IRREVERSIBILITY ANALYSIS OF MAGNETO-HYDRODYNAMIC NANOFLUID FLOW INJECTED THROUGH A ROTARY DISK

by

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The non-linear Navier-Stokes equations governed on the nanofluid flow injected through a rotary porous disk in the presence of an external uniform vertical magnetic field can be changed to a system of non-linear partial differential equations by applying similar parameter. In this study, partial differential equations are analytically solved by the modified differential transform method, Pade differential transformation method to obtain self-similar functions of motion and temperature. A very good agreement is observed between the obtained results of Pade differential transformation method and those of previously published ones. Then it has become possible to do a comprehensive parametric analysis on the entropy generation in this case to demonstrate the effects of physical flow parameters such as magnetic interaction parameter, injection parameter, nanoparticle volume fraction, dimensionless temperature difference, rotational Brinkman number and the type of nanofluid on the problem.

Key words: *fractal geometry, fractal porosity, polar bear, hollow hair entropy generation, injection, magneto-hydrodynamic flow, nanofluid, rotary porous disk*

Introduction

In order to describe and analyze some of physical systems in the term of mathematical modeling, non-linear equations are used. The flow due to rotating disks, one of the classical problems in fluid mechanics which has received much attention to in several industrial and engineering processes is one of these phenomena. The pioneering study of fluid flow due to an infinite rotating disk was carried by von Karman [1] which gave a formulation of the problem and then introduced his famous transformations that reduced the governing partial differential equations to ordinary differential equations. Many researchers have been done recently in the field of fluid flow toward a rotating disk causes in different physical phenomena. Rashidi *et al.* [2] presented an analytic solution of condensation film on inclined rotating disk.

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Also they solved steady flow over a rotating disk in porous medium by homotopy analysis method [3]. The magneto-hydrodynamic (MHD) is a magnetic field which can be induced currents in a movable conductive fluid and is used to heat, pump and stir liquid metals in industry [4]. The MHD systems are used effectively in many devices like power pumps, generators, accelerators, electrostatic filters, droplet filters, the design of heat exchangers, the cooling of reactors, *etc.* [5-7]. The attendance of magnetic fields ultimate the forces in which act on the fluid [8]. Homotopy solution has been applied for the unsteady 3-D MHD flow and mass transfer in a porous space in another study [9]. Rashidi *et al.* have been solved different problems of MHD flow [10-12]. Suspending an ultrafine solid particle in a convectional fluid causes an increase in the thermal conductivity. This is one of the most modern and applicable methods for increasing the coefficient of heat transfer [13, 14].

In recent years, many studies have been published on the applications and entropy generation rates of the second law of thermodynamics. Entropy generation analysis is used to optimize the thermal engineering devices for higher energy efficiency [15]. The performance of engineering equipment in the presence of the irreversibilities is reduced, and entropy generation is a measure of the level of the available irreversibilities in a process. In order to access the best design of thermal systems, one can employ the second law of thermodynamics by minimizing the irreversibility [15, 16]. Entropy generation is a criterion of the destruction of the available system work [17]. Also the entropy generation in off-centered stagnation flow towards a rotating disc has been analyzed parametrically by Rashidi *et al.* [18]. Here, we have considered the steady MHD nanofluid flow due to a rotary porous disk. The main goal of the present study is to solve the problem analytically by using modified differential transform method, DTM-Pade and do a comprehensive parametric analysis on the entropy generation.

Flow analysis and mathematical formulation

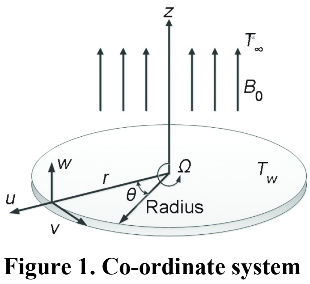

Figure 1. Co-ordinate system for the rotating disk flow

Let us consider the problem of the steady, laminar, 2-D, and incompressible nanofluid flow passing a porous rotating disk in the presence of an externally applied uniform vertical magnetic field. Figure 1 shows the physical model and geometrical co-ordinates.

As it has been showed in this figure the disk rotates with constant angular velocity Ω and it is placed at $z = 0$. The components of the flow velocity are (u, v, w) in the directions of increasing (r, θ, z), respectively. The external uniform magnetic field which applied perpendicular to the surface of the disk has a constant magnetic flux density which is assumed unchanged by taking small magnetic Reynolds number. The surface of the rotating disk is maintained at a uniform temperature T_w, while the temperature and pressure of the ambient nanofluid are T_∞ and P_∞, respectively.

The Navier-Stokes equations governing the motion of the mentioned problem take the form:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}+\frac{1}{\rho_{nf}}\frac{\partial P}{\partial r}=\upsilon_{nf}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\sigma B_0^2 u}{\rho_{nf}} \tag{2}$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=\upsilon_{nf}\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{\sigma B_0^2}{\rho_{nf}}v \tag{3}$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}+\frac{1}{\rho_{nf}}\frac{\partial P}{\partial z}=\upsilon_{nf}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right) \tag{4}$$

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right) \tag{5}$$

The terms $\mu_{nf} = \rho_{nf}\upsilon_{nf}$, ρ_{nf}, and α_{nf} in the mentioned equations are defined by [19, 20]:

$$\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}} \tag{6}$$

$$\alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}} \tag{7}$$

$$\rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s \tag{8}$$

$$(\rho C_p)_{nf}=(1-\varphi)(\rho C_p)_f+\varphi(\rho C_p)_s \tag{9}$$

$$\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\varphi(k_f-k_s)}{(k_s+2k_f)+\varphi(k_f-k_s)} \tag{10}$$

The boundary conditions are introduced as:

$$\begin{cases} u=0, \quad v=\Omega r, \quad w=w_0, \quad T=T_w, \quad \text{at} \quad z=0 \\ u\to 0, \quad v\to 0, \quad P\to P_\infty, \quad T\to T_\infty, \quad \text{at} \quad z\to\infty \end{cases} \tag{11}$$

To obtain the system of non-dimensional ordinary differential equations form of the above equations, we now introduce the following similarity transforms variables:

$$\eta=\sqrt{\frac{\Omega}{\nu_f}}z, \quad u=\Omega r F(\eta), \quad v=\Omega r G(\eta), \quad w=\sqrt{\Omega\nu_f}H(\eta)$$
$$p-p_\infty=2\mu_f\Omega P(\eta), \quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{12}$$

Substituting these transformations into eqs. (1)-(5), the non-linear ordinary differential equations have been derived as:

$$\frac{\mathrm{d}H}{\mathrm{d}\eta}+2F=0 \tag{13}$$

$$\mathrm{A}\frac{\mathrm{d}^2F}{\mathrm{d}\eta^2}-H\frac{\mathrm{d}F}{\mathrm{d}\eta}-F^2+G^2-MF=0, \quad \text{with} \quad \mathrm{A}=\frac{1}{(1-\varphi)^{2.5}\left(1-\varphi+\varphi\dfrac{\rho_s}{\rho_f}\right)} \tag{14}$$

$$\mathrm{A}\frac{\mathrm{d}^2 G}{\mathrm{d}\eta^2}-H\frac{\mathrm{d}G}{\mathrm{d}\eta}-2FG-MG=0 \tag{15}$$

$$\mathrm{B}\frac{1}{\mathrm{Pr}}\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2}-H\frac{\mathrm{d}\theta}{\mathrm{d}\eta}=0, \quad \text{with } \mathrm{B}=\frac{\dfrac{k_{nf}}{k_f}}{\dfrac{(\rho C_p)_{nf}}{(\rho C_p)_f}} \tag{16}$$

The boundary conditions become:

$$F(0)=0, \quad G(0)=1, \quad \theta(0)=1, \quad H(0)=W_S, \quad F(\infty)=0, \quad G(\infty)=0, \quad \theta(\infty)=0 \tag{17}$$

where $W_S = w_0/(\Omega v_f)^{1/2}$ shows a uniform injection ($W_S < 0$) at the disk surface, The boundary conditions (17) imply that both the radial F, the tangential G velocities and temperature θ vanish sufficiently far away from the rotating disk, *i. e.* sufficiently large η-values.

Entropy generation

It has been shown [21, 22, 23] the volumetric rate of local entropy generation of the nanofluid, in the presence of axial symmetry and magnetic field and according to the above assumptions, can be described as:

$$\dot{S}'''_{gen}=\frac{k_{nf}}{T_w^{\ 2}}[\nabla T]^2+\frac{\mu_{nf}}{T_w}\Phi+\frac{1}{T_w}[(J-QV)(E+V\times B)] \tag{18}$$

where

$$[\nabla \mathrm{T}]^2=\left[(T_r)^2+\left(\frac{1}{r}T_\theta\right)^2+(T_z)^2\right] \tag{19}$$

$$\Phi=2\left[(u_r)^2+\frac{1}{r^2}(v_\theta+u)^2+(w_z)^2\right]+\left(v_z+\frac{1}{r}w_\theta\right)^2+(w_r+u_z)^2+\left[\frac{1}{r}u_\theta+r\left(\frac{v}{r}\right)_r\right]^2 \tag{20}$$

$$J=\sigma(E+V\times B) \tag{21}$$

Since the electric force per unit charge, as compared to $V \times B$ is negligible and also the electric current is much greater than QV, eq. (18) can be further simplified as [16, 24]:

$$\dot{S}'''_{gen}=\underbrace{\frac{k_{nf}}{T_w^{\ 2}}(T_z)^2}_{Thermal\ irreversibility}+\underbrace{\frac{\mu_{nf}}{T_w}\left\{2\left[(u_r)^2+\frac{1}{r^2}u^2+(w_z)^2\right]+(v_z)^2+(u_z)^2+\left[r\left(\frac{v}{r}\right)_r\right]^2\right\}}_{Fluid\ friction\ irreversibility}+$$

$$+\underbrace{\frac{\sigma B_0^{\ 2}}{T_w}\left(u^2+v^2\right)}_{Joule\ dissipation\ irreversibility} \tag{22}$$

To analyze the entropy generation, the dimensionless form of it could be driven. The entropy generation number, dimensionless form of entropy generation rate, defined as the ratio between the actual entropy generation rate and the characteristic entropy generation rate. Here the entropy generation number (N_G) becomes [25]:

$$N_G = \frac{\dot{S}'''_{gen}}{\dfrac{k_{nf}\Omega \Delta T}{\nu_f T_w}} \tag{23}$$

$$\alpha\theta'(\eta)^2 + \text{Br}\left(\frac{3}{\text{Re}}H'(\eta)^2 + \overline{r}^2\{(F'(\eta)^2 + G'(\eta)^2) + M[F(\eta)^2 + G(\eta)]^2)\}\right) \tag{24}$$

Results and discussion

The non-linear differential eqs. (13)-(16) subject to the boundary conditions in eq. (17) have been solved analytically via DTM-Pade for value of η.

In order to verify the results of this study; a comparison between these DTM-Pade results for $M = 1.0$, $\varphi = 0.1$, $W_S = -1.0$, Pr = 1.0, Cu nanoparticles and previously published numerical ones of [25] has been presented in fig. 2(a). As can be seen a very good agreement has been obtained in this comparison. Since a complete parametric analysis on the self-similar functions has been done in [25], there is no need to repeat these studies here again. So in this study it has been focused on the parametric analysis of entropy generation.

The effect of variation in the parameter M on the dimensionless entropy generation for different has been illustrated in figs. 2(b) and (c). As can be seen, the variations of dimensionless entropy generation have been presented respect to both r/R ($\eta = 1.0$) and η. From this figure it is clear that the parameters M, η, and r/R have important effects on the dimensionless entropy generation. In each values of η and M with increasing in r/R the values of the dimensionless entropy generation increase. The effect of W_S on the dimensionless entropy generation has been shown in fig. 2(d).

It is clear at figs. 2(e) and (f) the parameters dimensionless temperature difference α and rotational Brinkman number (Br), have a constant effect on the dimensionless entropy generation in all values of η. These figures show that the bigger values of dimensionless temperature difference α and rotational Brinkman number causes bigger values of the dimensionless entropy generation regardless of the value of η. Figure 2(g) shows the effect of η on the dimensionless entropy generation. As can be seen by increasing in η, it means by tending to physical infinity, the values of the dimensionless entropy generation decrease which is acceptable physically. Figure 2(h) presents the effect of φ on the dimensionless entropy generation. Totally from these figures it is clear that by increasing in φ up to about $\eta = 0.5$ the dimensionless entropy generation value decreases, whereas for $\eta > 0.5$ its value increases. It means that, like M but more than this, φ has a different effect on the entropy generation depends on η. As one case, left figure shows the effect of φ in $\eta = 0.5$.

The polar presentation of the effect of nanofluid type, which in the case fig. 3(a) nanoparticles are Cu, in case fig. 3(b) are CuO, and the case fig. 3(c) are Al_2O_3, on the dimensionless entropy generation, can be seen in fig. 3. This figure is very understandable since these three figures plotted in the same legend values. As another aspect of this figure, the mentioned effect of r/R on the dimensionless entropy generation in all previous figures has been presented in polar co-ordinate on the disk.

Conclusions

In this paper, the steady nanofluid flow which is electrically conducting and incompressible, due to a rotary porous disk in the presence of an externally applied uniform vertical magnetic field has been solved analytically by the modified differential transform method,

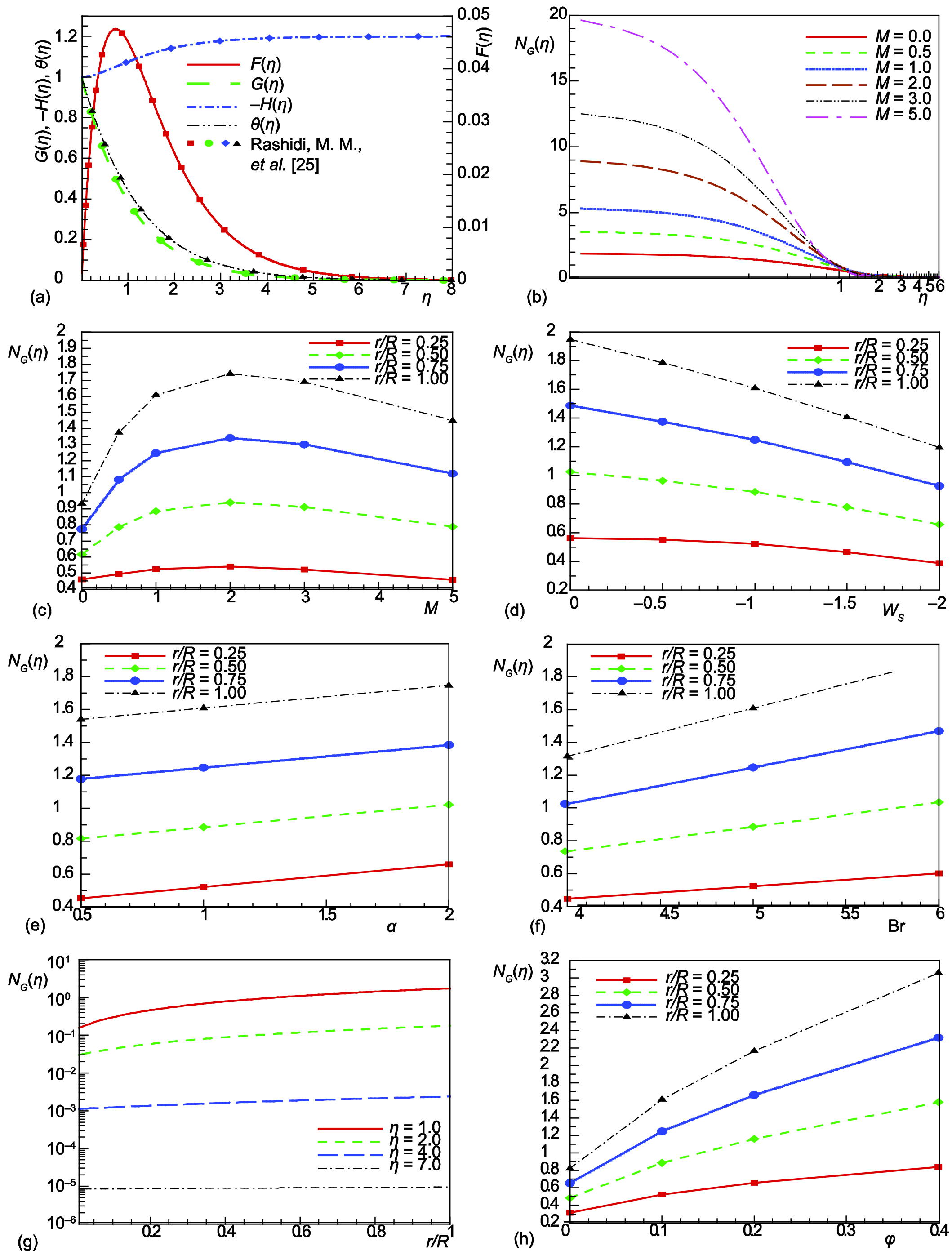

Figure 2. (a) Comparison of the results of DTM-Pade with [25], (b) Variation of N_G with η; (c) Variation of N_G with M; (d) Variation of N_G with W_S; (e) Variation of N_G with α; (f) Variation of N_G with Br; (g) Variation of N_G with r/R; (h) Variation of N_G with φ

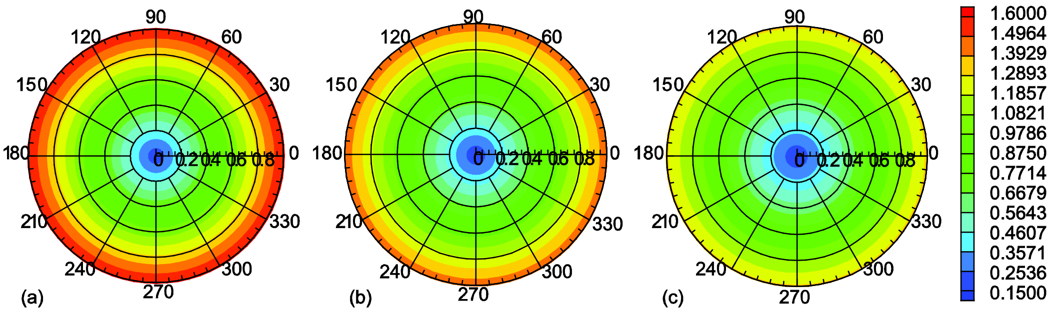

Figure 3. Distribution of N_G (η) for different nanoparticles with constant parameters; η = 1.0, W_S = –1.0, Pr = 1.0, α = 1.0, Br = 5.0, Re = 5.0, M = 1.0, φ = 0.1, (a) Cu, (b) CuO, (c) Al_2O_3
(for color image see journal web-site)

DTM-Pade. A comprehensive parametric analysis on are presented. Finally after finding the radial, tangential and axial velocity components as well as temperature distribution as the main goal of this study, a comprehensive parametric analysis on the dimensionless entropy generation has been done and effects of physical flow parameters such as magnetic interaction parameter, injection parameter, nanoparticle volume fraction, dimensionless temperature difference, rotational Brinkman number and the type of nanofluid on the problem have been showed. The results are contain of very important subjects which have great role in designing the rotary disk, an important and practical case in the industrial applications.

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