odd harmonics. The appearance of these harmonics can explained this. The signal, which is close to sinusoidal curve, has extrema for 1/4 and 3/4 of period. These extrema are represented on fig. 7 by the fundamental (fig. 7 shows even and odd harmonics for polyurethane foam testing, \(P = 80\) minutes and \(T^{\circ}_{\text{mean}} = 18\) °C). For odd harmonics, these extrema perfectly coincide while even harmonics extrema involve a phase shift.

Discussion of the results

The assessment of thermal diffusivity taking into account the contribution of harmonics is possible through damping. Indeed, with thermal signals which are modeled with Fourier series, the magnitude of signal is expressed by the following equation:

\[
\theta_j = \frac{T^{\circ}_{j,\text{max}} - T^{\circ}_{j,\text{min}}}{2}
\]
Moreover, the phase shift between thermal solicitation and thermal response can be graphically assessed, and thus can lead to the thermal diffusivity. Figure 8 presents the evolution of thermal diffusivity against the first six odd harmonics. As it can be see, the variation of thermal diffusivity over harmonics is very weak. The contribution of the 3-order harmonic, which is the most important of the whole harmonics, do not lead to a significant variation of the thermal diffusivity (less than 5%). This analysis tends to confirm the interest of fundamental for the assessment of thermal diffusivity through periodic state characterization.
Dissymmetrical tests

To confirm the preponderant influence of fundamental in the assessment of thermal diffusivity, some dissymmetrical tests were carried out on the four materials. Then, the purpose of these tests is to experimentally validate that dissymmetrical thermal signal modelled by fundamental lead to the same thermal diffusivity than the value obtained with symmetrical test. These tests were performed with the same periods which was used previously. Nevertheless, periods are dissymmetric: periods are composed by heating and cooling phases which are not similar. For polyurethane foam, the 80 minutes periods consists in a 50 minutes heating phase and a 30 minutes cooling phase. For each dissymmetrical period, an opposite period is applied on another test. In the same way, beechwood, marble and concrete tests were carried out with 120/60 minutes period, 40/20 minutes period and 50/30 minutes period, respectively. Exchanging sheets were controlled to 15 °C. As expected, signal source and thermal response are strongly dissymmetrical (see figure 9 which shows polyurethane foam tests). Thermal signals for a longer heating phase are presented on the left of fig. 9, while thermal signals for shorter heating phases are shown in the right.

As expected, fundamentals are not perfectly similar to experimental signals, regarding its dissymmetrical appearance. This deviation is less significant for thermal response. The analysis of the harmonic magnitudes is necessary to validate the exploitation of fundamental.
Nevertheless, the first four harmonics are sufficient to model experimental signals. Figure 10 shows line spectra of the first twelve harmonics, from dissymmetrical tests where the heating phase is shorter. Results concerning periods where heating phase is longer are not presented here to optimize the comprehensibility of spectra. No additional significant results are observed for these tests. For all tests, harmonic magnitudes drop between fundamental and 2-order harmonic, followed by regular and progressive decrease of magnitude.

Contrary to symmetrical tests, even harmonic contributions are not negligible. Indeed, 2-order harmonics show contributions which lie between 15% and 37%. Following harmonics lead to contributions less than 10%. Table 3 summarizes the results of thermal diffusivity obtained by the exploitation of fundamental. As it was done for symmetrical tests, an average of thermal diffusivity from damping and phase shift is realised and compared to previous results.

Thermal diffusivity is very close to the previous results. The discrepancy is less than 3% for polyurethane foam, beechwood and concrete, and less than 9% for marble. With this comparison, the fundamental is necessary and sufficient to characterize thermal diffusivity in periodic state, with symmetrical and dissymmetrical period.
This paper focuses on a sensitive analysis of the influence harmonics to assess the thermal diffusivity of building materials in a periodic state. In a first part, the thermal model is briefly presented, following by the thermophysical characterization of four building materials (polyurethane foam, beechwood, marble and concrete). The thermal conductivity measurements show weak linear evolution against mean temperature of sample. The thermal diffusivity of these materials is obtained by the exploitation of damping and phase shift of fundamental harmonics, resulting from a Fourier series decomposition of the experimental thermal signals. After establishing the optimal periods for each tested materials, the thermal diffusivity was assessed. For the four materials, the results are in accordance with those from the literature. Moreover, the preponderant influence of fundamental is highlighted through the evaluation of harmonic contributions. This paper shows that the fundamental is necessary and sufficient to establish the thermal diffusivity, regarding the weak evolution of thermal diffusivity against harmonics (less than 5%). This observation allows to carry out some dissymmetrical tests, where heating and cooling phases are dissymmetrical. In this case, the exploitation of damping and phase shift of fundamental leads to the assessment of thermal diffusivity which shows no significant deviation with symmetrical tests.

**Nomenclature**

- **\( a \)** – thermal diffusivity, \([\text{m}^2\text{s}^{-1}]\)
- **\( C \)** – contribution of harmonic
- **\( C_p \)** – calorific capacity \([\text{WK}^{-1}\text{m}^{-2}]\)
- **\( e \)** – thickness of the sample, \([\text{m}]\)
- **\( g, h \)** – functions of integral calculation
  - Simpson’s method
- **\( P_\text{o} \)** – optimal period, \([\text{s}]\)
- **\( T \)** – temperature, \([\text{K}]\)
- **\( t \)** – time, \([\text{s}]\)
- **\( x \)** – abscissa, \([\text{m}]\)

**Greek symbol**

- **\( \alpha, \beta \)** – coefficients of the trigonometric function of the Fourier series
- **\( \Gamma \)** – phase shift
- **\( A \)** – damping
- **\( \lambda \)** – thermal conductivity, \([\text{Wm}^{-1}\text{K}^{-1}]\)
- **\( \theta \)** – amplitude of the temperature signal, \([\text{°C}]\)
- **\( \phi \)** – heat flow, \([\text{Wm}^{-2}]\)
- **\( \eta \)** – dimensionless parameter defined in 5
- **\( \omega \)** – pulsation of the signal \([\text{s}^{-1}]\)

**Subscripts**

- **\( \text{i} \)** – experimental data
- **\( j \)** – number of harmonic
- **\( r \)** – thermal response
- **\( s \)** – signal source

**Table 3. Comparison of measured thermal diffusivity**

<table>
<thead>
<tr>
<th>Period</th>
<th>Diffusivity ( \times 10^{-7}) m(^2)/s</th>
<th>Average</th>
<th>Period</th>
<th>Diffusivity ( \times 10^{-7}) m(^2)/s</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polyurethane foam</td>
<td>From ( \Lambda(\eta) )</td>
<td>From ( \Gamma(\eta) )</td>
<td></td>
<td>From ( \Lambda(\eta) )</td>
<td>From ( \Gamma(\eta) )</td>
</tr>
<tr>
<td>80</td>
<td>9.68</td>
<td>9.43</td>
<td>9.56</td>
<td>60</td>
<td>8.90</td>
</tr>
<tr>
<td>50+30</td>
<td>9.66</td>
<td>9.55</td>
<td>9.61</td>
<td>40+20</td>
<td>9.60</td>
</tr>
<tr>
<td>30+50</td>
<td>9.44</td>
<td>9.44</td>
<td>9.44</td>
<td>20+40</td>
<td>8.92</td>
</tr>
<tr>
<td>Beechwood</td>
<td>1.69</td>
<td>1.89</td>
<td>1.79</td>
<td>80</td>
<td>12.67</td>
</tr>
<tr>
<td>120+60</td>
<td>1.71</td>
<td>1.89</td>
<td>1.80</td>
<td>50+30</td>
<td>12.84</td>
</tr>
<tr>
<td>60+120</td>
<td>1.70</td>
<td>1.89</td>
<td>1.79</td>
<td>30+50</td>
<td>12.87</td>
</tr>
<tr>
<td>Marble</td>
<td>9.43</td>
<td>9.43</td>
<td>9.44</td>
<td>60</td>
<td>8.83</td>
</tr>
<tr>
<td>Concrete</td>
<td>9.66</td>
<td>9.55</td>
<td>9.61</td>
<td>40+20</td>
<td>9.60</td>
</tr>
<tr>
<td>30+50</td>
<td>9.44</td>
<td>9.44</td>
<td>9.44</td>
<td>20+40</td>
<td>8.92</td>
</tr>
<tr>
<td>80</td>
<td>9.68</td>
<td>9.43</td>
<td>9.56</td>
<td>60</td>
<td>8.90</td>
</tr>
<tr>
<td>50+30</td>
<td>9.66</td>
<td>9.55</td>
<td>9.61</td>
<td>40+20</td>
<td>9.60</td>
</tr>
<tr>
<td>30+50</td>
<td>9.44</td>
<td>9.44</td>
<td>9.44</td>
<td>20+40</td>
<td>8.92</td>
</tr>
</tbody>
</table>
References


