THE DIFFUSION MODEL OF FRACTAL HEAT AND MASS TRANSFER IN FLUIDIZED BED A Local Fractional Arbitrary Euler-Lagrange Formula

by

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In this manuscript, the local fractional arbitrary Euler-Lagrange formula are utilized to address the diffusion model of fractal heat and mass transfer in a fluidized bed based on the Fick's law with local fractional vector calculus.

Key words: *diffusion equation, Fick' law, fluidized bed, local fractional vector calculus*

Introduction

A great interest in the theoretical study of heat transfer in fluidized beds has been shown during the past ten years. For example, the thermodynamics in fluidized bed with moist particles was proposed in [1-6]. The heat transfers of fluidized bed with geldart type-D particles, oxy-fuel fluidized bed boilers [7] and gas-solid fluidized beds [8] were presented. As one of models of fluidized beds [9, 10], the diffusion model was investigated by many authors, such as Tardos *et al.* [11], Hoebink *et al.* [12], Bukur *et al.* [13], Van Ballegooijen *et al.* [14], and Wang and Chen [15]. In [16], the hybrid Euler-Lagrange approach was considered to discuss the particle transport process of circulating fluidized bed. The Euler-Lagrange formula was applied to address the heat transfer of carrot cubes in a spout-fluidized-bed drier with moving boundaries [17].

In fact, the roughness surface of fractal materials can be designed [18, 19] to describe gas-solids flow in a circulating fluidized bed [20]. Fractal analysis of three-phase fluidized bed was suggested by Fan *et al.* [21] and Kikuchi *et al.* [22]. Recently, the local fractional vector calculus structured in [23] was utilized to describe fractal problems in various areas, such as quantum mechanics [24] and fluid mechanics [25-27]. The diffusion on solid [28] and its non-differentiable solution [29] were discussed.

The physical parameters of the classical diffusion model in [11-17] describing the heat and mass transfer in a fluidized bed is differentiable. However, the non-differentiable version of the diffusion model for the fractal heat and mass transfer in a fluidized bed is not considered. The present work develops the local fractional diffusion model of fractal heat and mass transfer in a fluidized bed and the local fractional arbitrary Euler-Lagrange formula.

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The diffusion model of fluidized bed *via* local fractional derivative

Fractal mass transfer in fluidized bed

Let the quantities $\vec{\mathrm{J}}(r,t)$ and $\theta(r,t)$ be the flux of solute and the potential related to the concentration in fluidized bed, respectively. Due to the local fractional vector surface integral A (from Appendix), the integral form of the local fractional Fick's law in fluidized bed becomes:

$$\oiint \vec{\mathrm{J}}(r,t)\mathrm{d}\vec{\mathrm{S}}^{(\beta)} = -\oiint D(\theta)\nabla^{\alpha}\theta(r,t)\mathrm{d}\vec{\mathrm{S}}^{(\beta)} \tag{1}$$

which leads to:

$$\oiint [\vec{\mathrm{J}}(r,t) + D(\theta)\nabla^{\alpha}\theta(r,t)]\mathrm{d}\vec{\mathrm{S}}^{(\beta)} = 0 \tag{2}$$

where $D(\theta)$ is a fractal diffusion coefficient in fluidized bed.

For any $\mathrm{d}\vec{\mathrm{S}}^{(\beta)}$ in (2) the differential form of the local fractional Fick's law in fluidized bed can be written as [28]:

$$\vec{\mathrm{J}}(r,t) = D(\theta)\nabla^{\alpha}\theta(r,t) \tag{3}$$

If $D(\theta) = D,$ (4) changes [28]:

$$\vec{\mathrm{J}}(r,t) = -D\nabla^{\alpha}\theta(r,t) \tag{4}$$

where D is a fractal diffusion coefficient in fluidized bed.

Using the local fractional volume integral B (from Appendix), the local fractional conservation of fractional mass in fluidized bed results in:

$$\frac{\mathrm{d}^{\alpha}}{\mathrm{d}t^{\alpha}}\iiint \theta(r,t)\mathrm{d}V^{(\gamma)} = -\oiint \vec{\mathrm{J}}(r,t)\mathrm{d}\vec{\mathrm{S}}^{(\beta)} \tag{5}$$

From (5) and C (from Appendix), we get:

$$\iiint \left[\frac{\mathrm{d}^{\alpha}\theta(r,t)}{\mathrm{d}t^{\alpha}} + \nabla^{\alpha}\vec{\mathrm{J}}(r,t)\right]\mathrm{d}V^{(\gamma)} = 0 \tag{6}$$

For any $\mathrm{d}V^{(\gamma)}$ in (6) the differential form of local fractional conservation of fractional mass in fluidized bed is expressed as:

$$\frac{\mathrm{d}^{\alpha}\theta(r,t)}{\mathrm{d}t^{\alpha}} + \nabla^{\alpha}\vec{\mathrm{J}}(r,t) = 0 \tag{7}$$

Submitting (4) into (7) gives the diffusion model for mass transfer in fluidized bed:

$$\frac{\mathrm{d}^{\alpha}\theta(r,t)}{\mathrm{d}t^{\alpha}} + \nabla^{\alpha}[-D\nabla^{\alpha}\theta(r,t)] = 0 \tag{8}$$

or

$$\frac{d^{\alpha}\theta(r,t)}{dt^{\alpha}} - D\nabla^{2\alpha}\theta(r,t) = 0 \tag{9}$$

subject to the initial-boundary conditions:

$$\theta(r,0)=\theta_0(r) \tag{10}$$

$$-D\nabla^\alpha\theta(r,t)\Big|_{S^{(\beta)}}=\eta[\theta_s(r,t)-\theta_e(r,t)], \quad t>0 \tag{11}$$

where $\theta_e(r,t)$ is the equilibrium moisture content in fluidized bed and $\nabla^{2\alpha}=\nabla^\alpha\nabla^\alpha$. If the fractal dimension α is equal to 1, the diffusion model for fractal mass transfer in fluidized bed (9) becomes classical one [15, 17]. We notice that, using the fractal complex transform [30], the classical diffusion model for mass transfer in fluidized bed is transferred into (9).

Fractal heat transfer in fluidized bed

The local fractional Fourier law in fluidized bed can be written as [23]:

$$\phi(r,t)=-\mu\nabla^\alpha T(r,t) \tag{12}$$

where μ is a constant and $T(r, t)$ is a non-differentiable temperature at the point $r\in V^{(\gamma)}$, time $t\in T$.

The first law of thermodynamics in fractal media states [23]:

$$\iiint_{V^{(\gamma)}}\rho_\alpha c_\alpha\frac{\partial^\alpha T(r,t)}{\partial t^\alpha}\,\mathrm{d}V^{(\gamma)}=\iint_{S^{(\beta)}}\phi(r,t)\mathrm{d}\vec{\mathrm{S}}^{(\beta)} \tag{13}$$

where ρ_α and c_α are the density and the specific heat of the fractal material, respectively.

Due to C (from Appendix) and (12), (13) can be rewritten as:

$$\iiint_{V^{(\gamma)}}\rho_\alpha c_\alpha\frac{\partial^\alpha T(r,t)}{\partial t^\alpha}\,\mathrm{d}V^{(\gamma)}=\iiint_{V^{(\gamma)}}\mu\nabla^{2\alpha}T(r,t)\mathrm{d}V^{(\gamma)} \tag{14}$$

which yields:

$$\iiint_{V^{(\gamma)}}\left[\mu\nabla^{2\alpha}T(r,t)-\rho_\alpha c_\alpha\frac{\partial^\alpha T(r,t)}{\partial t^\alpha}\right]\mathrm{d}V^{(\gamma)}=0 \tag{15}$$

For any $\mathrm{d}V^{(\gamma)}$ in (6) fractal heat transfer in fluidized bed is suggested as:

$$\mu\nabla^{2\alpha}T(r,t)-\rho_\alpha c_\alpha\frac{\partial^\alpha T(r,t)}{\partial t^\alpha}=0 \tag{16}$$

subject to the initial-boundary conditions:

$$T(r,0)=T_0(r) \tag{17}$$

$$-\mu\nabla^\alpha T(r,t)=\varsigma[T_s(r,t)-T_a(r,t)]-\lambda\{D\nabla^\alpha[\rho\theta(r,t)]\}, \qquad t>0 \tag{18}$$

where λ is a medium property in fluidized bed. If the fractal dimension α is equal to 1, the diffusion model for heat transfer in fluidized bed (16) becomes classical one [15, 17]. In another way, the classical diffusion model for mass transfer in fluidized bed is transferred into (16) using the fractal complex transform [30].

The local fractional arbitrary Euler-Lagrange formula

The fractal mass transfer in fluidized bed is written in the weak form:

$$\iiint \left[\frac{\mathrm{d}^{\alpha}\theta(r,t)}{\mathrm{d}t^{\alpha}} - D\nabla^{2\alpha}\theta(r,t) \right] \theta^{*}(r,t)\mathrm{d}V^{(\gamma)} = 0 \tag{19}$$

which leads to:

$$\iiint \frac{\mathrm{d}^{\alpha}\theta(r,t)}{\mathrm{d}t^{\alpha}} \theta^{*}(r,t)\mathrm{d}V^{(\gamma)} = -\oiint \vec{\mathrm{J}}(r,t)\theta^{*}(r,t)\mathrm{d}\vec{\mathrm{S}}^{(\beta)} \tag{20}$$

where $\theta^{*}(r, t)$ is an arbitrary non-differentiable test function.

Using (4), we obtain the fractal mass transfer in fluidized bed:

$$\iiint \frac{\mathrm{d}^{\alpha}\theta(r,t)}{\mathrm{d}t^{\alpha}} \theta^{*}(r,t)\mathrm{d}V^{(\gamma)} = \oiint D\nabla^{\alpha}\theta(r,t)\theta^{*}(r,t)\mathrm{d}\vec{\mathrm{S}}^{(\beta)} \tag{21}$$

Following D (from Appendix), the right side of (21) can be written as:

$$\oiint D\nabla^{\alpha}\theta(r,t)\theta^{*}(r,t)\mathrm{d}\vec{\mathrm{S}}^{(\beta)} = \iiint_{V^{(\gamma)}} [D\theta^{*}(r,t)\nabla^{2\alpha}\theta(r,t) + D\nabla^{\alpha}\theta(r,t)\nabla^{\alpha}\theta^{*}(r,t)]\mathrm{d}V^{(\gamma)} \tag{22}$$

which results in:

$$\iiint \frac{\mathrm{d}^{\alpha}\theta(r,t)}{\mathrm{d}t^{\alpha}} \theta^{*}(r,t)\mathrm{d}V^{(\gamma)} = -\oiint D\nabla^{\alpha}\theta(r,t)\theta^{*}(r,t)\mathrm{d}\vec{\mathrm{S}}^{(\beta)} + \iiint_{V^{(\gamma)}} D\nabla^{\alpha}\theta(r,t)\nabla^{\alpha}\theta^{*}(r,t)\mathrm{d}V^{(\gamma)} \tag{23}$$

such that, using (11), we have the local fractional arbitrary Euler-Lagrange formula of fractal mass transfer in fluidized bed:

$$\iiint \frac{\mathrm{d}^{\alpha}\theta(r,t)}{\mathrm{d}t^{\alpha}} \theta^{*}(r,t)\mathrm{d}V^{(\gamma)} = \oiint \eta[\theta_{s}(r,t) - \theta_{e}(r,t)]\theta^{*}(r,t)\mathrm{d}\vec{\mathrm{S}}^{(\beta)} + \\ + \iiint_{V^{(\gamma)}} D\nabla^{\alpha}\theta(r,t)\nabla^{\alpha}\theta^{*}(r,t)\mathrm{d}V^{(\gamma)} \tag{24}$$

The weak form of the heat transfer in fluidized bed reads as:

$$\iiint \left[\mu\nabla^{2\alpha}T(r,t) - \rho_{\alpha}c_{\alpha} \frac{\partial^{\alpha}T(r,t)}{\partial t^{\alpha}} \right] T^{*}(r,t)\mathrm{d}V^{(\gamma)} = 0 \tag{25}$$

where $T^{*}(r, t)$ is an arbitrary non-differentiable test function.

From (25) we have:

$$\iiint T^{*}(r,t)\mu\nabla^{2\alpha}T(r,t)\mathrm{d}V^{(\gamma)} = \iiint \rho_{\alpha}c_{\alpha}T^{*}(r,t) \frac{\partial^{\alpha}T(r,t)}{\partial t^{\alpha}} \mathrm{d}V^{(\gamma)} \tag{26}$$

From D, the left side of (26) can be written as:

$$\iiint T^{*}(r,t)\mu\nabla^{2\alpha}T(r,t)\mathrm{d}V^{(\gamma)} = \oiint_{S^{(\beta)}} T^{*}(r,t)\mu\nabla^{\alpha}T(r,t)\mathrm{d}\vec{\mathrm{S}}^{(\beta)} -$$

$$-\iiint_{V^{(\gamma)}} \mu\nabla^{\alpha}T(r,t)\nabla^{\alpha}T^{*}(r,t)\mathrm{d}V^{(\gamma)} \tag{27}$$

which, applying (18), yields:

$$\iiint T^{*}(r,t)\mu\nabla^{2\alpha}T(r,t)\mathrm{d}V^{(\gamma)} = -\oint\!\!\!\oint_{S^{(\beta)}} T^{*}(r,t)\left\{\varsigma[T_{s}(r,t)-T_{a}(r,t)]-\lambda\{D\nabla^{\alpha}[\rho\theta(r,t)]\}\right\}\mathrm{d}S^{(\beta)} -$$

$$-\iiint_{V^{(\gamma)}} \mu\nabla^{\alpha}T(r,t)\nabla^{\alpha}T^{*}(r,t)\mathrm{d}V^{(\gamma)} \tag{28}$$

Submitting (28) into (26), we obtain:

$$\iiint \rho_{\alpha}c_{\alpha}T^{*}(r,t)\,\frac{\partial^{\alpha}T(r,t)}{\partial t^{\alpha}}\,\mathrm{d}V^{(\gamma)} =$$

$$= -\oint\!\!\!\oint_{S^{(\beta)}} T^{*}(r,t)\left\{\varsigma[T_{s}(r,t)-T_{a}(r,t)]-\lambda\{D\nabla^{\alpha}[\rho\theta(r,t)]\}\right\}\mathrm{d}\vec{\mathrm{S}}^{(\beta)} -$$

$$-\iiint_{V^{(\gamma)}} \mu\nabla^{\alpha}T(r,t)\nabla^{\alpha}T^{*}(r,t)\mathrm{d}V^{(\gamma)} \tag{29}$$

which is the local fractional arbitrary Euler-Lagrange formula of fractal heat transfer in fluidized bed.

Concluding remarks

From pure mathematical view point, the local fractional diffusion model of fractal heat and mass transfer in fluidized bed is investigated in the framework of the local fractional vector calculus. Using the local fractional Gauss and Green theorems of the fractal vector field and considering an arbitrary non-differentiable test function, the local fractional arbitrary Euler-Lagrange formula of fractal heat and mass transfer in fluidized bed are also discussed. This result is an extended version of the arbitrary Euler-Lagrange formula using the non-differentiable characteristics of the functions in the field.

Appendix

The local fractional vector surface integral of $\vartheta(r)$ on fractal surface $\vec{\mathrm{S}}^{(\beta)}$ is defined as [23, 25, 27]:

$$\iint \vartheta(r_{P})\mathrm{d}\vec{\mathrm{S}}^{(\beta)} = \lim_{N\to\infty}\sum_{P=1}^{N}\vartheta(r_{P})\vec{\mathrm{n}}_{P}\Delta\vec{\mathrm{S}}_{P}^{(\beta)} \tag{A}$$

with N elements of area with a unit normal local fractional vector $\vec{\mathrm{n}}_{P}$, $\Delta S_{P}^{(\beta)}\to 0$ as $N\to\infty$ for $\beta = 2\alpha$.

The local fractional volume integral of $\vartheta(r)$ on fractal volume $V^{(\gamma)}$ is defined as [23, 25, 27]:

$$\iiint \vartheta(r_{P})\mathrm{d}V^{(\gamma)} = \lim_{N\to\infty}\sum_{P=1}^{N}\vartheta(r_{P})\Delta V_{P}^{(\gamma)} \tag{B}$$

with N elements of volume $\Delta V_P^{(\gamma)} \to 0$ as $N \to \infty$ for $\gamma = 3\beta/2 = 3\alpha$.

For $\gamma = 3\beta/2 = 3\alpha,\ 1 > \alpha > 0,$ the local fractional Gauss theorem of the fractal vector field reads as [23, 25, 27]:

$$\iiint\limits_{V^{(\gamma)}} \nabla^{\alpha} \vartheta \mathrm{d}V^{(\gamma)} = \oiint\limits_{S^{(\beta)}} \vartheta \mathrm{d}\vec{\mathrm{S}}^{(\beta)} \tag{C}$$

For $\gamma = 3\beta/2 = 3\alpha,\ 1 > \alpha > 0,$ the local fractional Green theorem of the fractal vector field states [23, 25]:

$$\oiint\limits_{S^{(\beta)}} \theta \nabla^{\alpha} \vartheta \mathrm{d}\vec{\mathrm{S}}^{(\beta)} = \iiint\limits_{V^{(\gamma)}} (\theta \nabla^{2\alpha} \vartheta + \nabla^{\alpha} \vartheta \nabla^{\alpha} \theta) \mathrm{d}V^{(\gamma)} \tag{D}$$

Nomenclature

c_α – specific heat
D – a fractal diffusion coefficient
r – space co-ordinates, [m]
$T(r, t)$ – temperature, [K]
t – time, [s]

Greek symbols

α – time fractal dimensional order, [–]
$\theta(r, t)$ – concentration, [–]
ρ_α – density

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