THE USE OF RELATIVE INVERSE THERMAL ADMITTANCE FOR THE CHARACTERIZATION AND OPTIMIZATION OF FIN-WALL ASSEMBLIES

by

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The concept of relative inverse thermal admittance applied to the convective fin-wall assembly optimization of longitudinal rectangular fins under 2-D heat conduction is presented in this work. Since heat transfer at the fin tip is taken into account, it is not always possible to optimize the previous cited geometry. This is relevant in optimization processes and because of this has been displayed in several graphs. Here, different values for convective conditions at the fin and wall surfaces are used and the influence of the h_w/h_f *ratio in optimum geometry is determined. The fin effectiveness is used as the fundamental parameter to prove that the fin is fulfilling the objective of increasing heat dissipation. Once the optimum thickness has been obtained, the Biot number is easily calculated and the fin effectiveness for an isolated fin and the fin-wall assembly can be determined graphically. The optimization process is carried out through a set of universal graphs in which the range of parameters covers most of the practical cases a designer will find. The concept of relative inverse thermal admittance is applied in a general form and emerges as an easy used tool for optimizing fin-wall assemblies.*

Key words: *optimization, fin-wall assembly, relative inverse thermal admittance*

Introduction

Many papers have been published on fin-wall assembly characterization and optimization, using different wall and fin geometries and under a variety of boundary conditions. The main difficulty involved in such studies is the large number of geometrical and thermal parameters that must be taken into account for a detailed and complete analysis. In this work, we focus on the influence of the ratio between the heat transfer coefficients of the naked wall and fin for the optimization of these assemblies using the concept of relative inverse admittance Alarcon *et al*. [1] and Luna-Abad and Alhama [2]. Heat transfer at the fin tip is assumed even though this prevents finding the optimal geometry in some cases, as mentioned by several authors, including, Yeh [3], and Chung *et al*. [4].

With regards to optimization, it is worth to mention several papers. Bar-Cohen [5] use analytical 1-D models for the design of longitudinal fin thickness in fin arrays under natu-

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ral and forced convection conditions. Aziz [6], fins with rectangular and triangular profile are optimized under convection boundary conditions and adiabatic tip fin. This work is based on the assumption of unknown base temperature, unlike others works. Aziz and Kraus [7] have optimized several fin profiles under radiation and convection boundary conditions. In this work other early results are used. They are used 1-D conduction models and different ambient temperatures are taken into account for radiation and convection-radiation. Mutual radiation between fins and base are considered. Emissivity and conductivity are temperature dependent. Yeh [8] compared the results of optimum finned surfaces with longitudinal rectangular fins obtained through 1-D and 2-D models, assuming heat transfer at the fin tips. Alarcon *et al*. [9] used the network method to optimize rectangular fin-wall assemblies under different geometrical and thermal conditions.

In the present work, the performance coefficient *relative inverse thermal admittance*, already used in the optimization of rectangular fins by Luna-Abad *et al*. [10], is now applied to the optimization of the fin-wall assembly under convection boundary conditions, using the network method as a numerical tool, Gonzalez-Fernandez [11] and Luna-Abad [12]. Relative inverse thermal admittance has been demonstrated to be a suitable coefficient for characterizing these assemblies since it satisfies the requirements of a universal performance coefficient, Alarcon *et al*. [1]. Here, optimized universal curves based on this parameter are obtained for the first time, these curves, formed of optimal points and calculated by 2-D models, can be interpreted and managed directly. The parameters needed to start the design are the fin volume and the *convection coefficient/conductivity ratio*, although it is possible to use other pairs of parameters such as fin volume and effectiveness, or fin transversal Biot number and effectiveness. The quasi-universal optimization curves obtained cover the wide range of thermal and geometrical parameters of real sets. The influence of the ratio between heat transfer coefficients of wall and fin is also studied. The optimization protocols proposed are direct and the numerical solution obtained by network method makes the errors negligible even for configurations that work under 2-D conditions. The range of parameters is so wide that fin effectiveness reaches values close to unity and permits us to obtain the limit values of the Biot number. We use the effectiveness as defined by Razelos [13], for fin-wall assemblies, not as is commonly used for isolated fins.

To streamline the graphical presentation of the results, the $h_\mathrm{w}/h_\mathrm{f}$ ratio is partitioned into three intervals. Finally, examples of fin-wall optimization are explained to help the reader understand optimization based on inverse relative admittance.

Relative inverse thermal admittance

This parameter was introduced by Alarcon *et al*. [1, 9] as a dimensionless, universal performance coefficient suitable for the analysis and design of fins. Its evaluation requires knowledge of the specific inverse admittance, y_r, and the specific inverse admittance of optimal fins with different shapes (rectangular, annular, spine, *etc*.) and/or several materials, $y_\mathrm{r,opt}$. For brevity, the term *thermal* will be suppressed hereinafter. For a given type of fin, the relative inverse admittance, y_rel, is defined as the ratio $y_\mathrm{r}/y_\mathrm{r,opt}$. The specific inverse admittance refers to the mass (or volume) unity of a particular fin and is equal to the ratio between the total heat dissipated to the surrounding fluid (or transferred to other elements) and the temperature difference between the fin base and the surrounding fluid:

$$y_\mathrm{r} = \frac{q_f}{(T_b - T_\infty)V_f} \tag{1}$$

The fin geometry, for which the specific inverse admittance reaches a maximum, immediately provides the value of $y_{\mathrm{r,opt}}$. In contrast with classical performance parameters, such as efficiency, effectiveness, and input admittance, relative inverse admittance is a coherent parameter for fin design, [1-2, 10, 12]. However, since this work refers to fin-wall assemblies, relative inverse admittance is defined as the ratio between heat transferred by a real fin-wall assembly and the maximum heat that would be transferred for a fin-wall assembly of different geometry but maintaining the total volume of the fin and the volume of the wall.

The 2-D mathematical model

Assuming, (1) the convection is very high at the primary surface, (2) the conductivity has the same value for the fin and wall, and (3) the heat transfer coefficient is the same for the fin surface and fin tip, the problem is governed by the equations:

– fin

$$\frac{\partial^2 \Phi_{\mathrm{f}}}{\partial z^2} + \frac{\partial^2 \Phi_{\mathrm{f}}}{\partial y^2} = 0, \quad 0 < y < e, \quad w < z < w + L \tag{2}$$

$$-k\frac{\partial \Phi_{\mathrm{f}}}{\partial y} = h_{\mathrm{f}}\Phi_{\mathrm{f}}, \quad y = e, \quad w < z < w + L \tag{3}$$

$$\frac{\partial \Phi_{\mathrm{f}}}{\partial y} = 0, \quad y = 0, \quad w < z < w + L \tag{4}$$

$$T_{\mathrm{w}} = T_{\mathrm{f}}, \quad 0 < y < e, \quad z = w \tag{5}$$

$$-k\frac{\partial \Phi_{\mathrm{f}}}{\partial z} = -k\frac{\partial \Phi_{\mathrm{w}}}{\partial z}, \quad 0 < y < e, \quad z = w \tag{6}$$

$$-k\frac{\partial \Phi_{\mathrm{f}}}{\partial z} = h_{\mathrm{f}}\Phi_{\mathrm{f}}, \quad 0 < y < e, \quad z = w + L \tag{7}$$

– wall

$$\frac{\partial^2 \Phi_{\mathrm{w}}}{\partial z^2} + \frac{\partial^2 \Phi_{\mathrm{w}}}{\partial y^2} = 0, \quad 0 < y < b, \quad 0 < z < w \tag{8}$$

$$-k\frac{\partial \Phi_{\mathrm{w}}}{\partial z} = h_{\mathrm{w}}\Phi_{\mathrm{w}}, \quad e < y < b, \quad z = w \tag{9}$$

$$\frac{\partial \Phi_{\mathrm{w}}}{\partial y} = 0, \quad y = 0, \quad 0 < z < w \quad \text{and} \quad y = b, \quad 0 < z < w \tag{10}$$

$$\Phi_{\mathrm{w}} = 1, \quad 0 < y < b, \quad z = 0 \tag{11}$$

These equations are numerically solved by network simulation method, a numerical technique that has been successfully applied in a variety of linear and non-linear problems [14, 15]. Geometrical and thermal parameters for fin-wall assembly are shown in fig. 1.

Results and discussion – influence of $h_{\mathrm{w}}/h_{\mathrm{f}}$

The basic parameter whose influence in the optimization processes will be analyzed is the ratio between the wall and fin heat transfer coefficients. The question to be addressed

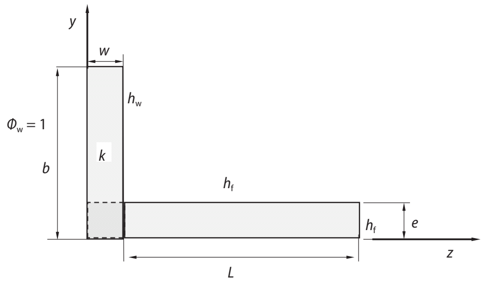

Figure 1. Fin-wall assembly: geometrical and thermal parameters

is: *for a given volume of material*, what are the dimensions that will maximize the heat dissipated from a fin-wall assembly taking into consideration the assumptions mentioned previously. The wall thickness of the assembly is maintained constant. Due to the amount of information that has to be managed, universal curves are obtained for three typical values of the ratio h_w/h_f.

Case 1, $h_w/h_f = 1$

This is the ratio considered in most works related with fin-wall assembly characterization and optimization. The wall height and width are considered as constant, and take 0.01 and 0.001 m., respectively. To cover a significant spectrum of real values, volume parameter (fin volume), V_f, extends from 1E–6 to 1E–4 m^3, while the h_f/k ratio, another input parameter for the design, runs from 0.1 to 1E3 m^{-1}. Since wall height and wall thickness are assumed to be constant parameters, a change in the volume of the fin-wall assembly is equivalent to a change in the volume of the fin without wall. Optimum values for fin thickness are first obtained by running a large quantity of assemblies for a large set of pairs of values [V_f – (h_f/k)]. The results are the optimum points represented in fig. 2, using the fin volume as parameter, or, alternatively, in figs. 3 and 4, using h_f/k as parameter. Note that the relative maximum that appears in these curves, fig. 2, moves towards the right and downwards, as fin volume decreases. For h_f/k values of the order

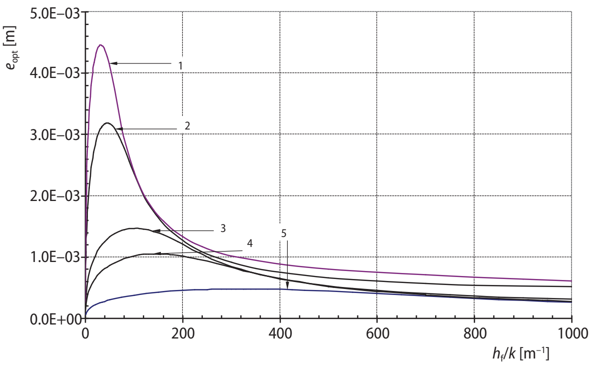

Figure 2. Optimum thickness *vs.* h_f/k ratio with fin volume as parameter: (1) V_f = 1E–4 m³, (2) V_f = 5E–5 m³, (3) V_f = 1E–5 m³, (4) V_f = 5E–6 m³, (5) V_f = 1E–6 m³, and h_w/h_f = 1

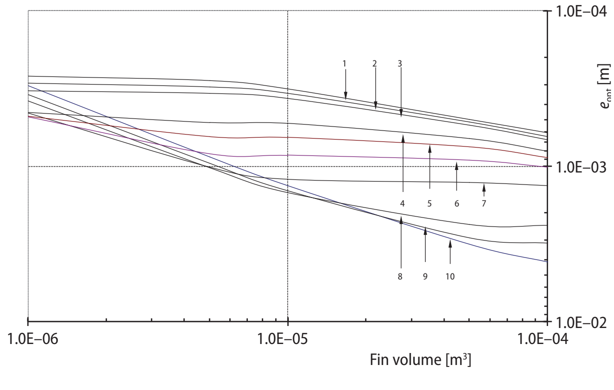

Figure 3. Optimum thickness *vs.* fin volume with h_f/k ratio as parameter: (1) h_f/k = 1000 m^{-1}, (2) h_f/k = 900 m^{-1}, (3) h_f/k = 800 m^{-1}, (4) h_f/k = 500 m^{-1}, (5) h_f/k = 400 m^{-1}, (6) h_f/k = 300 m^{-1}, (7) h_f/k = 200 m^{-1}, (8) h_f/k = 100 m^{-1}, (9) h_f/k = 75 m^{-1}, (10) h_f/k = 50 m^{-1}, and h_w/h_f = 1

of 10 m^{-1} or less, curves of e_{opt} as a function of volume are straight lines in a logarithmic scale, fig. 4, while for higher values of h_f/k, fig. 3, curves separate into two or three straight sections for the whole range of volumes. Note that these curves are formed by optimum points.

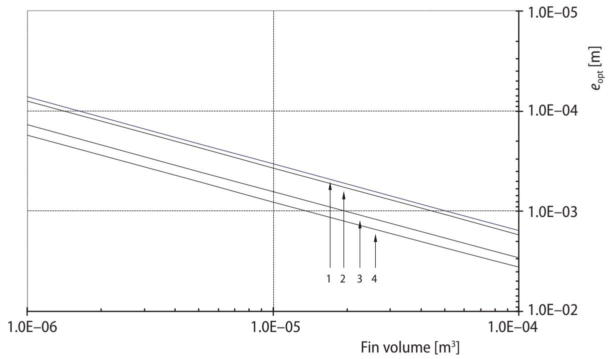

Figure 4. Optimum thickness *vs.* fin volume with h_f/k ratio as parameter: (1) h_f/k = 0.75 m^{-1}, (2) h_f/k = 1 m^{-1}, (3) h_f/k = 5 m^{-1}, (4) h_f/k = 10 m^{-1}, and h_w/h_f = 1

The existence of a maximum in fig. 2 means that, for two different materials and convection conditions (two different h_f/k ratios), the same geometry, *i. e*. the same value of thickness, provides an optimum fin-wall assembly, this is interesting information when the input parameter for the optimization is the thickness of the fin, [8]. Given that the value of h_f/k falls with the Biot number and, in turn, with increasing effectiveness, the choice will always be a low h_f/k ratio, which means working within the region of the curves of fig. 2 located to the left of the maximum. In the same figure, note that in a ranges of 70-200, 200-400, and 500-1000 m^{-1} for the h_f/k ratio and 1E–4 to 5E–5, 1E–5 to 5E–5, and 5E–5 to 1E–6 m^3, for the fin volume, the optimum thickness hardly varies. The effectiveness versus $Bi_{t,f}$, with the fin volume as parameter, is shown in fig. 5. Here, a minimum value of 2 for fin effectiveness has been imposed, meaning that, limiting values of the h_f/k ratio, e_{opt}, $Bi_{t,f}$ are presented in tab. 1.

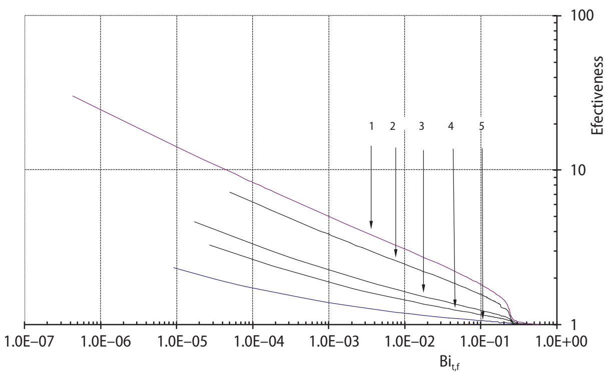

Figure 5. Effectiveness of fin-wall assembly *vs.* $Bi_{t,f}$: (1) V_f = 1E–6 m^3, (2) V_f = 5E–6 m^3, (3) V_f = 1E–5 m^3, (4) V_f = 5E–5 m^3, and (5) V_f = 1E–4 m^3

Table 1. Limit values of optimum points for a fin effectiveness 2

Fin volume [m^3]	h_f/k [m^{-1}]	e_{opt} [m]	$Bi_{t,f}$
1.E–06	219.8	4.63E–04	1.02E–01
5.E–06	130.42	1.05E–03	1.37E–01
1.E–05	103.57	1.47E–03	1.52E–01
5.E–05	59.193	3.11E–03	1.79E–01
1.E–04	45.983	4.23E–03	1.95E–01

In contrast, when the effectiveness is defined for the fin-wall assembly (heat dissipated by the fin-wall assembly and heat dissipated by the naked wall without fin, [13]) and the classical definition of the Biot number is used, the curves are clearly different for each volume in the whole range of values of $Bi_{t,f}$, fig. 5.

Note that values of fin-wall assembly effectiveness are much lower than those of the fin effectiveness. Several optimum examples of fin-wall assembly are shown in tab. 2 for a given wall height and thickness.

Table 2. Optimum fin-wall assembly for different fin volumes and h_f/k ratios: $h_f = h_w$, $b = 0.01$ m, and $w = 0.001$ m

h_f/k [m^{-1}]	Fin volume [m^3]	e_{opt} [m]	$Bi_{t,f}$ $(=h_f e_{opt}/k)$	Effectiveness
1	1.E–06	8.00E–05	8.00E–05	1.77
	5.E–06	2.33E–04	2.33E–04	2.30
	1.E–05	3.70E–04	3.70E–04	2.64
	5.E–05	1.09E–03	1.09E–03	3.77
	1.E–04	1.72E–03	1.72E–03	4.47
100	1.E–06	3.79E–04	3.79E–02	1.11
	5.E–06	1.02E–03	1.02E–01	1.16
	1.E–05	1.47E–03	1.47E–01	1.18
	5.E–05	2.36E–03	2.31E–01	1.20
	1.E–04	2.39E–03	2.39E–01	1.20

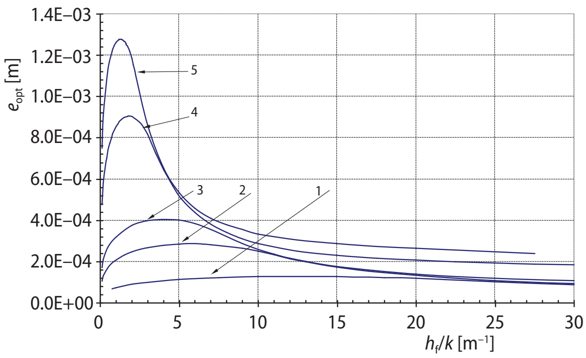

Figure 6. Optimum thickness *vs.* h_f/k ratio with fin volume as parameter: (1) V_f = 1E–4 m^3, (2) V_f = 5E–5 m^3, (3) V_f = 1E–5 m^3, (4) V_f = 5E–6 m^3, (5) V_f = 1E–6 m^3, and h_w/h_f = 10

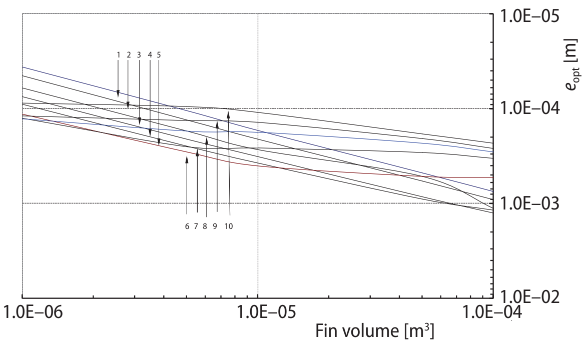

Figure 7. Optimum thickness *vs.* fin volume, with h_f/k ratio as a parameter: (1) h_f/k = 0.1 m^{-1}, (2) h_f/k = 0.2 m^{-1}, (3) h_f/k = 0.5 m^{-1}, (4) h_f/k = 1 m^{-1}, (5) h_f/k = 2 m^{-1}, (6) h_f/k = 5 m^{-1}, (7) h_f/k = 10 m^{-1}, (8) h_f/k = 15 m^{-1}, (9) h_f/k = 20 m^{-1}, (10) h_f/k = 30 m^{-1}, and h_w/h_f = 10

Case 2, $h_w/h_f > 1$

For this case, the optimum thickness of the fin as a function of the h_f/k ratio, with the volume as parameter, is shown in fig. 6. An h_w/h_f ratio of 10 has been chosen. Again, although a maximum appears in these curves, all the points are optimal, however, the points located to left of the maximum have higher effectiveness than those located towards the right, for the same optimum thickness, and it may not be suitable to use fins. This graph alone or, alternatively, the optimum thickness as a function of the fin volume with h_f/k as parameter, fig. 7, is sufficient to easily carry out the optimization as mentioned in the former case. Finally, fin-wall assembly effectiveness *vs.* $Bi_{t,f}$ with fin volume as parameter is represented in fig. 8 in order to ensure that the fin-wall assembly is suitable.

Case 3, $h_w/h_f < 1$

As is known, the value of h which is generally much lower at the fin base than at the fin surface or tip, depends on the fluid velocity, which differs on the wall to which the fin is attached, on the surface of the fin and at the tip, depending on fin orientation with respect to the fluid and whether there is a laminar or turbulent flow or whether the fin is subjected to natural or forced convection. So, it is reasonable to take a higher value for h_f than for h_w. As in the previous cases, optimum thickness is plotted against the h_f/k ratio using fin volume as parameter and for an h_w/h_f ratio of 0.5. For this value, a region where it is not possible to find an optimum geometry emerges when heat at the fin tip is considered, fig. 9. Notice that in early cases, in the range of chosen values, this does not occur. For h_f/k values to the left of the mentioned region, effectiveness values are higher

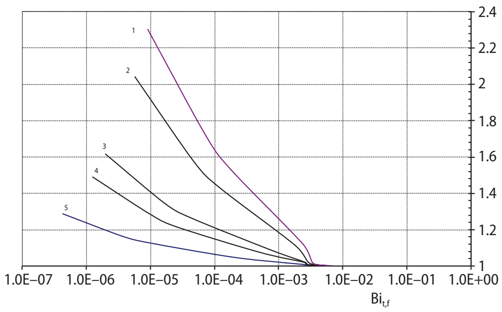

Figure 8. Effectiveness of fin-wall assembly *vs.* Bi$_{t,f}$ with volume as parameter: (1) V_f = 1E–4 m^3, (2) V_f = 5E–5 m^3, (3) V_f = 1E–5 m^3, (4) V_f = 5E–6 m^3, (5) V_f = 1E–6 m^3, and h_w/h_f = 10

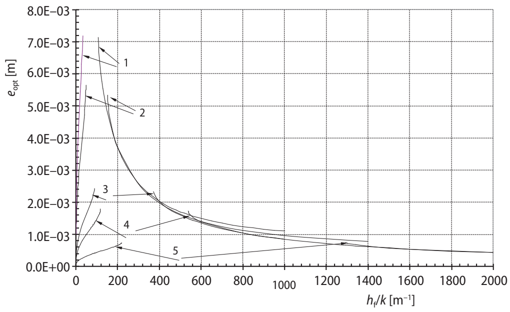

Figure 9. Optimum thickness as a function of h_f/k ratio using fin volume as parameter: (1) V_f = 1E-4 m^3, (2) V_f = 5E-5 m^3, (3) V_f = 1E-5 m^3, (4) V_f = 5E-6 m^3, (5) V_f = 1E-6 m^3, and h_w/h_f = 0.5

than the values at the right side. Figure 10 provides the same information as fig. 9 using h_f/k as parameter, these curves can be accurately adjusted by means of potential functions.

Finally, fig. 11, which represents fin-wall assembly effectiveness as a function of Bi$_{t,f}$ using fin volume as parameter, shows, on the one hand, a blank region that appears at the center of the range of Bi$_{t,f}$ and, on the other hand, that the effectiveness values are sufficiently low towards the right to assume 2-D conduction in this range. Limit values of the design parameters that define the blank region are shown in tab. 3.

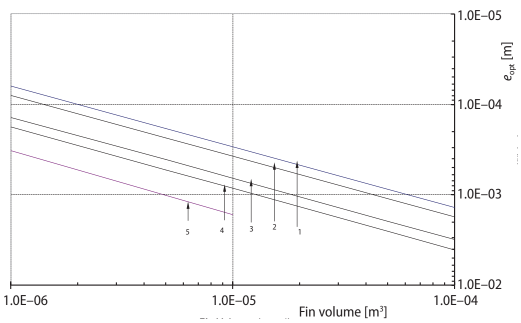

Figure 10. Optimum thickness *vs.* fin volume, with h_f/k ratio as a parameter: (1) h_f/k = 0.5 m^{-1}, (2) h_f/k = 1 m^{-1}, (3) h_f/k = 5 m^{-1}, (4) h_f/k = 10 m^{-1}, (5) h_f/k = 50 m^{-1}, and h_w/h_f = 0.5

Table 3. Parameter values that define the blank area where optimization is not possible. h_w/h_f = 0.5

Volume [m^3]	h_f/k [m^{-1}]	e_{opt} [m]	Bi$_{t,f}$	Effectiveness
1.E–06	220	7.370E–04	0.162	0.978
	1290	7.430E–04	0.958	0.583
5.E–06	117.7	1.799E–03	0.212	1.109
	538.24	1.725E–03	0.928	0.742
1.E–05	89	2.436E–03	0.217	1.193
	370	2.334E–03	0.864	0.798
5.E–05	47.5	5.657E–03	0.269	1.374
	152.16	5.348E–03	0.814	0.893
1.E–04	33	7.188E–03	0.237	1.575
	106	7.147E–03	0.758	0.947

The optimum geometry has been obtained and plotted in fig. 12, in which the optimum thickness versus h_f/k ratio is shown, using fin volume as parameter and for h_w/h_f = 0.1. In this case, a blank area where optimum points disappear also exists, limit values that define this area are shown in tab. 4. The effectiveness for optimal geometry has also been represented as a function of Bi$_{t,f}$, fig. 13.

Optimization process

Starting from the fin volume, h_f, h_w, and k, fig. 2 ($h_w = h_f$), fig. 6 ($h_w/h_f = 10$), fig. 9 (h_w/h_f = 0.5) and, fig. 12 (h_w/h_f = 0.1) provide optimum thickness for the fin-wall assembly for which maximum

Table 4. Limit points that define the blank region where optimization is not possible. $h_w/h_f = 0.1$

Fin volume [m³]	h_f/k [m⁻¹]	e_{opt} [m]	$Bi_{t,f}$	L/e_{opt}	Effectiveness
1.E–06	102.5	6.091E–04	6.243E–02	3.710E–01	1.56
5.E–06	50.2	1.392E–03	6.987E–02	3.875E–01	1.92
1.E–05	37	1.928E–03	7.134E–02	3.717E–01	2.11
5.E–05	18.2	4.250E–03	7.735E–02	3.613E–01	2.46
1.E–04	13.66	6.420E–03	8.770E–02	4.122E–01	2.40

heat will be transferred. The L_{opt} can be obtained through $L_{opt} = V_f/e_{opt}$ and the interfin space can be deduced. The $Bi_{t,f}$ is determined by means of e_{opt} and h_f/k, while fin-wall assembly effectiveness is obtained through curves of fig. 5 (when $h_w = h_f$), fig. 8 (when $h_w/h_f = 10$), fig. 11 ($h_w/h_f = 0.1$), and fig. 13 ($h_w/h_f = 0.5$). The optimization process has been represented in fig. 14.

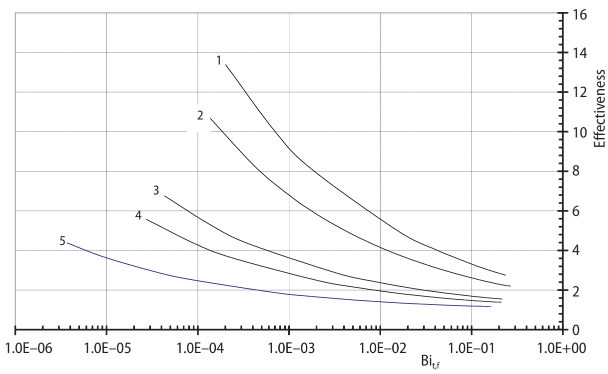

Figure 11. Fin-wall assembly effectiveness as a function of $Bi_{t,f}$: (1) $V_f = 1E–4$ m³, (2) $V_f = 5E–5$ m³, (3) $V_f = 1E–5$ m³, (4) $V_f = 5E–6$ m³, (5) $V_f = 1E–6$ m³, and $h_w/h_f = 0.5$

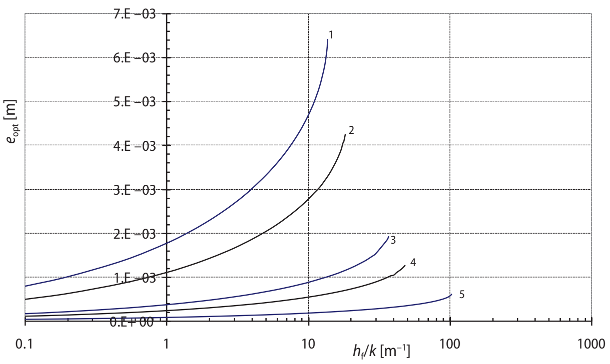

Figure 12. Optimum thickness as a function of h_f/k ratio using fin volume as parameter: (1) $V_f = 1E-4$ m³, (2) $V_f = 5E–5$ m³, (3) $V_f = 1E–5$ m³, (4) $V_f = 5E–6$ m³, (5) $V_f = 1E–6$ m³, and $h_w/h_f = 0.1$

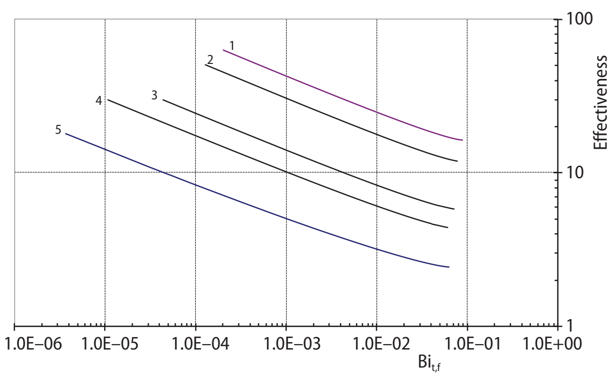

Figure 13. Fin-wall assembly effectiveness as a function of $Bi_{t,f}$ using fin volume as parameter: (1) $V_f = 1E–4$ m³, (2) $V_f = 5E–5$ m³, (3) $V_f = 1E–5$ m³, (4) $V_f = 5E–6$ m³, (5) $V_f = 1E–6$ m³, and $h_w/h_f = 0.1$

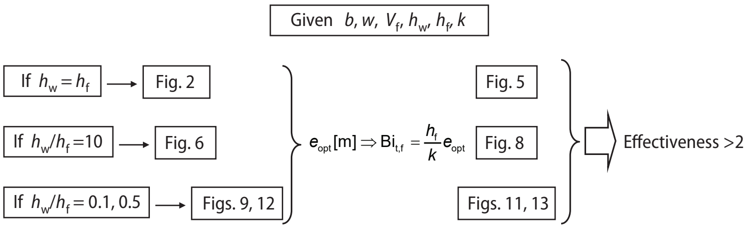

Figure 14. Optimization diagram of the fin-wall assembly, input data: fin volume, h_w, h_f, and k and a constant wall length and thickness are assumed

The use of relative inverse admittance is presented as a definitive parameter for optimization of extended surfaces, isolated fins and fin-wall assembly.

Conclusions

The parameter, relative inverse admittance, has been successfully applied to characterize and optimize fin-wall assembly sets with rectangular fins and different convection conditions on wall and fin surfaces, taking into account the heat dissipated by the tip. The 2-D conduction hypothesis is assumed to cover the complete range of geometrical and thermal parameters of the assemblies, always ensuring that the set improves the heat dissipated by the naked wall by using the classical concept of effectiveness applied to a fin-wall assembly, a performance coefficient whose value is incorporated in the universal optimization curves presented. The ratio between convection coefficients on wall and fin surfaces has been used as a parameter that separates the results into three categories.

In short, the optimum thickness for a fin-wall assembly has been determined and plotted *vs.* the h_f/k ratio in the form of universal curves. The regions where it is not possible to find the optimal geometry within these curves have been defined. Fin-wall assembly effectiveness, as well as the relation of this parameter with optimum Biot number, is always determined to ensure that the assemblies work as dissipative devices.

Nomenclature

b – height of the wall, [m]
$Bi_{t,f}$ – transversal Biot number, ($= eh_f/k$), [–]
e – semi thickness of fin, [m]
h – convective heat transfer coefficient, [$Wm^{-2}K^{-1}$]
k – thermal conductivity, [$Wm^{-1}K^{-1}$]
L – fin length, [m]
q – heat flow, [W]
T – temperature, [K]
V_0 – volume of fin-wall assembly, [m^3]
V_f – volume of fin, [m^3]
V_w – volume of wall, [m^3]
w – width of the wall, [m]
y_r – specific inverse admittance, [Wm^{-3}]
y_{rel} – relative inverse admittance, ($= y_r/y_{r,opt}$), [–]
$y_{r,opt}$ – specific inverse admittance in the optimum condition, [–]
y, z – co-ordinates

Greek symbols

Φ – dimensionless temperature, [$= (T - T_\infty)/(T_b - T_\infty)$], [–]

Subscripts:

b – base of fin
f – relative to the fin
opt – optimum
w – relative to the wall
∞ – refers to the surrounding fluid temperature

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