EXPERIMENTAL VERIFICATION OF APPROXIMATE SOLUTION OF THE INVERSE STEFAN PROBLEM OBTAINED BY APPLYING THE INVASIVE WEED OPTIMIZATION ALGORITHM

by

Edyta HETMANIOK**, Damian SLOTA, and Adam ZIELONKA

Institute of Mathematics, Silesian University of Technology, Gliwice, Poland

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The paper proposes a procedure for solving the inverse Stefan problem consisted in reconstruction of the function describing the heat transfer coefficient on the basis of temperature measurements. Elaborated method is based on two procedures: solution of the appropriate direct Stefan problem by using the finite difference method combined with the alternating phase truncation method and minimization of some functional with the aid of invasive weed optimization algorithm. For verifying the effectiveness of investigated algorithm the experimental data obtained in the solidification of aluminum are used.

Key words: *artificial intelligence, evolutionary algorithms, invasive weed optimization algorithm, inverse Stefan problem*

Introduction

Direct Stefan problem is a mathematical model characterizing the thermal processes in which the phase transition takes place like, for example, solidification of pure metals, melting of ice, freezing of water or deep freezing of food articles. Processes of that kind are described with the aid of heat conduction equation completed by the particular initial conditions, boundary conditions and the Stefan condition defined on the boundary between two phases [1]. Evaluation of such system of equations enables to determine the temperature distribution and the freezing front location representing the solution of direct Stefan problem. However goal of this paper is to solve the inverse Stefan problem. Term "inverse problem" serves for denoting the task in which some input information is missing and must be reconstructed, like, for instance, the form of initial condition, boundary conditions or parameters of material. Thus, for making the inverse Stefan problem possible to solve, the missing information must be compensated by some supplementary information which can be the known location of freezing front, its velocity in normal direction or values of temperature in the selected points of domain. Difficulty in solving the inverse Stefan problem is caused by the fact that it belongs to the group of ill posed problems, which means that finding its solution in analytical way is almost impossible, moreover, the solution may not exist at all or it exists but may be neither unique nor stable [2]. Therefore the constant research are engaged to develop some efficient procedures for determining the approximate solution of inverse Stefan problem [3-8], and any new proposals of methods useful for finding solution of this problem are interesting and welcome.

* Corresponding author; e-mail: edyta.hetmaniok@polsl.pl

Authors of this paper have already dealt with the direct and inverse Stefan problem by using the analytical methods [9, 10] as well as the approximate methods applying the algorithms of artificial intelligence inspired by the behaviors from real word [11-13] where the artificial bee colony algorithm, ant colony optimization algorithm and selected immune algorithms are used, respectively. In the current paper we continue the subject by employing the invasive weed optimization (IWO) algorithm – one of the recently invented evolutionary algorithms based on the tactics used by weed colony for finding a suitable place for spreading over and growth. The algorithm has been first introduced by Mehrabian and Lucas [14], in dynamic and control systems theory, and till now it has found a number of practical applications, for example, in tuning of a robust controller [14], analysis of electricity markets [15], antenna configuration [16] and others. Investigated inverse Stefan problem consists in reconstruction of the function describing the heat transfer coefficient and the proposed approach is based on two procedures. The first procedure serves for solving the direct Stefan problem, associated with the discussed inverse one, by using the finite difference method combined with the alternating phase truncation method [5, 7, 17]. The second procedure consists in minimizing an essential functional with the aid of IWO algorithm. For verifying the effectiveness of investigated method we use the experimental data obtained in the solidification of aluminum, similarly as we have already done in [13] verifying the usefulness of selected immune algorithms. Original contribution of the authors consists in the adaptation of IWO algorithm to make it a part of the procedure useful for solving the inverse Stefan problem and in the proper selection of parameters of IWO algorithm for its practical applications. The values of parameters were selected on the basis of our previous calculations made for the well-known benchmark functions typical for optimization tasks, as well as on our previous experiences get in solving similar problems of considered kind, for example [18].

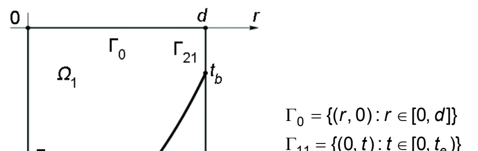

Figure 1. Domain of the problem

Problem formulation

Domain of the discussed problem is shown in fig. 1. Region $\Omega = [0, d] \times [0, t^*]$, presented there, is divided into two subregions: Ω_1 and Ω_2 taken by the liquid and solid phase, respectively, and its boundary is divided into five parts like it can be seen in the figure. Symbol Γ_g denotes the interface (freezing front), location of which is described by function $r = \xi(t)$.

In considered two-phase axisymmetric Stefan problem the distributions T_k of temperature satisfy inside regions Ω_k (k = 1, 2) the heat conduction equation:

$$c_k \, \rho_k \frac{\partial T_k}{\partial t}(r, t) = \frac{1}{r} \frac{\partial}{\partial r}\left[\lambda_k r \frac{\partial T_k}{\partial r}(r, t) \right] \tag{1}$$

where c_k, ρ_k, and λ_k are the specific heat, mass density, and thermal conductivity coefficient in liquid phase (k = 1), and solid phase (k = 2), respectively, whereas t and r refer to the time and spatial location. Thus, solving of the Stefan problem consists in determination of functions T_k fulfilling eq. (1) as well as the initial condition on boundary Γ_0 ($T_0 > T^*$):

$$T_1(r, 0) = T_0 \tag{2}$$

the boundary conditions of the second kind on boundaries Γ_{1k} $(k = 1, 2)$:

$$\frac{\partial T_k}{\partial r}(0, t) = 0 \tag{3}$$

and the boundary conditions of the third kind on boundaries Γ_{2k} $(k = 1, 2)$:

$$-\lambda_k \frac{\partial T_k}{\partial r}(d, t) = \alpha(t)[T_k(d, t) - T_\infty] \tag{4}$$

where α denotes the heat transfer coefficient, T_0 is the initial temperature, and T_∞ means the ambient temperature. Moreover, on interface Γ_g the condition of temperature continuity and the Stefan condition must be satisfied to ensure compatibility of the temperatures and heat fluxes on the freezing front:

$$T_1[\xi(t), t] = T_2[\xi(t), t] = T^* \tag{5}$$

$$L\rho_2 \frac{\mathrm{d}\xi}{\mathrm{d}t} = -\lambda_1 \left.\frac{\partial T_1(r,t)}{\partial r}\right|_{r=\xi(t)} + \lambda_2 \left.\frac{\partial T_2(r,t)}{\partial r}\right|_{r=\xi(t)} \tag{6}$$

where T^* denotes the solidification temperature, and L describes the latent heat of fusion.

Goal of this paper lies in solving the inverse Stefan problem, which means in this case an identification of function α describing the heat transfer coefficient on boundaries Γ_2. The additional information, thanks to which the solution of inverse problem will be possible to do in general, is given by the temperature measurements in selected points of the solid phase $(r_i, t_j) \in \Omega_2$:

$$T_2(r_i, t_j) = U_{ij}, \qquad i = 1, ..., N_1, \qquad j = 1, ..., N_2$$

where N_1 denotes the number of sensors and N_2 is the number of measurements taken from each sensor.

Proposed method of solution is the following: for the given form of coefficient α we solve the direct Stefan problem, given by eqs. (1)–(6), by using the finite difference method connected with the alternating phase truncation method [5, 7, 17]. In this way we obtain the values of temperature T_{ij} associated with the given α. For determining the form of sought coefficient α, that is for reconstructing as best as possible its real values, we minimize functional:

$$J(\alpha) = \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} (T_{ij} - U_{ij})^2 \tag{7}$$

representing the differences between given measurements U_{ij} and temperatures T_{ij} calculated in the successive stages of the procedure for fixed α. For minimizing functional defined in eq. (7) we apply the invasive weed optimization algorithm.

Invasive weed optimization algorithm

IWO algorithm is one of the recently discovered evolutionary algorithms representing the group of artificial intelligence algorithms inspired by the mechanisms of biological evolution [19]. Weeds are the unwanted plants disturbing the growth of desired plants in agriculture areas controlled by people, like farm fields and gardens. Moreover, the word "weeds" describes some plants that quickly reproduce and grow very aggressively and invasively. Each invading weed uses the free piece of field and grows independently to flowering weed, ready for producing new weeds. Number of produced seeds depends on the adaptation of a weed to the conditions. The better adaptation to the environment, the faster growth and better possibilities for producing seeds. Thus, the weaker weeds are eliminated and the best adapted ones grow more and more intensively.

Described natural process is imitated in the following way: particular solutions of the considered problem play the role of single weed. Measure of the adaptation is given by the fitness function depending on the problem under consideration. Individuals with the best value of fitness function create the most of seeds disseminated in some distance which gives them the possibility to find better place with better adaptation. The process is continued the assumed number of times and hopefully the obtained plant with the best value of fitness function is the closest to the optimal solution. The conception is realized in the following steps [14]:

(1) Random generation of n individuals composing the initial population.

(2) For each individual the value of fitness function is calculated by formula:

$$fit(\mathbf{x}) = \frac{1}{1 + f(\mathbf{x})} \tag{8}$$

where f denotes the minimized function.

(3) Determination of the number of seeds for each individual.
The number of seeds S_j of the given individual means its chances for reproduction, therefore it should to be the greater, the better is the adaptation of considered individual. Thus, the number of seeds is expressed in the following way:

$$S_j = S_{\min} + \left\lfloor (fit_j - fit_{\min}) \frac{S_{\max} - S_{\min}}{fit_{\max} - fit_{\min}} \right\rfloor \tag{9}$$

where fit_j, $fit_{\max}$, and $fit_{\min}$ denote the values of fitness function of the given individual, the best and the worst population member, respectively, $S_{\max}$ and $S_{\min}$ describe the assumed admissible maximal and minimal number of seeds and $\lfloor \cdot \rfloor$ is the integer part.

(4) Dispersion of seeds over the region and creation of new individuals.
New individual $\mathbf{x} = [x_1, ..., x_n]$ is created by the random generation of x_i, but such that the seed "falls on the ground" at the determined distance from the senior individual. Distance of the admissible seeds' fly is described by the normal distribution with zero mean value and standard deviation decreasing in each iteration according to the formula:

$$\sigma_i = \left(\frac{I_{\max} - i}{I_{\max}} \right)^3 (\sigma_{\text{init}} - \sigma_{\text{fin}}) + \sigma_{\text{fin}} \tag{10}$$

where $I_{\max}$ represents the maximal number of iterations, i is the number of current iteration, whereas σ_{init} and σ_{fin} denote the assumed initial and final values of standard deviation.

(5) Calculation of the values of fitness function for new individuals created in the previous step.

(6) Selection of n best adapted individuals from among the new individuals and members of the former population. The n selected best individuals create the new population.

(7) Steps 2-6 are repeated $I_{\max}$ times.

Experimental verification

For verifying the elaborated procedure serving for reconstruction of the boundary conditions in the inverse Stefan problem we used the experimental data obtained in the process of solidification of aluminum EN AW-Al99.5. The experiment was executed with the use of UMSA (universal metallurgical simulator and analyzer) equipment serving for analysis of thermal processes in metals [20, 21]. In the experiment four cylinder samples, each one of 18 mm diameter and 20 mm height, were used. The charge material was melted down in the induction crucible furnace and cast into a graphite chill-mould of 25 mm diameter. Next, the material was mechanically worked out for adopting it to the required dimensions. In two first samples the thermocouple was located in the axis of sample, whereas in two next samples the thermocouple was placed in the distance of 4.5 mm away from the axis. The bottom and top surfaces of the samples were thermally insulated. During the experiment three rounds of melting and solidification processes of the sample material were performed, which means that from each sample three distributions of temperature were obtained.

For the cause of the region geometry and the thermal symmetry, we decided to model the problem with the aid of two-phase axisymmetric one-dimensional Stefan problem expressed by means of eqs. (1)-(6) for the following values of parameters: λ_1 = 104 W/mK, λ_2 = 240 W/mk, c_1 = 1290 J/kgK, c_2 = 1000 J/kgK, ρ_1 = 2380 kg/m^3, ρ_2 = 2679 kg/m^3, L = 390000 J/kg, T^* = 930 K, T_∞ = 298 K, T_0 = 1013 K. Solution of investigated task is the reconstructed form of function α describing the heat transfer coefficient and depending on various number of parameters:

$$\alpha(t) = \alpha(t; \alpha_1, \alpha_2, ..., \alpha_k), \qquad k \in \{1, 3, 6, 10\}$$

For approximating this function we used the Bezier curves [22] (for k = 1 the heat transfer coefficient was approximated with constant function).

Developed method consists of two procedures. First of them, required for solving the appropriate direct Stefan problem, is the finite difference method associated with the alternating phase truncation method [5, 7, 17] performed for the mesh of steps $\Delta t = 0.1$ and Δr = d/500. The second one is needed for minimizing functional (7) in order to determine such values of the sought coefficient that the differences between the measured and reconstructed values of temperature will be as small as possible. For this purpose we used the IWO algorithm executed for the following values of parameters: maximal and minimal number of seeds $S_{\max} = 5$, $S_{\min} = 1$, initial and final standard deviation $\sigma_{\text{init}} = 5$, $\sigma_{\text{fin}} = 0.1$. These optimal values of parameters are the results of several testing calculations and our previous experiences. Maximal numbers $I_{\max}$ of iterations as well as the numbers n of individuals in one population, needed to obtain satisfying results, were different for different numbers k of parameters characterizing the reconstructed function α – for $k = 1$ we used $n = 10$ and $I_{\max} = 50$, for $k = 3$ we had $n = 10$ and $I_{\max} = 100$, for $k = 6$ we took $n = 20$ and $I_{\max} = 150$, and finally, for $k = 10$ we used $n = 40$ and $I_{\max} = 300$. The initial population was randomly selected from the range [0, 100]. Another fact, which should be taken into account, is the heuristic nature of IWO algorithm, which means that each execution of the procedure can give slightly different results. Therefore we evaluated the calculations for 10 times in each considered case and we accepted the best of obtained results as the approximate values of reconstructed coefficient. Let us also notice that each execution of the procedure requires to solve for many times the appropriate direct Stefan problem.

Obtained distributions of the heat transfer coefficient reconstructed by applying the elaborated procedure for various numbers of parameters are presented in fig. 2.

Reconstructions of the cooling curve calculated for the cases of function α identified for one parameter and ten parameters, respectively, are illustrated in fig. 3. The figure presents comparison of the temperature distribution obtained for measurement data and calculated for the reconstructed function α (for one and ten parameters, respectively) together with the absolute error of this reconstruction. Mean and maximal values of relative and absolute errors of the cooling curve reconstruction obtained for various numbers of parameters characterizing sought function α are compiled in tab. 1.

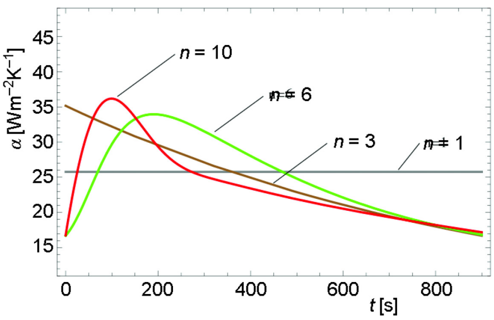

Figure 2. Distribution of the heat transfer coefficient reconstructed for various number of parameters characterizing function *α*

Table 1. Errors of the cooling curve reconstructions for various numbers of parameters characterizing function *α*

α	1	3	6	10
δ_{mean} [%]	4.764	0.714	0.301	0.189
δ_{max} [%]	10.439	6.120	1.565	1.136
Δ_{mean} [K]	27.314	3.162	2.062	1.162
Δ_{max} [K]	69.550	42.842	14.330	10.450

Presented results show that the increasing number of parameters characterizing the reconstructed heat transfer coefficient α implies more and more better reconstruction of cooling curve. Values of the mean and maximal errors collected in tab. 1, absolute and relative as well, decrease with the increasing number of parameters. Illustration of the cooling curve reconstruction shown in fig. 3 indicates the significant differences between the reconstruction received for one and for ten parameters describing function α. Further increase of the number of parameters does not improve significantly the quality of reconstruction, it only extends the time of calculations. Obviously, the bigger number of reconstructed parameters requires the

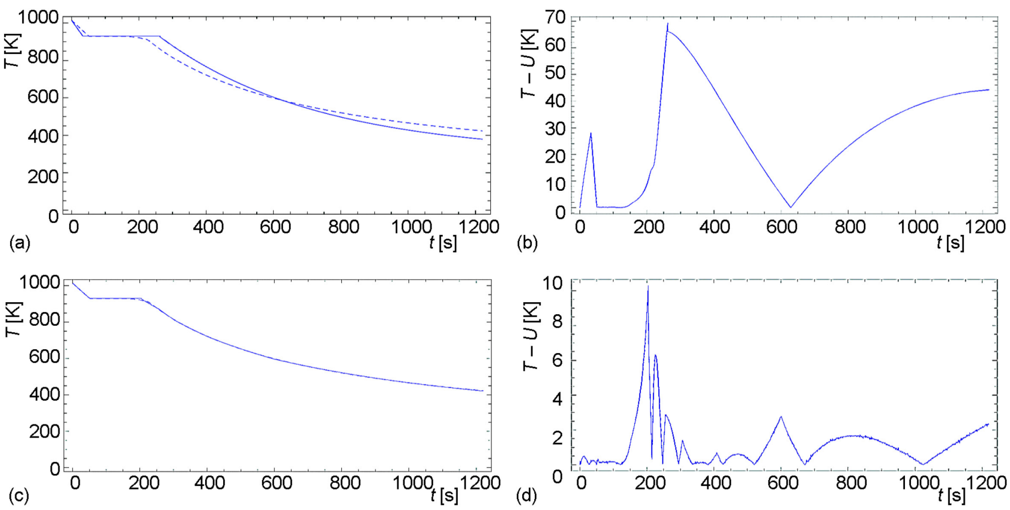

Figure 3. Reconstruction of the cooling curve (dashed line – measurement data, solid line – reconstructed curve) and absolute error of this reconstruction obtained for one parameter, figs. (a) and (b), and for ten parameters, figs. (c) and (d), characterizing function *α*

bigger number of individuals and iterations in IWO algorithm to obtain satisfying results. This is important conclusion because each execution of elaborated procedure requires in fact to solve the direct problem associated with the considered inverse problem. Therefore one should reasonably balance between the expected exactness of the results and the acceptable time of computations.

We have mentioned that as the approximate values of reconstructed parameters we accepted the best ones of results obtained in 10 executions of the procedure. Dispersion of computed values of minimized functional was very small in all experiments (in cases of identifying one, three, six, and ten parameters the dispersions were equal to 0.001%, 0.003%, 1.673%, and 1.465%, respectively), which confirms stability of elaborated procedure.

Conclusions

The paper presented a proposal of the procedure serving for identification of the heat transfer coefficient and reconstruction of the cooling curve on the basis of known measurements of temperature in the process described with the aid of inverse Stefan problem. Essential part of the method consists in minimization of the functional expressing the error of approximate solution and realized by using the invasive weed optimization algorithm. Function describing the heat transfer coefficient depended on various numbers of parameters, increasing number of which ensured better reconstruction of measurement data. Executed calculations showed that ten parameters is enough to obtain satisfying results, especially in the face of fact that increasing number of parameters requires more individuals and iterations in the IWO algorithm which implies the longer time of computations. Summing up, presented results indicate that the applied mathematical model connected with the elaborated procedure ensures the very good reconstruction of experimental data.

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Nomenclature

c – specific heat, [Jkg^{-1}K^{-1}]
d – length, [m]
fit – fitness function
I_{max} – maximal number of iterations in IWO
J – minimized functional
L – latent heat of fusion, [Jkg^{-1}]
n – number of individuals in IWO
N_1 – number of sensors
N_2 – number of measurements
r – spatial location, [m]
S – number of seeds
T – temperature, [K]
T_{ij} – computed temperature, [K]
T_0 – initial temperature, [K]
T^* – solidification temperature, [K]
T_∞ – ambient temperature, [K]
t – time, [s]
t^* – length of the time interval, [s]
U_{ij} – measured temperature, [K]
$\mathbf{x}$ – individual in IWO

Greek symbols

α – heat transfer coefficient, [Wm^{-2}K^{-1}]
Γ – boundary of the region
Δ – absolute error
δ – relative percentage error, [%]
λ – thermal conductivity, [Wm^{-1}K^{-1}]
ρ – mass density, [kgm^{-3}]
σ – standard deviation in IWO
ξ – location of freezing front
Ω – region of the problem

Subscripts

1 – liquid phase
2 – solid phase
init – initial
fin – final
max – maximal
min – minimal

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