A NEW NUMERICAL INVESTIGATION OF SOME THERMO-PHYSICAL PROPERTIES ON UNSTEADY MHD NON-DARCIAN FLOW PAST AN IMPULSIVELY STARTED VERTICAL SURFACE

by

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Original scientific paper
DOI: 10.2298/TSCI15S1S49M

The behaviour of unsteady non-Darcian magnetohydrodynamic fluid flow past an impulsively started vertical porous surface is investigated. The effect of thermophoresis due to migration of colloidal particles in response to a macroscopic temperature gradient is taken into account. It is assumed that both dynamic viscosity and thermal conductivity are linear functions of temperature. The governing equations are non-dimensionalized by using suitable similarity transformation which can unravel the behaviour of the flow at short time and long time periods. A novel iteration scheme, called bivariate spectral local linearization method is developed for solving the corresponding systems of highly non-linear partial differential equations. The results of the numerical solutions obtained are presented graphically and analyzed for the effects of the various important parameters entering into the problem on velocity, temperature, and concentration field within the boundary layer.

Key words: *bivariate Lagrange interpolation, variable thermal conductivity, unsteady flow, non-Darcian flow, impulsive stretching sheet, thermophoresis*

Introduction

Theoretical study of viscous incompressible fluid flow past semi-infinite heated vertical plate is very significant due to its huge applications in industry such as food and polymer processing. The flow due to stretching of a flat surface was first reported by Crane [1]. Brown and Riley [2] presented an analysis that covers three distinct phases in the temporal development of the free convection flow past a suddenly heated semi-infinite vertical plate. In all the three stages, the unsteadiness in the flow field arises due to the step-change in wall temperature. Ingham [3] extended the mathematical model of [2] by considering a special case where wall temperature T_w is suddenly raised to $T_w = T_\infty + Ax^m$. Stewartson [4, 5] investigated the motion of fluid within boundary layer which arises when a semi-infinite flat plate is impulsively started from rest with velocity *U*. Dennis [6] and Watkins [7] investigated the boundary layer flow development of a viscous fluid on a semi-infinite flat plate due to impulsive motion of the free stream. Williams and Rhyne [8] argued extensively that the viscous flow within the boundary layer develops slowly, reaching a fully

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developed steady flow only after some period of time. The boundary layer development occurs in two stages. It was observed that for small time τ, the convective acceleration plays only a minor role in the flow development. For large time τ the fluid flow is dominated by convective acceleration. Impulsively started flow has been described in Smith [9], and Nath *et al.* [10]. In industrial systems, fluids can be subjected to extreme conditions such as high temperature, pressure, and external heating. Each of these factors may lead to high temperature being generated within the fluid. According to [11, 12], it is a well-known fact that properties which are most sensitive to temperature rise are viscosity and thermal conductivity. Mukhopadhyay[13] adopted the temperature dependent viscosity model of Batchelor [11] and presented the effect of variable fluid viscosity on flow along a symmetric wedge. The effect of thermophoresis on laminar flow over cold inclined plate with variable properties was reported by Jayaraj [14]. Bakier [15] studied the combination of thermophoresis, radiation effects, Forchheimer quadratic (inertial) drag and surface mass flux on coupled heat and mass transfer of a micropolar liquid in a saturated porous medium. Recently, Animasaun [16] presented the effects of some thermo-physical parameters on non-Darcian MHD dissipative Casson fluid flow along linearly stretching vertical surface with migration of colloidal particles in response to macroscopic temperature gradient.

Upon using the scaling developed in [8], it seems hard to obtain analytic solutions of unsteady boundary layer flows which are valid for all time. Nath *et al.* [10] examined the asymptotic behaviour of the solution for a steady case and later obtained the solution for both unsteady and steady cases by using an implicit finite difference scheme. To obtain this kind of solution over the time domain $0 \le \xi \le 1$, Pop *et al.* [17] adopted numerical solution by using the Keller-box. Dennis [6] applied perturbation techniques which only provide valid solutions for small time. Recently, Liao [18] explained the limitations of the numerical approach adopted in [10, 19] and hence obtained analytic solutions uniformly valid for all dimensionless time $0 \le \tau < +\infty$ in the whole region $0 \le \eta < +\infty$ using homotopy analysis method (HAM). In contrast, spectral methods are computationally less expensive than HAM and more accurate than either finite difference methods or Keller-box particularly for problems with smooth transition from small-time solution to large-time solution. The method proposed in this study employs the Chebyshev spectral collocation method to discretize both the space and time variables. Considering that spectral method approaches require a few grid points to give highly accurate approximate solutions, the proposed method used in this study is found to be very effective and computationally fast. So far, no attempt has been made to present the effect of temperature-dependent fluid viscosity and thermal conductivity, thermophoresis particle deposition on convective heat and mass transfer of non-Darcian flow along an isothermal vertical porous surface. A combination of these effects is studied in this work. In order to unravel these effects, a reliable and efficient method of solution is required. The dimensionless governing equations are solved using a new approach called the bivariate spectral local linearisation method. This method is developed as an extension of the local linearisation method that has been successfully applied on systems ordinary differential equations [20]. In this work we extend the use of the method to systems of partial differential equations. It is hoped that the results obtained will not only provide useful information for industrial applications, but also serve as a complement to the previous related studies.

Mathematical problems

Consider the flow of a viscous incompressible fluid flow past a vertical porous surface with the effects of energy flux caused by a composition gradient. Due to the nature of the fluid flow in consideration, at sufficiently high velocity the relationship between flow rate and potential gradient is assumed non-linear. This effect is incorporated by including a second order velocity term to Darcy equation which represents the microscopic inertial effect in momentum equation. It is assumed that a uniform magnetic field of strength B_o is applied in the perpendicular direction towards the flow. For boundary layer analysis it is found that the temperature gradient along the plate is much lower than the temperature gradient normal to the surface; hence, the component of thermophoretic velocity along the porous surface is negligible compared to the component of its velocity normal to the surface [21-23]. In this study, a case where the fluid is emerging out of a slit at origin $(x = 0, y = 0)$ and moving along the vertical wall is considered. The x-axis is taken along the surface in the vertically upward direction and the y-axis is taken as normal to the plate. Initially, at time $t \le 0$ the plate and the adjacent fluid are at the same temperature T_∞ and concentration C_∞ in a stationary condition. At $t > 0$ the immediate fluid layer next to the vertical porous surface is given an impulsive motion in the vertical direction against the gravitational field with velocity $u_w = ax$ and both the temperature and concentration level near the plate are raised from T_∞ to T_w and C_∞ to C_w, respectively. The surface of the plate is assumed to have an arbitrary constant temperature T_w. The unsteadiness in the flow field is caused by impulsively induced motion of immediate fluid layer on the vertical surface and also by sudden increase in the surface temperature and concentration. The density variation and the buoyancy effects are taken into consideration, so that the Boussinesq approximation for both the temperature and concentration gradient is adopted. Under these assumptions along with boundary layer approximations, the flow is governed by the system of equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left[\mu(T)\frac{\partial u}{\partial y}\right] + g\beta(T - T_\infty) + \\ + g\beta^*(C - C_\infty) - \frac{\mu(T)u}{K\rho} - \frac{b^* u^2}{K} - \frac{\sigma B_o^2 u}{\rho} \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho C_P}\frac{\partial}{\partial y}\left[\kappa(T)\frac{\partial T}{\partial y}\right] + \frac{D_m K_t}{C_P C_S}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + \frac{\partial}{\partial y}[V_T(C - C_\infty)] = D_m\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The associated boundary conditions are:

$$t \le 0: \quad u = 0, \quad v = 0, \quad T = T_\infty, \quad C = C_\infty, \quad \text{for all } y \tag{5}$$

$$t > 0: u = u_w, \quad v = -V(x), \quad T = T_w, \quad C = C_w, \quad \text{at} \quad y = 0 \tag{6}$$

$$t > 0: \quad u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty \tag{7}$$

In this investigation, dynamic viscosity (μ) and thermal conductivity (κ) of the fluid are assumed to vary as a linear function of temperature. This assumption is incorporated in order to account for the influence of dynamic viscosity, and thermal conductivity which changes due to the contact between immediate fluid layer, and vertical heated surface under consideration. Hence, we adopted the mathematical model of temperature dependent viscosity and thermal conductivity as:

$$\mu(T) = \mu^*[1 + b(T_w - T)], \qquad \kappa(T) = \kappa^*[1 + \gamma(T - T_\infty)] \tag{8a, b}$$

The relations (9) are introduced to the transform u, v, T, C, and V_T, respectively, as:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \theta = \frac{T - T_\infty}{T_W - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_W - C_\infty},$$

$$V_T = -\frac{K^{TH}}{T_{\text{Ref}}}\frac{\partial T}{\partial y}, \quad \lambda = -\frac{K^{TH}(T_W - T_\infty)}{\vartheta T_{\text{Ref}}} \tag{9a, b, c, d, e, f}$$

In formulating the problem of the boundary layer development which is impulsively set into motion, one encounters the problem of determining the appropriate scaling for the problem. The solution for small dimensionless time τ is similar in the scaled co-ordinate $y(\vartheta t)^{-1/2}$ (*i. e.* the solution exists for small time), while the solution for large time is similar in the scaled co-ordinate $y(u_w/\vartheta t)^{1/2}$ (*i. e.* the solution exists for large time). This implies that we have to find a scaling of the y-co-ordinate which behaves like $y(\vartheta t)^{-1/2}$ for small time and as $y(u_w/\vartheta t)^{1/2}$ for large time. Furthermore, it is convenient to choose time scale ξ so that the region of time integration may become finite. According to [8], such similarity transformations are:

$$\eta = y\sqrt{\frac{a}{\vartheta\xi}}, \quad \psi(x, y) = xf(\eta, \xi)\sqrt{a\vartheta\xi}, \quad \xi = 1 - \mathrm{e}^{-\tau}, \quad \tau = at \tag{10a, b, c, d}$$

Using (8)-(10) in (1)-(7), eq. (1) is identically satisfied, and (2)-(7) reduced to:

$$[1 + \omega - \omega\theta]\frac{\partial^3 f}{\partial \eta^3} - \omega\frac{\partial \theta}{\partial \eta}\frac{\partial^2 f}{\partial \eta^2} + \frac{1}{2}\eta(1 - \xi)\frac{\partial^2 f}{\partial \eta^2} + \xi f\frac{\partial^2 f}{\partial \eta^2} + \xi\omega \mathrm{Gr}_t\theta + \xi\omega \mathrm{Gr}_s\phi$$

$$-\xi\left(1 + \frac{\mathrm{F_s}}{\mathrm{Da}}\right)\frac{\partial f}{\partial \eta}\frac{\partial f}{\partial \eta} - \xi P_p[1 + \omega - \omega\theta]\frac{\partial f}{\partial \eta} - \xi M\frac{\partial f}{\partial \eta} = \xi(1 - \xi)\frac{\partial^2 f}{\partial \eta \partial \xi} \tag{11}$$

$$[1 + \varepsilon\theta]\frac{\partial^2 \theta}{\partial \eta^2} + \varepsilon\frac{\partial \theta}{\partial \eta}\frac{\partial \theta}{\partial \eta} + \Pr \mathrm{Df}\frac{\partial^2 \phi}{\partial \eta^2} + \Pr\frac{1}{2}\eta(1 - \xi)\frac{\partial \theta}{\partial \xi} +$$
$$+ \Pr \xi f\frac{\partial \theta}{\partial \eta} = \Pr \xi(1 - \xi)\frac{\partial \theta}{\partial \xi} \tag{12}$$

$$\frac{\partial^2 \phi}{\partial \eta^2} + \mathrm{Sc}\xi f\frac{\partial \phi}{\partial \eta} + \mathrm{Sc}\frac{\eta}{2}(1 - \xi)\frac{\partial \phi}{\partial \eta} - \mathrm{Sc}\lambda\phi\frac{\partial^2 \theta}{\partial \eta^2} - \mathrm{Sc}\lambda\frac{\partial \theta}{\partial \eta}\frac{\partial \phi}{\partial \eta} = \mathrm{Sc}\xi(1 - \xi)\frac{\partial \phi}{\partial \xi} \tag{13}$$

subject to:

$$f(0, \xi) = S, \quad \frac{\partial f(0, \xi)}{\partial \eta} = 1, \quad \theta(0, \xi) = \phi(0, \xi) = 1 \tag{14a, b, c}$$

where governing parameters are:

$$\omega = b(T_w - T_\infty), \quad \varepsilon = \gamma(T_w - T_\infty), \quad \mathrm{Gr}_t = \frac{g\beta}{a^2 xb}, \quad \mathrm{Gr}_s = \frac{g\beta^*}{a^2 xb},$$

$$\mathrm{F_s} = \frac{b^*}{x}, \quad \mathrm{Da} = \frac{K}{x^2}, \quad P_P = \frac{\vartheta}{aK}, \quad M = \frac{\sigma B_O^2}{a\rho}, \quad \mathrm{Pr} = \frac{\vartheta}{\alpha},$$

$$\mathrm{Du} = \frac{D_m k_t (C_w - C_\infty)}{\vartheta C_P C_S (T_w - T_\infty)}, \quad \mathrm{Sc} = \frac{\vartheta}{D_m}, \quad \lambda = -\frac{K^{TH}}{\vartheta T_{\mathrm{Ref}}}(T_W - T_\infty), \quad \text{and} \quad S = \frac{V(x)}{\sqrt{\vartheta a \xi}}$$

The physical quantities of interest in this problem are Skin friction coefficient, Nusselt number, and Sherwood number defined as:

$$C_f = \frac{\tau_w}{\rho u_e^2} = \frac{\vartheta}{(ax)^2} \left.\frac{\partial u}{\partial y}\right|_{y=0}, \quad \mathrm{Nu} = \frac{-\kappa x}{\kappa(T_w - T_\infty)} \left.\frac{\partial T}{\partial y}\right|_{y=0}, \quad \mathrm{Sh} = \frac{-D_m x}{D_m(C_w - C_\infty)} \left.\frac{\partial C}{\partial y}\right|_{y=0} \tag{15a, b, c}$$

Using the transformation variables in eqs. (10a, b, c, d), and $\mathrm{Re} = ax/\vartheta$, we obtain the following dimensionless quantities which are proportional to local skin friction coefficient, local heat transfer and local mass transfer:

$$f''(0, \xi) = \sqrt{\xi} C_f \sqrt{\mathrm{Re}}, \quad -\theta'(0, \xi) = \frac{\sqrt{\xi}}{\sqrt{x\,\mathrm{Re}}}\mathrm{Nu}, \quad -\phi'(0, \xi) = \frac{\sqrt{\xi}}{\sqrt{x\,\mathrm{Re}}}\mathrm{Sh} \tag{16a, b, c}$$

Bivariate spectral local linearization

We use the idea of decoupling systems of equations that was called local linearisation method (LLM) in Motsa [20]. This approach applies quasi-linearisation in one variable per equation. We remark that in [20] the method was applied in systems of non-linear ODE. In this work, the LLM is extended to systems of PDE. The local linearisation scheme corresponding to eqs. (11)-(13) becomes:

$$\alpha_{0,r}(\eta, \xi) f'''_{r+1} + \alpha_{1,r}(\eta, \xi) f''_{r+1} + \alpha_{2,r}(\eta, \xi) f'_{r+1} +$$
$$+ \alpha_{3,r}(\eta, \xi) f_{r+1} - \xi(1-\xi)\frac{\partial f'_{r+1}}{\partial \xi} = K_{1,r}(\eta, \xi) \tag{17}$$

$$\beta_{0,r}(\eta, \xi)\theta''_{r+1} + \beta_{1,r}(\eta, \xi)\theta'_{r+1} + \beta_{2,r}(\eta, \xi)\theta_{r+1} - \mathrm{Pr}\,\xi(1-\xi)\frac{\partial \theta_{r+1}}{\partial \xi} = K_{2,r}(\eta, \xi) \tag{18}$$

$$\phi''_{r+1} + \sigma_{1,r}(\eta, \xi)\phi'_{r+1} + \sigma_{2,r}(\eta, \xi)\phi_{r+1} - \mathrm{Sc}\xi(1-\xi)\frac{\partial \phi_{r+1}}{\partial \xi} = K_{3,r}(\eta, \xi) \tag{19}$$

where the coefficients are defined as:

$$\alpha_{0,r} = a + \omega - \omega\theta_r, \quad \alpha_{1,r} = -\omega\theta'_r + \frac{1}{2}\eta(1-\xi) + \xi f_r, \quad \alpha_{3,r} = \xi f''_r, \quad \sigma_{2,r} = -\mathrm{Sc}\lambda\theta''_{r+1},$$

$$\beta_{0,r} = 1 + \varepsilon\theta_r, \quad \alpha_{2,r} = -\xi P_P[a + \omega - \omega\theta_r] - \xi M - 2\xi\left(1 + \frac{\mathrm{F_s}}{\mathrm{Da}}\right) f'_r, \quad R_{3,r} = 0,$$

$$\beta_{1,r} = 2\varepsilon\theta_r' + \frac{1}{2}\,\mathrm{Pr}\,\eta(1-\xi) + \mathrm{Pr}\,\xi f_{r+1}, \quad \beta_{2,r} = \varepsilon\theta_r'', \quad \sigma_{1,r} = \mathrm{Sc}\xi f_{r+1} + \frac{\mathrm{Sc}}{2}\eta(1-\xi) - \mathrm{Sc}\lambda\theta_{r+1}',$$

$$R_{1,r} = \xi\left[f_r f_r'' - \left(1+\frac{\mathrm{F_s}}{\mathrm{Da}}\right)f_r' f_r'\right] - \xi\omega(\mathrm{Gr}_t\theta_r + \mathrm{Gr}_s\phi_r), \quad R_{2,r} = \varepsilon(\theta_r\theta_r'' + \theta_r'\theta_r') - \mathrm{Pr}\,\mathrm{Du}\phi_r''$$

To solve the linearised eqs. (17)-(19), an approximate solution defined in terms of bivariate Lagrange interpolation polynomials in sought. For example, the approximate solution for $f(\eta, \xi)$ takes the form:

$$f(\eta, \xi) \approx \sum_{m=0}^{N_x}\sum_{j=0}^{N_t} f(\tau_m, \zeta_j)L_m(\tau)L_j(\zeta) \tag{20}$$

where the functions $L_m(\tau)$ and $L_j(\zeta)$ are the well-known characteristic Lagrange cardinal polynomials. Equation (26) interpolates $f(\eta, \xi)$ at the collocation points (known as Chebyshev-Gauss-Lobatto) defined by:

$$\tau_1 = \cos\left(\frac{\pi i}{N_x}\right), \qquad \zeta_i = \cos\left(\frac{\pi i}{N_t}\right), \qquad i = 0, 1, \ldots N_x;\; j = 0, 1, \ldots N_t \tag{21a, b}$$

The choice of collocation points (21) makes it possible for the Chebyshev spectral collocation method to be used as a solution procedure. The Chebyshev collocation method requires that the domain of the problem be transformed to $[-1, 1]\times[-1, 1]$. Accordingly, linear transformations have been used to transform $\eta \in [0, \eta_\infty]$, and $\xi \in [0, L_t]$ to $\tau \in [-1, 1]$, and $\zeta \in [-1, 1]$, respectively. Here η_∞ is a finite value that is introduced to facilitate the application of the numerical method at infinity and L_t is the largest value of ξ. Substituting eq. (20) in eqs. (17)-(19) and making use of the derivatives formulas for Lagrange functions at Gauss--Lobatto points given in [24, 25] results in:

$$B_1^{(i)}F_i - 2\xi_i(1-\xi_i)\sum_{j=0}^{N_t} d_{i,j}DF_j = K_{1,i} \tag{22a}$$

$$B_2^{(i)}\Theta_i - 2P_r\xi_i(1-\xi_i)\sum_{j=0}^{N_t} d_{i,j}\Theta_j = K_{2,i} \tag{22b}$$

$$B_3^{(i)}\Phi_i - 2Sc\xi_i(1-\xi_i)\sum_{j=0}^{N_t} d_{i,j}\Phi_j = K_{3,i} \tag{22c}$$

where

$$B_1^{(i)} = \alpha_{0,i}D^3 + \alpha_{1,i}D^2 + \alpha_{2,i}D + \alpha_{3,i}, \; B_2^{(i)} = \beta_{0,i}D^2 + \beta_{1,i}D + \beta_{2,i}, \; B_3^{(i)} = D^2 + \sigma_{1,i}D + \sigma_{2,i}$$

Equations (22a, b, c) can be solved in succession after being converted to matrix equations of size $(N_t+1)(N_x+1)\times(N_t+1)(N_x+1)$. Starting from a given initial guess, the approximate solutions for $f(\eta, \xi)$, $\theta(\eta, \xi)$, and $\phi(\eta, \xi)$ are obtained by iteratively solving the matrix equations.

Discussions

The approximate numerical solutions of the dimensionless governing eqs. (11)-(13) were solved using the bivariate spectral local linearisation method (BSLLM) as described in the previous section. In this section, we present the results of the numerical computations for the velocity, temperature, species concentration profiles and also for important physical properties of the flow for various input parameters. Grid independence tests revealed that $N_x = 60$ and $N_t = 15$ collocation points in the η and ξ domain, respectively, were sufficient to give accurate and consistent results. A further increase in the number of collocation points did not result in a change in the computed results. Furthermore, the minimum number of iterations required to give results that are consistent to within a tolerance level of 10^{-7} were used. The value of η_∞ was set to be 20. It is worth mentioning that when $\xi = 0$ (initial unsteady stage) and $\xi = 1$ (final steady stage), eqs. (11)-(13) reduce to ordinary differential equations which can easily be solved using ODE solvers such as Matlab's bvp4c solver. In order to validate the accuracy of BSLLM, the approximate numerical results can be compared against the bvp4c results for the limiting cases when $\xi = 0$ and $\xi = 1$. The comparison of the two solutions using $\mathrm{Gr_t} = \mathrm{Gr_s} = 1$, $\mathrm{F_s} = \mathrm{Da} = 1$, $P_p = S = 0.3$, $M = 0.1$, $\mathrm{Du} = \varepsilon = 0.2$, $\lambda = 0.1$, Sc = 0.62, and Pr = 0.72, at various values of ω is shown in tab. 1. The results are found to be in good agreement.

Table 1. Variation of skin friction $f''(0, \xi)$ and Nusselt number $-\theta'(0, \xi)$ with ω at initial unsteady stage ($\xi = 0$) and at final steady stage ($\xi = 1$)

ω	$f''(0, \xi = 0)$BSLL	$f''(0)$bvp4c	$-\theta'(0, \xi = 0)$BSLL	$-\theta'(0)$bvp4c
0	–0.564190	–0.564190	0.395624	0.395624
0.2	–0.579950	–0.579950	0.395624	0.395624
0.4	–0.595423	–0.595423	0.395624	0.395624
ω	$f''(0, \xi = 0)$BSLL	$f''(0)$bvp4c	$-\theta'(0, \xi = 0)$BSLL	$-\theta'(0)$bvp4c
0	–1.581386	–1.581386	0.425741	0.425739
0.2	–1.385371	–1.385371	0.468162	0.468162
0.4	–1.215487	–1.215487	0.493087	0.493087

Table 2. Variation of local mass transfer rate (Sherwood number) $-\varphi'(0, \xi)$ with Dufour number at various values of ξ

Du	$-\varphi'(0, \xi = 0)$	$-\varphi'(0, \xi = 0.5)$	$-\varphi'(0, \xi = 1)$
0	0.461906	0.524138	0.592831
0.5	0.458702	0.520006	0.592145
1.0	0.455573	0.515987	0.591115

It is further observed from tab. (1) that the local skin friction decreases with increase in magnitude of temperature dependent viscous parameter (ω) whereas no significant effect is observed on local heat transfer at the wall during initial unsteady stage. As the magnitude of ω increases, the temperature of the fluid increases which further enhance the internal heat energy and reduces the intermolecular forces that holds molecules strongly. This accounts for the reduction in skin friction coefficient (*i. e.* reduces drag coefficient at the wall).The local heat transfer rate which remains constant with an increase in the magnitude of ω can be traced to the omission of convective acceleration at the initial unsteady stage. As $\xi \to 1$, convective acceleration gradually replaces unsteady acceleration which dominates at initial unsteady stage; hence it is observed that both $f''(0, \xi = 1)$ and $-\theta'(0, \xi = 1)$ are increasing function of ω at the final steady stage. Table 2 exhibits the nature of local mass transfer rate (Sherwood number) with Dufour number parameter at various values of dimensionless time ξ.

At all time $(0 \le \xi \le 1)$, it is found that local mass transfer rate decreases with increasing Dufour parameter. It is further observed that maximum local mass transfer rate is obtained in the absence of energy flux due to composition gradient at all time within the domain $(0 \le \xi \le 1)$. The negligible decrease in $-\phi'(0, \xi = 0)$ with an increase in the rate of energy flux due to composition gradient can be traced to the migration of particles from the hot region (at the wall $\eta = 0, \xi$) to the cold region (at free stream $\eta = \infty, \xi$) due to temperature gradient which is known as thermophoretic force.

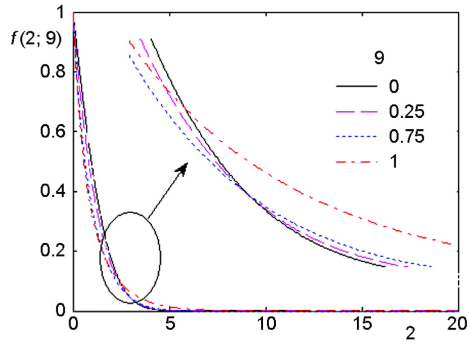

Figure 1. Velocity profiles for different ξ

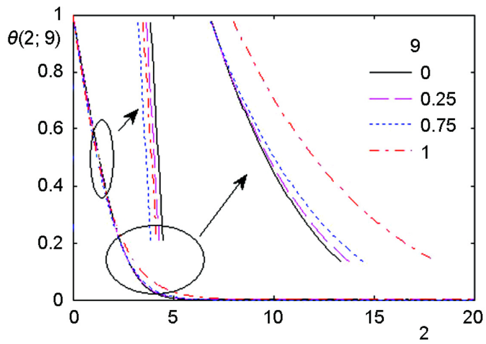

Figure 2. Temperature profiles for different ξ

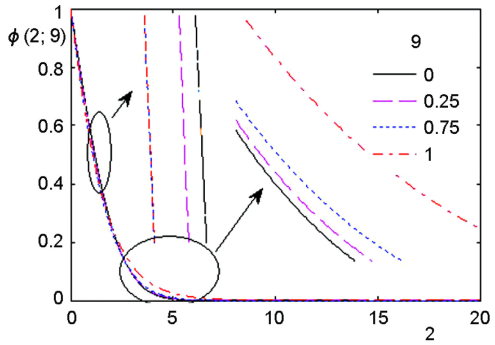

Figure 3. Concentration profiles for different ξ

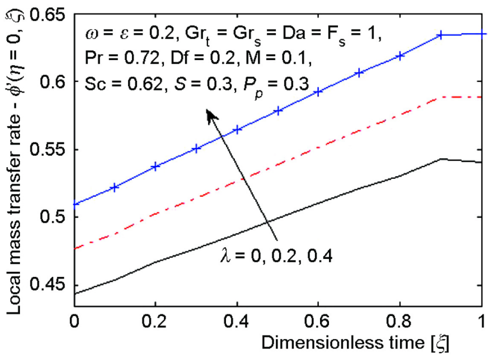

Figure 4. Local mass transfer rate against ξ

Figures 1, 2 and 3 illustrate the velocity, temperature, and concentration profiles of the fluid flow in consideration at various dimensionless time when $\mathrm{Gr_t} = \mathrm{Gr_s} = 1$, $M = 0.1$, $\lambda = 0.1$, $\mathrm{Sc} = 0.62$, $\mathrm{F_s} = \mathrm{Da} = 1$, $P_p = S = 0.3$, $\mathrm{Du} = \varepsilon = 0.2$, and $\mathrm{Pr} = 0.72$. From these figures, it is observed that all profiles decrease with ξ near the wall and increase thereafter before each profiles asymptotically tends to zero. This result clearly indicates that unsteady acceleration term which exists in the absence of buoyancy terms when $\xi = 0$ opposes the transport phenomena. This influence varnishes gradually as $\xi \to 1$ and subdues with convective acceleration together with buoyancy terms which enhance the fluid flow as $\eta \to \infty$.The combined influence of dimensionless time ξ and thermophoretic parameter λ on local mass transfer is shown in fig. 4. It is seen that local mass transfer rate increases with ξ and λ. This behaviour in the fluid flow can be traced to the presence of energy flux caused by composition gradient.

Conclusions

A new spectral collocation method (BSLLM) has been developed and implemented to solve the problem of unsteady heat and mass transfer past a semi-infinite vertical plate with diffusion-thermo and thermophoresis effects in the presence of suction. The fluid was assumed to have temperature dependent dynamic viscosity and thermal conductivity. Validation of the proposed numerical method for accuracy was established by comparing the present results against results obtained using Matlab's bvp4c solver for special cases where the governing equations reduce to ordinary differential equations. The results from this study indicate that the effects of temperature dependent viscous parameter (ω) is to decrease and increase local skin friction at initial unsteady and final steady stage, respectively. Heat transfer rate is more pronounced with an increase in ω at the final stage. The proposed BSLLM method can be extended to solve other types of non-linear PDE systems with fluid mechanics applications or problems from other disciplines of science and engineering which are defined in terms of systems of non-linear PDE.

Acknowledgments

This work is based on the research supported in part by the National Research Foundation of South Africa (Grant No: 85596)

Nomenclature

a – stretching index, [s^{-1}]
C – concentration of the fluid, [–]
C_p – specific heat capacity, [$Jkg^{-1}K^{-1}$]
Da – local Darcy parameter, [–]
Du – Dufour number, [–]
F_s – local Forchheimer parameter, [–]
g – acceleration due to gravity, [ms^{-2}]
Gr_t – modified thermal Grashof number, [–]
M – magnetic field parameter, [–]
P_p – porosity parameter, [–]
Pr – Prandtl number, [–]
S – suction parameter, [–]
Sc – Schmidt number, [–]
Sh – Scherwood number, [–]
T – temperature of the fluid, [K]
t – time, [s]
u, v – velocity components, [ms^{-1}]
V_T – thermophoretic velocity, [ms^{-1}]

Greek symbols

ϑ – kinematic viscosity, [m^2s^{-1}]
κ – thermal conductivity, [$WK^{-1}m^{-1}$]
λ – thermophoretic parameter, [–]
μ – dynamic viscosity, [$kgm^{-1}s^{-1}$]
ρ – density of the fluid, [kgm^{-3}]
σ – electric conductivity, [$Wm^{-2}K^{-4}$]
τ_w – shear stress, [–]

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Paper submitted: October 10, 2014
Paper revised: January 1, 2015
Paper accepted: February 2, 2015