MODELING AND IDENTIFICATION OF HEAT EXCHANGER PROCESS USING LEAST SQUARES SUPPORT VECTOR MACHINES

by

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In this paper, Hammerstein model and non-linear autoregressive with eXogeneous inputs (NARX) model are used to represent tubular heat exchanger. Both models have been identified using least squares support vector machines based algorithms. Both algorithms were able to model the heat exchanger system without requiring any a priori assumptions regarding its structure. The results indicate that the blackbox NARX model outperforms the NARX Hammerstein model in terms of accuracy and precision.

Key words: Hammerstein model, heat exchanger, identification, support vector machine

Introduction

In many engineering processes, heat exchangers are important thermal devices, such as power and chemical plants, oil refineries, gas turbines, boilers, and turbofans, etc. Turbulent steam heat exchanger (TSHE) is one of the most common types of heat exchangers. The general function of the TSHE is to transfer heat from a hot fluid flow to a cold fluid flow, in most cases through an intermediate metallic wall and without moving parts. From a modeling approach, heat exchangers are complex systems involving ill-defined dynamics, non-linearities and time-varying characteristics [1-5]. The non-linear behavior and complexity of heat exchangers make the control of a heat exchanger a complex process because of many phenomena such as leakage, friction, temperature-dependent flow properties, contact resistance, unknown fluid properties, etc. [6, 7]. The goal of system identification is to find mathematical equation that gives approximation to the actual behavior of a real system [8]. In [9], author gave subjective views on some essential feature in the area of non-linear models identification. Fu and Li [10], surveyed traditional methods of linear system identification and modern methods of non-linear system identification, fuzzy logic, genetic algorithm, swarm algorithm, multi innovation algorithm, and hierarchical algorithms in hopes of bringing benefits to related researchers and engineers. Identification of non-linear systems is very challenging research area. Having an accurate system model is important but it is not easy to identify.

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Since the model is an approximation to the true system, there is trade-off between the simplicity of the model and the accuracy of its prediction. Block structure models, series of static non-linearities and dynamic linear system might be compromise [11].

In this paper, Hammerstein model and non-linear autoregressive with eXogeneous inputs (NARX) model are used to represent tubular heat exchanger. Hammerstein models, cascade of memoryless non-linearity followed by a dynamic linear block is widely used to represent many practical systems [12-14]. Many identification algorithms have been developed for Hammerstein models since Narendra’s method [15]. Bako et al. [16] proposed a recursive subspace identification algorithm for state space Hammerstein models using least squares support vector machines (LS-SVM) to estimate the non-linear part of the system and ordinary least squares for recovering the linear part.

Jalaleddini and Kearney [17] extended Verhagen identification algorithm for Hammerstein system to estimate directly the coefficient of the basis function expansion of the non-linearity and the state space model of the linear component and applied to stretch reflex identification. In [18], authors proposed an identification algorithm for Hammerstein systems with a symmetric two segment piecewise linear non-linearities using improve particle swarm optimization algorithm. Simulation showed that the calculation complexity of the proposed algorithm is less than both the over parameter method and iteration algorithm. Most of these methods assumes apriori knowledge about the structure of non-linearity. Support vector machines (SVM) and LS-SVM have the ability in approximating non linear function without requiring apriori structural information [19]. Goethals et al. [20] proposed a method for the identification of Hammerstein models based on LS-SVM.

Leontaritis and Billings [21] introduced input-output parametric models for non-linear systems called black box NARX models. The NARX model is widely used for identification of non-linear single input single output systems [22]. Many papers have been published to propose algorithms for modeling and identification of black box NARX systems [23, 24].

In this work, two LS-SVM identification algorithms proposed in [20] and [24] are used to identify a Hammerstein model and a black box NARX model for heat exchanger process. The two models are going to be compared with each other in terms of accuracy and precision.

**Standard LS-SVM regression algorithm**

The quadratic $\varepsilon$-insensitive loss function is selected in the LS-SVM regression. The optimization problem of the LS-SVM regression is formulated:

$$
\min \ J(w, \xi) = \frac{1}{2} w^T w + \frac{c}{2} \sum_{i=1}^{N} \xi_i^2
$$

subject to

$$
y_i = w \phi(x_i) + b + \xi_i, i = 1, \ldots, N
$$

The Lagrangian is defined:

$$
L(w, b, \xi, a, c) = \frac{1}{2} w^T w + \frac{c}{2} \sum_{i=1}^{N} \xi_i^2 - \sum_{i=1}^{N} a_i [w \phi(x_i) + b + \xi_i - y_i]
$$

where $a_i$ are Lagrange multipliers.

From the optimality conditions:
\[
\frac{\partial L}{\partial w} = 0, \quad \frac{\partial L}{\partial b} = 0, \quad \frac{\partial L}{\partial \xi_i} = 0, \quad \frac{\partial L}{\partial a_i} = 0
\]

we have:

\[
w = \sum_{i=1}^{N} a_i \varphi(x_i), \quad \sum_{i=1}^{N} a_i = 0, \quad a_i = c \xi_i, \quad w \varphi(x_i) + b + \xi_i - y_i = 0
\]

From eq. (3), the optimization problem can be rewritten:

\[
\begin{bmatrix}
0 \\
1 \\
\vdots \\
1
\end{bmatrix}
\begin{bmatrix}
1 & \cdots & 1 \\
K(x_i, x_j) + \frac{1}{c} & \cdots & K(x_i, x_j) + \frac{1}{c} \\
\vdots & \ddots & \vdots \\
K(x_i, x_j) + \frac{1}{c} & \cdots & K(x_i, x_j) + \frac{1}{c}
\end{bmatrix}
\begin{bmatrix}
b \\
a_1 \\
\vdots \\
a_N
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
y_1 \\
\vdots \\
y_N
\end{bmatrix}
\]

(4)

where \(K(x_i, x_j)\) is kernel satisfying Mercer’s condition [19] defined as \(K(x_i, x_j) = \varphi(x_i)^T \varphi(x_j)\).

The LS-SVM model is:

\[
f(x) = \sum_{i=1}^{N} a_i K(x, x_i) + b
\]

(5)

The LS-SVM for system identification

This section reviews two algorithms developed by Goethals et al. [20], and Falck et al. [24]. In the first paper [20], authors proposed a technique for identification of Hammerstein systems. In [24], an identification algorithm for fully black-box NARX models has been considered.

**Identification of Hammerstein models using LS-SVM**

The identification approach summarized in this section assumes a model structure of the form:

\[
y_i = \sum_{i=1}^{n} a_i y_{i-i} + \sum_{j=1}^{m} b_j f(u_{i-j}) + e_i
\]

(6)

with \(u_t, y_t, \in \mathbb{R}, \{ (u_n, y_n) \}\) a set of input and output measurements and \(e_t\) the equation error which is assumed to be white and \(n\) and \(m\) denote the order of the autoregressive part and the exogenuous part, respectively. The following structure is assumed for the static nonlinearity \(f\):

\[
f(u) = w^T \varphi(u) + b_0
\]

Hence, eq. (6) can be rewritten:

\[
y_i = \sum_{i=1}^{n} a_i y_{i-i} + \sum_{j=1}^{m} b_j \left[ w^T \varphi(u_{i-j}) + b_0 \right] + e_i
\]

(7)

The proposed algorithm focuses on finding estimates for the linear parameters \(a_i, i = 1, \ldots, n\), \(b_j, j = 1, \ldots, m\), and the static non-linearity \(f\).
Algorithm 1. The algorithm for NARX identification of Hammerstein systems using LS-SVM can be summarized:

– Rewrite eq. (6):

$$\hat{y}_t = \sum_{i=1}^{n} a_i y_{t-i} + \sum_{j=1}^{m} w_j \varphi(u_{t-j}) + d + e_t$$  \hspace{1cm} (8)

– Find estimates for $a_i$, $i = 1, \ldots, n$ and $d$ by solving the following set of linear equations:

$$
\begin{bmatrix}
0 & 0 & 1_{N-r+1}^T & 0 \\
0 & 0 & 0 & 1^T_p \\
1 & 0 & K^0 + c^{-1}I & K^0 \\
0 & 0 & K^0I & I_{N-1}^T_{m+1}
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta_f \\
d \\
\gamma_f
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
$$  \hspace{1cm} (9)

where

$$\gamma_f = \begin{bmatrix}
y_{t-1} \\
y_{t-2} \\
\vdots \\
y_{t-n}
\end{bmatrix}$$

$$K(p, q) = \sum_{j=0}^{m} K(u_{p+j-1}, u_{q+j-1})$$

with $r = \text{max}(m, n) + 1$.

– Obtain estimates for $b_j$, and $f$ from rank one approximation of:

$$
\begin{bmatrix}
b_0 \\
\vdots \\
b_m
\end{bmatrix}
\begin{bmatrix}
f(u) \\
f(u) \\
f(u)
\end{bmatrix}^T
= 
\begin{bmatrix}
\alpha_N & \cdots & \alpha_r \\
\alpha_N & \cdots & \alpha_r \\
0 & \cdots & \alpha_N \\
\alpha_N & \cdots & \alpha_r \\
\alpha_N & \cdots & \alpha_r
\end{bmatrix}
\begin{bmatrix}
K(N, 1) & K(N, 2) & \cdots & K(N, N) \\
K(N-1, 1) & K(N-1, 2) & \cdots & K(N-1, N) \\
\vdots & \vdots & \ddots & \vdots \\
K(r-m, 1) & K(r-m, 2) & \cdots & K(r-m, N)
\end{bmatrix}
\begin{bmatrix}
\beta_0 \\
\vdots \\
\beta_m
\end{bmatrix}
+ \sum_{i=1}^{N} K(t, 1)$$  \hspace{1cm} (10)

with $\hat{f}(u)$ an estimate for $f(u) = f(u) - (1/N) \sum_{i=1}^{N} f(u_i)$. After obtaining estimates of $b_j$, estimate for $\sum_{i=1}^{N} f(u_i)$ can be obtained.
\[ \sum_{i=1}^{N} f(u_i) = \frac{Nd}{\sum_{j=0}^{m} b_j} \]

hence

\[ f(u) = f(u) + \left(1 - \frac{1}{N} \sum_{i=1}^{N} f(u_i) \right) \]

– Use the input sequence \([u_1, u_2, \ldots, u_{n-1}]\) and the estimates of the response of the non-linearity to this input \([f(u_1), f(u_2), \ldots, f(u_{n-1})]\), to train a LS-SVM to approximate the non-linear function \(f\).

**Identification algorithm for fully black-box NARX models**

The NARX model is expressed:

\[ y_t = f(y_{t-1}, \ldots, y_{t-n}, u_t, u_{t-1}, \ldots, u_{t-m}) + e_t \] (11)

where \(y_t \in \mathbb{R}\) and \(u_t \in \mathbb{R}\) denote the measured output and input at time, \(t\), respectively and \(f: \mathbb{R}^{n+m+1} \rightarrow \mathbb{R}\) is an unknown function. The equation error \(e_t\) is assumed to be white noise with zero mean and finite variance.

The function, \(f\), in eq. (11) is modeled as a LS-SVM represented in eq. (5). In the sequel a proposed algorithm to estimate the function, \(f\), is presented.

**Algorithm 2.** The algorithm for identification of NARX models using LS-SVM can be summarized:

– Select model orders \(n\) and \(m\).
– Select regularization parameter \(c\) and a kernel function \(K\) (and its parameters).
– Solve the linear system in eq. (4).

**Process description**

The process chosen in this paper is TSHE. It is one of the most common types of heat exchangers. The general function of the TSHE is to transfer heat from a hot fluid flow to a cold fluid flow, in most cases through an intermediate metallic wall and without moving parts. The basic component of a heat exchanger can be viewed as a tube with one fluid (steam) running through it and another fluid (liquid) flowing outside. The structure of the TSHE is shown in fig. 1, where liquid is heated by pressurized saturated steam through a Cu tube.

In this paper, a parallel flow tubes design has been

**Figure 1. Structure of TSHE**
considered. The main motivation for such design is its significance role in various industrial sectors such as process control industry (food manufacturing), superheating processes and thermal power plants [25].

**Mathematical model of TSHE**

A mathematical model of TSHE, shown in fig. 1, is presented in this section. The water is heated by pressurized saturated steam through a Cu tube. The controlled variable is the outlet liquid temperature, $\beta_0(t)$. Among the input variables, the liquid flow rate, $q(t)$, is selected as the control variable, whereas the steam temperature, $\beta_i(t)$, and the inlet liquid temperature, $\beta_h(t)$, are disturbances. The tube is described by a linear co-ordinate $x$, which measures the distance of a generic section from the inlet. Here we assume that the liquid and metal temperatures ($\beta_i$ and $\beta_m$) are functions only of time and the axial co-ordinate $x$, whereas the saturated steam temperature, $\beta_s(t)$, is uniform and independent of the shape of the tube. The $D_i$ and $D_e$ are the internal and the external diameter of the tube, respectively. The liquid speed, $v(t) = q(t)/\mu_1$, where $q(t)$ is the liquid flow rate and $\mu_1$ is the liquid density, is assumed to be uniform in the tube. The process equations due to [26] is given by:

$$
\begin{align*}
\tau_1 \frac{\partial \beta_i(t,x)}{\partial t} + \tau_1 v(t) \frac{\partial \beta_i(t,x)}{\partial t} + \partial_i(t,x) - \beta_m(t,x) = 0 \\
T_m \frac{\partial \beta_m(t,x)}{\partial t} + \beta [\beta_m(t,x) - \beta_i(t,x)] + \partial_m(t,x) - \partial_s(t,x) = 0
\end{align*}
$$

(12)

The coefficients $\tau_1$, $T_m$, and $\beta$ are computed:

$$
\tau_1 = \frac{\mu_1 c_1}{\alpha_i \pi D_i^2}, \quad T_m = \frac{\mu_m c_m}{\alpha_s \pi D_e^2}, \quad \beta = \frac{\alpha_i D_i}{\alpha_s D_e}
$$

where $\mu_1$ and $\mu_m$ are linear densities (mass per length unit), $c_1$ and $c_m$ are specific heats, $\alpha_i$ is the liquid/metal heat transfer coefficient, and $\alpha_s$ is the steam/metal heat transfer coefficient. Assuming that the specific heats $c_1$ and $c_m$ are constant, coefficient $T_m$ is also constant, whereas $\tau_1$ and $\beta$ depend on the liquid speed $v(t)$ through tube with coefficient $\alpha_i$ according to [27]:

$$
\alpha_i = K v^n, \quad \mu = 0.8
$$

The quantitative features of the considered heat exchanger in this paper are defined: the tube length in the heat exchanger where the steam flows through is 2.44 meter, with an inner diameter $D_i = 0.0547$ m and outer diameter $D_e = 0.0613$ m, the nominal heat transfer coefficients $\sigma_i = 754$ J/m$^2$K and $\sigma_s = 3510$ J/m$^2$K, the specific heats are $c_1 = 1$ kcal/kg°C and $c_m = 0.094$ kcal/kg°C, respectively, and the linear densities (mass per length unit) are given by 0.223 and 0.532 kg/m [25].

**The LS-SVM modeling of TSHE**

To test performance of the proposed algorithm, the heat exchanger system presented in [25] is considered. The data were downloaded from the DAISY data base for system identification, see [28]. The data were created: the steady-state corresponding to $v = 0.3$ m/s, $\beta_i = 65$ °C, $\beta_m = 98.765$ °C, $\beta_s = 120$ °C is taken as the nominal point. A sample of 4000 input-output data points was generated at the rate of 1 Hz. The input signal $u(k)$ is made up of 100 steady-state samples in the nominal point, followed by 100 Gaussian distributed samples
centred on the nominal steady-state speed, 600 beta-distributed samples with $a = 2.142$, $b = 1.415$, weighted towards low speed values, 1200 beta-distributed samples with $a = 1.621$, $b = 3.829$, which privilege high speeds, and finally 2000 uniformly distributed samples. The disturbances $\delta_h$ and $\delta_r$ are kept constant and equal to their nominal values. The first 1000 samples were reserved for testing and the rest for training. The training and testing data sets are shown in figs. 2 and 3, respectively.

The NARX Hammerstein model LS-SVM identification algorithm presented in the section Identification of Hammerstein models using LS-SVM, and the LS-SVM identification algorithm for fully black-box NARX model presented in the section Identification algorithm for fully black-box NARX models were employed to the training data. The LS-SVM hyperparameters and the NARX orders ($n$ and $m$) were chosen based on cross-validation method.

**Case 1: The NARX Hammerstein model**

The NARX Hammerstein model LS-SVM identification algorithm presented in the section Identification of Hammerstein models using LS-SVM is applied to the training data. The hyperparameters governing the algorithm were selected based on cross-validation test as
follows. The ARX orders were selected as \( n = 6 \) and \( m = 0 \) as presented in tab. 1 and the LS-SVM hyperparameters were chosen to be \( C = 0.1 \) and \( \sigma = 0.1 \), and \( \sigma = 0.01 \) as shown in tab. 2.

Figure 4 shows Hammerstein model identification algorithm estimate together with measured output for the test data. The identified model accounted for 80.93% of the actual output variance.

Table 1. Case 1, RMSE between true and estimated output for test data set using various values for ARX orders \((n, m)\); the other hyperparameters were fixed as \( C = 10 \) and \( \sigma = 0.1 \)

<table>
<thead>
<tr>
<th>( C )</th>
<th>( \sigma )</th>
<th>( n )</th>
<th>( m )</th>
<th>( RMSE )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.01</td>
<td>5</td>
<td>0</td>
<td>11.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>0</td>
<td>10.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>0</td>
<td>10.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>0</td>
<td>10.93</td>
</tr>
<tr>
<td>10</td>
<td>0.1</td>
<td>6</td>
<td>0</td>
<td>10.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>0</td>
<td>10.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0</td>
<td>11.30</td>
</tr>
</tbody>
</table>

Table 2. Case 1, RMSE between true and estimated output for test data set using various values of LS-SVM hyperparameters were fixed as \( C \) and \( \sigma \); the ARX orders were fixed as \( n = 6 \) and \( m = 0 \)

<table>
<thead>
<tr>
<th>( C )</th>
<th>( \sigma )</th>
<th>( n )</th>
<th>( m )</th>
<th>( RMSE )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.01</td>
<td>6</td>
<td>0</td>
<td>14.72</td>
</tr>
<tr>
<td>0.1</td>
<td>0.01</td>
<td>6</td>
<td>0</td>
<td>10.56</td>
</tr>
<tr>
<td>1</td>
<td>0.01</td>
<td>6</td>
<td>0</td>
<td>10.74</td>
</tr>
<tr>
<td>10</td>
<td>0.01</td>
<td>6</td>
<td>0</td>
<td>10.69</td>
</tr>
<tr>
<td>100</td>
<td>0.01</td>
<td>6</td>
<td>0</td>
<td>10.70</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>6</td>
<td>0</td>
<td>10.56</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>6</td>
<td>0</td>
<td>11.06</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>6</td>
<td>0</td>
<td>13.13</td>
</tr>
</tbody>
</table>
| | 30.20 | 6 | 0 |}

Comparison between Case 1 and Case 2

The LS-SVM identification algorithm for fully black-box NARX model presented in the section Identification algorithm for fully black-box NARX models is employed to the training data. The hyperparameters were chosen based on cross-validation method which resulted in \( C = 10000 \), and \( \sigma = 100 \) as shown in tab. 4. The NARX orders \( n \) and \( m \) were chosen to be 9
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and 6, respectively, tab. 3. The black-box NARX model identification algorithm estimate
together with measured output for the test data is shown in fig. 5. The algorithm gave 97.57%
fit of the identified model to the testing data.

Table 3. Case 2, RMSE between true and estimated output for test data set using various
values for ARX orders \((n, m)\), the other hyperparameters were fixed as
\(C = 10\) and \(\sigma = 10\)

<table>
<thead>
<tr>
<th>(C)</th>
<th>(\sigma)</th>
<th>(n)</th>
<th>(m)</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>7</td>
<td>0</td>
<td>4.93</td>
</tr>
<tr>
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<td>4.88</td>
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<td></td>
<td></td>
<td>10</td>
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<td>4.78</td>
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<tr>
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<td>11</td>
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<td>4.82</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>(C)</th>
<th>(\sigma)</th>
<th>(n)</th>
<th>(m)</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>9</td>
<td>0</td>
<td>2.87</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>5</td>
<td></td>
<td>2.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td></td>
<td>2.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td></td>
<td>2.54</td>
</tr>
</tbody>
</table>

Table 4. Case 2, RMSE between true and estimated output for test data set using various values of LS-
SVM hyperparameters were fixed as \(C\) and \(\sigma\); the ARX orders were fixed as
\(n = 9\) and \(m = 6\)

<table>
<thead>
<tr>
<th>(C)</th>
<th>(\sigma)</th>
<th>(n)</th>
<th>(m)</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
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<td>6</td>
<td>23.90</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>1000</td>
<td>39.76</td>
</tr>
<tr>
<td></td>
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<td>9</td>
<td>6</td>
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<td>9</td>
<td>6</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100000</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>

Figure 5. Outlet temperature of the simulated process (1) and
the Black box NARX model prediction (2)
(for color image see journal web site)

Comparison between Case 1 and Case 2

Table 5 represents statistical measures of performance of NARX Hammerstein model
and black-box NARX model. The performance measures which are used to evaluate the
two cases are defined as follows.
– Sum squared error (SSE) of testing samples
\[ SSE = \sum_{i=1}^{N}(y_i - \hat{y}_i)^2 \] Where \(y_i\) and \(\hat{y}_i\) denote
the measured and estimated outputs, respectively.
– The SSE/SST, which is defined as the ratio between SSE and sum squared deviation of testing samples \( \text{SST} = \sum_{i}^{N_i} (y_i - \bar{y})^2 \). Where \( \bar{y} \) denotes the arithmetic mean of the measured output.
– The SSR/SST, which is defined as the ratio between sum squared deviation that can be explained by the estimator and sum squared deviation of testing samples \( \text{SSR} = \sum_{i}^{N_i} (\hat{y}_i - \bar{y})^2 \).
– Percent fit equal to

$$
100 \left( 1 - \frac{\|y - \hat{y}\|}{\|y - \bar{y}\|} \right).
$$

Table 5. The SSE, SSE/SST, SSR/SST, and percent fit of Case 1 and Case 2

<table>
<thead>
<tr>
<th>Data set</th>
<th>Regressor</th>
<th>SSE</th>
<th>SSE/SST</th>
<th>SSR/SST</th>
<th>Percent fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat exchanger</td>
<td>Case 1</td>
<td>111.55</td>
<td>0.0364</td>
<td>0.9854</td>
<td>80.93</td>
</tr>
<tr>
<td></td>
<td>Case 2</td>
<td>1.81</td>
<td>5.93e-4</td>
<td>0.9967</td>
<td>97.57</td>
</tr>
</tbody>
</table>

It is important to note that the smaller SSE, the better the approximating function fits the data. Also, a lower value of SSE/SST reflects precision in agreement between the estimated and regressor values, while higher value of SSR/SST shows higher statistical information being accounted by regressor. Moreover, the higher value of percent fit indicates the better match between the actual system and the identified model. It is clear from tab. 5 that the black-box NARX model outperforms the NARX Hammerstein model in all aspects.

**Conclusion**

In this paper, two models have been used to represent TSHE, black-box NARX model, and NARX Hammerstein model. Both models have been identified using LS-SVM based algorithms. Both algorithms were able to model the heat exchanger system without requiring any apriori assumptions regarding its structure. The results indicated that the black-box NARX model outperforms the NARX Hammerstein model in terms of accuracy and precision.

**Nomenclature**

- \( c_1, c_m \) – specific heats, [kcal kg\(^{-1}\) °C\(^{-1}\)]
- \( D_i, D_o \) – inner and outer diameters, [m]
- \( m \) – autoregressive part order, [-]
- \( n \) – autoregressive part order, [-]
- RMSE – root mean square error, [-]
- SST – sum squared deviation, [-]
- SSE – sum squared error, [-]
- SSR – sum squared deviation, [-]
- \( v(t) \) – liquid speed, [ms\(^{-1}\)]

**Acronyms**

- LS-SVM – least squares support vector machines
- NARX – non-linear autoregressive with eXogeneous inputs
- SVM – support vector machines
- TSHE – tubular steam heat exchanger

**Greek symbols**

- \( \alpha_1 \) – liquid/metal heat transfer coefficient, [J m\(^{-2}\) s\(^{-1}\) °C]
- \( \alpha_s \) – steam/metal heat transfer coefficient, [J m\(^{-2}\) s\(^{-1}\) °C]
- \( \delta_l \) – liquid temperatures, [°C]
- \( \delta_{10}(t) \) – outlet liquid temperature, [°C]
- \( \delta_{2}(t) \) – steam temperature, [°C]
- \( \delta_m \) – metal temperatures, [°C]
- \( \delta_{3}(t) \) – inlet liquid temperature, [°C]

**References**

[13] Lv, X., Ren, X., Non-Iterative Identification and Model Following Control of Hammerstein Systems with Asymmetric Dead-Zone Non-Linearities, IET Control Theory A, 6 (2012), 1, pp. 84-89