# ESTIMATION OF RADIATIVE PARAMETERS IN PARTICIPATING MEDIA USING SHUFFLED FROG LEAPING ALGORITHM

## by

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The transient radiative transfer in 1-D homogeneous media with ultra-short Gaussian pulse laser irradiated was investigation by the finite volume method. The concept of optimal detection distance was proposed. The radiation characteristic was studied thoroughly. Afterwards, a memetic meta-heuristic shuffled frog leaping algorithm was introduced to inverse transient radiative problems. It is demonstrated that the extinction coefficient and scattering albedo can be retrieved accurately even with noisy data in a homogeneous absorbing and isotropic scattering plane-parallel slab. Finally, a technique was proposed to accelerate the inverse process by reducing the searching space of the radiative parameters.

Key words: inverse problem, shuffled frog leaping, transient radiative transfer, turbid media

# Introduction

The problem of transient radiative transfer in participating media has attracted increasing interest over the last decade due to the recent developments in applications that involve extremely small time scales. Lasers with pulse durations of pico- to femto-seconds were utilized to investigate the properties of scattering and absorbing media in optical tomography, remote sensing, and combustion product analysis [1, 2]. The analysis of time-resolved optical signals is important for the biomedical imaging and probing applications involving detection of tumors in tissues [3, 4]. Such studies are critical for predicting the optical properties of tissues and inhomogeneity tumors. Pico-second time-resolved light scattering is a promising method for determining optical properties of absorbing scattering media. Many of the difficulties of traditional time integrated methods for determining optical properties can be overcome with the time-resolved measurements.

To solve the transient radiative equation, several numerical strategies have been developed, which include discrete ordinate method [5-7], finite volume method (FVM) [8-10], integral equation model (IE) [11], finite-element method [12, 13], and Monte Carlo method [14]. For the inverse problem, a wide variety of techniques have been successfully applied to inverse radiative analysis. Traditional algorithms including conjugate gradient method [15, 16], Levenberg-Marquardt method [17, 18] were proved to be effective in solving the inverse radiation problems. However, all these traditional methods depend on the initial value, iteration

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step, and the differentiability of the objective function. Furthermore, the gradients are difficult to be solved accurately by numerical simulation in some cases [19]. In addition, when the objective function is non-linear and non-differentiable, direct search approaches are the methods of choice. Recently, a large number of intelligent optimization algorithms, such as the particle swarm optimization (PSO) [19, 20], the ant colony optimization [21, 22], and the generic algorithm [23], were applied in radiative inverse problems. At the heart of every intelligent method, the strategy is to generate variations of parameter vectors. Once a vector is generated, a decision must be made whether to accept the newly derived parameters or not. All basic direct search methods use the greedy criterion to make this decision. Under the greedy criterion, a new parameter vector is accepted if and only if it reduces the value of the objective function.

The shuffled frog leaping (SFL) algorithm, proposed by Eusuff and Lansey [24] in 2003, is a memetic meta-heuristic algorithm which combines the local search tool of the PSO [25] and the idea of mixing information from parallel local searches to move toward a global solution [26]. It is based on the evolution of memes carried by interactive individuals and global exchange of information among the virtual population. The concepts of memetic are proposed to indicate an analogous to the gene in the context of culture, language, and behavior evolution [27]. After ten years of development, it has been widely applied in water distribution network design [24], line sequencing problems [28], unit commitment problems [29], and flow shop scheduling problems [30].

In the present work, the FVM method was used to simulate the transient radiative transfer of turbid media. The motivation of this study is to gain a fundamental understanding of the unique features of transient radiative transfer within the participating media. The SFL algorithm was applied to retrieve the extinction coefficient,  $\beta$ , and single scattering albedo,  $\omega$ , in a 1-D participating media. Furthermore, the correlations between the optical properties and the time resolved radiative signals [31] were utilized to reduce the searching space of the radiative parameters.

# Principle of SFL algorithm

The SFL is a memetic meta-heuristic algorithm which is based on the evolution of memes carried by interactive individuals and global exchange of information among the virtual population. The concept of memetic comes from the word *meme* which can be considered as the unit of culture evolution. The sample of virtual frogs which stands for a possible solution constitutes a population. The SFL algorithm involves a randomly generated initial virtual frog population  $\vec{P} = \{\vec{X}_1, \vec{X}_2, ..., \vec{X}_N\}$ . Each frog is represented by a 1-D vector  $\vec{X}_i$ , which denotes a potential solution of the inverse problem. To be specific, the *i*<sup>th</sup> frog in a *S* dimension problem is expressed as  $\vec{X}_i = [x_{i1}, x_{i2}, ..., x_{is}]$ . After the generation of the initial population, the frogs are evaluated and ranked in a descending order of the value of fitness function and the frog with the global best fitness identified as  $\vec{X}_g$  which is also the last frog. Subsequently, the entire population is divided into *m* memeplexes, each containing *n* frogs. The memeplexes can be considered as parallel frog cultures for the same goal. Each frog culture proceeds towards their goal independently. The *k*<sup>th</sup> memeplex  $\vec{M}^k$  is defined as [24]:

$$\vec{\mathbf{M}}^{k} = \left\{ \vec{\mathbf{X}}_{k+m(l-1)} \in \vec{\mathbf{P}} \middle| l = 1, 2, ..., n \right\}, \quad k = 1, 2, ..., m$$
(1)

The frogs with the best and worst fitness of each memeplex are identified as  $\vec{X}_b$  and  $\vec{X}_w$ . After the previous preparation, the evolution process is conducted to search the solution of the inverse problem. However, not all the frogs change its own position in the searching pro-

cess, but only the ones with the worst fitness in each memeplex. The corresponding position of worst frog is adjusted as [24]:

$$D = r(\vec{\mathbf{X}}_{\mathrm{b}} - \vec{\mathbf{X}}_{\mathrm{w}}) \tag{2}$$

$$\overline{\mathbf{X}'}_{\mathrm{w}} = \overline{\mathbf{X}}_{\mathrm{w}} + D, \qquad |D| \le D_{\max} \tag{3}$$

where *D* is the step size of the worst frog. The new position of the worst frog is denoted by  $X'_w$ . The parameters *r* represents a random number between 0 and 1. The  $D_{max}$  is the maximum step size of the frog. The fitness function of new generated position will be calculated after the updating. If the new frog is better, the worst frog will be replaced. Otherwise, repeat the execution of eqs. (2) and (3) with respect to the global best frog. In other words, in the calculation of the step size, *D*, the best frog in this memeplex  $\vec{X}_b$  is replaced by the global best frog  $\vec{X}_g$ . This operation is going to be repeated for  $E_{max}$  times before taking the next step. If there is still no improvement of the new frog, a new random frog will be generated to replace the worst one. Then, after this process is done in all the memeplexes, all the frogs are mixed together and re-divided into *m* memeplexes. The information of each memeplex is exchanged between one another through the shuffling process. The whole process is repeated till the stop criterions is reached, for example, the shuffling times reaches a pre-defined number or the fitness function reduced to a pre-defined small value. The pseudocode for SFL algorithm can be summarized [24]:

- Step 1. Initialization: Set the initial constant parameters, *i. e.* the numbers of memeplexes *m*, the number of frogs in each memeplexes *n*, *etc*. Then generate all the initial frogs position randomly.
- Step 2. Evaluation and ranking: Calculate the fitness of each frog and rank them in a descending order.
- Step 3. Grouping: Divide the frogs into *m* memeplexes according to eq. (1).
- Step 4. Local searching: Generate the new position of the worst frog in each memeplex by using eqs. (2) and (3). If the position of the new frog is better, then use it to replace the worst frog. Otherwise, repeat this step by replacing X<sub>b</sub> with X<sub>g</sub>.
- Step 5. Calculate the fitness of the new generated frog: If the position of the new frog is better, then use it to replace the worst frog and go to the next step. Otherwise go to Step 4 until the maximum evolution iterations  $E_{max}$  is reached.
- Step 6. Repeating: Go to Step 3 until one of the stop criteria is reached. The stop criteria are defined: 1 the iteration number reaches the maximum iteration number or 2 the best fitness is smaller than a preset small value  $\varepsilon_0$ .

## Analysis

#### Physical model

Figure 1 shows the schematic of a collimated laser-pulse irradiates upon a homogeneous absorbing and anisotropic scattering but non-emitting plane-parallel slab. The upper surface of the slab is exposed to a normally collimated incident Gaussian pulse laser. The lower surface is assumed to be black and cold, which means it is regarded as a blackbody with temperature of absolute zero. The refractive index of the slab is set to be uniform and equal to that of the surroundings. As a result, the incident light is transmitted through the front surface and the internal reflection can be ignored. The non-emission assumption is valid in many situations where short pulse laser or light source irradiates upon a cold medium or the temperature disturbance caused by the incident pulse in the medium can not be immediately revealed within the

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Figure 1. Schematic of a collimated laser-pulse irradiates upon a homogeneous absorbing and isotropic scattering but non-emitting plane-parallel slab

ticipating medium can be written [32]:

time scale of the radiative transport. The incident radiation intensity of Gaussian pulse with a pulse width,  $t_p$ , can be expressed:

$$I_{c}(t) = I_{0} \exp\left[-4\ln 2\left(\frac{t}{t_{p}} - 3\right)^{2}\right] \cdot \left[H(t) - H(t - 6t_{p})\right]$$
(4)

where  $I_0$  is the peak radiation intensity of the incident laser. The H(t) denotes Heaviside function.

The transient radiative transfer equation (TRTE) in 1-D par-

$$\frac{1}{c}\frac{\partial I(x,\mu,t)}{\partial t} + \mu \frac{\partial I(x,\mu,t)}{\partial x} = -\kappa_{a}I(x,\mu,t) - \sigma_{s}I(x,\mu,t) + \frac{\sigma_{s}}{4\pi}\int_{4\pi}I(x,\mu',t)\Phi(\mu',\mu)d\Omega'$$
(5)

where  $\mu$  and  $\mu$ ' denote the direction cosine of the scattering and incoming direction, respectively, I – the radiative intensity in the  $\mu$  direction at time t of position x, and c – the speed of light in the medium. The absorption and scattering coefficients are denoted by  $\kappa_a$  and  $\sigma_s$ , respectively,  $\Phi(\mu', \mu)$  represents the scattering phase function between incoming direction  $\mu$ ' and scattering direction  $\mu$ , and  $\Omega'$  is the solid angle in the direction  $\mu'$ .

The boundary conditions of the slab can be express [33]:

$$I^{+}(0,\mu,t) = \begin{cases} I_{c}(t), & \mu = 1\\ 0, & 0 < \mu < 1 \end{cases}$$
(6)

$$I^{-}(L,\mu,t) = 0, \qquad -1 < \mu < 0 \tag{7}$$

where L denotes the thickness of the media,  $I^+(0, \mu, t)$  and  $I^-(L, \mu, t)$  are the incident intensity of the left boundary and the intensity out of the right boundary, respectively.

The TRTE was solved by FVM [10] in the present work. The problem under consideration was the transient radiative transfer within a 1-D absorbing and scattering cold medium. Comparison between the FVM and [31] were found to be in good agreement [33]. Since the time interval,  $c\Delta t$ , has little effect on the numerical results as long as it satisfies  $c\Delta t \le \Delta z$  [34], where  $\Delta z = L/N_x$ , it is set equal to  $\Delta z$  in the inverse process. On the condition that the time step was set as  $c\Delta t = \Delta z$ , no obvious difference was observed in the time-resolved reflectance when the number of control volume  $N_x$  and direction  $N_\theta$  were beyond 150 and 30. Therefore,  $N_x$  and direction  $N_\theta$  were set as 150 and 30, respectively, in the inverse problem.

#### Similarity criteria

In many previous researches, the dimensionless time  $t^* = \beta ct$  was used for describing the time-resolved transmittance and reflectance signals. But it is found that for some cases of TRTE, different optical properties can lead to the same transmittance and reflectance results. In this section, two cases of different radiative properties are investigated. In Case 1, the thickness of the media, L, was set as 1.0 m. The extinct coefficient,  $\beta$ , was 1.0 m<sup>-1</sup> and the incident pulse width  $t_p = 10$  ns. For Case 2, the parameters were set as L = 0.1 m,  $\beta = 10$  m<sup>-1</sup>, and  $t_p = 1$  ns. The scattering albedo,  $\omega$ , was set the same as 0.9 in both cases. Clearly, in the two cases the optical thickness  $\tau = \beta L$  and dimensionless pulse width  $\beta ct_p$  were set as the same value, respectively. The transmittance and reflectance signals in the function of dimensionless time  $\beta ct$  and time ctare shown in fig. 2. It demonstrates that when  $\beta L$  and  $\beta ct_p$  are the same, the same results will be obtained with different L and  $t_p$ . In our previous paper, it has been pointed out that when  $ct_p/L = \text{constant}$ , the same transmittance and reflectance signals can be obtained with the same optical thickness and scattering albedo [35], which is equivalent to the description in this work. Thus, for the direct problems, this characteristic of TRTE can be used to save the computational time in different similar cases. However, in the inverse problem, the transmittance and reflectance are used to obtain the radiative properties or the thickness of the media. If different situations can lead to the same results, then the radiative properties or the thickness will not be estimated correctly. Therefore, ct rather than  $\beta ct$  is recommended to describe the time-resolved transmittance and reflectance signals of the media irradiated by the short pulse laser.



Figure 2. The transmittance (T) and reflectance (R) signals in the function of  $\beta ct$  and ct

## Optimal detection distance of reflectance

The time-resolved hemispherical reflectances of isotropic media with different radiative parameters are shown in fig. 3. It is worth noting that when other parameters are identical, the reflectance signals increase with the increase of the optical thickness. This is owing to the backward scattering of the media. However, when the optical thickness,  $\tau$ , reaches a certain critical value,  $t_{\rm cr}$ , the reflectance signals will retain the same even if  $\tau$  continues increasing. In another word, for the optical thick media, the reflectance signal is no longer relevant to the optical thickness which means the information of the media thickness can not be retrieved from the time-resolved reflectance signals. The critical optical thickness,  $t_{\rm cr}$ , can be defined as optimal detection distance (ODD). When the optical thickness reaches,  $t_{\rm cr}$ , the time-resolved reflectance signals can not be used independently as the known information while retrieving  $\tau$ . Furthermore, it is obvious that there is a positive relevant between the scattering albedo  $\omega$  and ODD, and a negative relevant between  $\beta$  and ODD.

## **Results and discussions**

To demonstrate the validity of the SFL algorithm in the inverse problem of transient radiative transfer, the extinction coefficient,  $\beta$ , and single scattering albedo,  $\omega$ , in a 1-D absorbing and scattering medium with black back surface were retrieved simultaneously in the



Figure 3. The time-resolved hemispherical reflectance of isotropic media with different radiative parameters

present study. All the simulations were performed on an Intel Core i7-3770 PC. In parameter estimation, a detailed examination of the sensitivity coefficients can provide considerable insight into the estimation problem. These coefficients can lead to improved experimental design. The sensitivity coefficient is the first derivative of a dependent variable, such as time-resolved transmittance or reflectance, with respect to an independent variable. If the sensitivity coefficients are either small or correlated with one another, the estimation problem is difficult to be solved and very sensitive to measurement errors. The sensitivity coefficient is defined:

$$S_{m_i}(\rho_j) = \frac{\partial \rho_j}{\partial m_i} \bigg|_{m_i = m_0} = \frac{\rho_j \left( m_0 + m_0 \Delta \right) - \rho_j \left( m_0 - m_0 \Delta \right)}{2m_0 \Delta}$$
(8)

where  $m_i$  denotes the independent variable which, in this work, stands for  $\beta$  and  $\omega$ ,  $\Delta$  represents a tiny change, and  $r_i$  denotes the transmittance or reflectance signals.

The sensitivity coefficients of  $\beta$  and  $\omega$  ( $S_{\beta}$  and  $S_{\omega}$ ) for reflectance signals are shown in fig. 4. The medium thickness was set as L = 0.1 m and the incident pulse width was set as  $ct_{p} = 0.2$  m. The tiny change,  $\Delta$ , was taken as 0.5%. It can be seen that when  $\beta$  was set as 1.0 and 10.0 m<sup>-1</sup>, the corresponding sensitivity coefficients increase if  $\omega$  increases from 0.1 to 0.95. However, when  $\omega$  was set as 0.98 and 0.3, which represent strong and low scattering media respectively, the sensitivity coefficients decrease while  $\beta$  varies from 0.1 to 5.0 m<sup>-1</sup>. Furthermore, the sensitivity coefficients are relatively larger when ct is in the interval of [0.5, 1.5 m] which are chosen as the sampling span in the inverse problem.

In the model depicted in section *Phisical model*, four combinations of single scattering albedo,  $\omega$ , and extinction coefficient,  $\beta$ , were investigated to evaluate the performance of SFL



Figure 4. The sensitivity coefficients of  $\beta$  and  $\omega$  ( $S_{\beta}$  and  $S_{\omega}$ ) for reflectance signals

which were  $1 - \beta = 1.0 \text{ m}^{-1}$ ,  $\omega = 0.35$ ,  $2 - \beta = 0.5 \text{ m}^{-1}$ ,  $\omega = 0.95$ ,  $3 - \beta = 0.1 \text{ m}^{-1}$ ,  $\omega = 0.75$ , and  $4 - \beta = 0.75 \text{ m}^{-1}$ ,  $\omega = 0.5$ , respectively. The incident pulse width  $ct_p$  was set as 0.2 m. The number of the grids  $N_x$  and polar angles  $N_{\theta}$  were set as 150 and 30, respectively. The thickness of the medium was taken as 0.1 m. The hemisphere reflectance as the function of ct and dimensionless time  $\beta ct$  are shown in fig. 5.



The objective function is defined:

$$F(\vec{a}) = \frac{1}{2} \int_{t_{s1}}^{t_{s2}} \left[ \rho_{est} \left( \vec{a} \right) - \rho_{mea} \left( \vec{a} \right) \right]^2 dt \quad (9)$$

Figure 5. Time-resolved hemispherical reflectance R(t) in the function of time ct

where  $\rho_{\text{mea}}(\vec{a})$  is measured time-resolved reflectance signals, which will be simulated by the forward model using the FVM, and  $\rho_{\text{est}}(\vec{a})$  is the estimated time-resolved reflectance for an estimated vector  $\vec{a} = (a_0, a_1, ..., a_N)^T$ , which in the present work is  $\vec{a} = (\beta, \omega)^T$ . Moreover,  $t_{s1}$  and  $t_{s2}$  are start and end point of sampling span, respectively. When the minimum value of  $F(\vec{a})$  reaches a preset small value  $\varepsilon$ , the inverse process stops and the estimated  $\beta$  and  $\omega$  are output as the inverse results, tab. 1.

Random standard deviation was added to radiative signals computed from the direct model to demonstrate the effects of measurement errors on the retrieval results. The given relation is used in present inverse analysis:

$$R_{\rm mea} = R_{\rm ori} + \sigma \, {\rm rand}() \tag{10}$$

where  $R_{\text{mea}}$  is the measured value,  $R_{\text{ori}}$  represents the original value of the measured signal, and rand() is a normal distribution random variable with zero mean and unit standard deviation. The standard deviation of the measured reflectance  $\sigma$ , for a  $\gamma\%$  measured error at 99% confidence is determined by:

(

$$\sigma = \frac{R_{\rm ori}\gamma\%}{2.567} \tag{11}$$

Table 1. Estimated results of  $\beta$  and  $\omega$  using SFL algorithm with and without measurement errors

Eman	Case	Estimate	ed result	Relative error [%]		
LIIOI	$(\beta, \omega)$	$\beta$ [m <sup>-1</sup> ]	ω	β	ω	
0	(1.0, 0.35)	1.0000	0.3500	0.004	0.004	
	(0.5, 0.95)	0.5000	0.9499	0.007	0.010	
	(0.75, 0.5)	0.7501	0.4999	0.017	0.015	
1	(1.0, 0.35)	0.9996	0.3500	0.036	0.010	
	(0.5, 0.95)	0.4997	0.9504	0.064	0.039	
	(0.75, 0.5)	0.7497	0.5001	0.039	0.014	
3	(1.0, 0.35)	0.9990	0.3500	0.099	0.013	
	(0.5, 0.95)	0.4988	0.9507	0.233	0.075	
	(0.75, 0.5)	0.7489	0.5003	0.144	0.050	
5	(1.0, 0.35)	0.9989	0.3500	0.106	0.018	
	(0.5, 0.95)	0.4982	0.9513	0.351	0.139	
	(0.75, 0.5)	0.7482	0.5004	0.246	0.087	

where 2.576 arises because 99% of a normally distributed population is contained within  $\pm 2.576$  standard deviation of the mean.

It is evident that the radiative properties of the media can be obtained accurately even with measurement errors. However, there are still some problems need to be solved. First, the searching space of  $\beta$  and  $\omega$  are set based on experience or prior information. Second, the computational time is relatively large which is partially caused by the large searching space of  $\beta$  and  $\omega$ .

To solve these problems, the correlations between the maximum hemispherical reflectance  $R_{max}$ ,  $\omega$ ,  $\beta ct_p$ , and

 $\beta L$ , were utilized to the inverse procedure. The single scattering albedo and extinction coefficient can be obtained simply by solving the equations [31]:

$$R_{\max}(\beta,\omega) = 0.05\beta\omega ct_{p}\ln(\beta L) + (-0.8\beta ct_{p} + 0.375)\beta\omega ct_{p} \quad \text{when} \quad \beta L < \beta ct_{p} \quad (12)$$

$$R_{\max}(\beta,\omega) = 0.156\beta\omega ct_{p} \quad \text{when} \quad \beta L > \beta ct_{p} \tag{13}$$

Considering the fact that  $\sigma_s = \beta \omega$ , eqs. (12) and (13) can be written:

$$R_{\max}(\beta,\sigma_{s}) = 0.05\sigma_{s}ct_{p}\ln(\beta L) + (-0.8\beta ct_{p} + 0.375)\sigma_{s}ct_{p} \quad \text{when} \quad \beta L < \beta ct_{p} \quad (14)$$

$$R_{\max}(\beta, \sigma_{s}) = 0.156\sigma_{s}ct_{p} \text{ when } \beta L > \beta ct_{p}$$
(15)

Consequently, in condition that the media thickness L is constant, if two peak values of the reflectance signal  $R_{\text{max}}$  corresponding to two different  $ct_p$ ,  $\beta$ , and  $\omega (= \sigma_s/\beta)$  can be retrieved from eqs. (14) and (15). These correlations provide a rapid way to obtain  $\beta$  and  $\omega$  of the 1-D turbid mediums. Then the inverse problem was transformed into a simpler problem of solving the binary quadratic equations. In the present, L was set as 0.1 m and the incident pulse widths  $ct_p$  were set as 0.2 m and 0.01 m, respectively.

As shown in tab. 2, the relations developed by Smith *et al.* [31] are very fast in retrieving the optical properties, but the retrieval accuracy is relatively poor. For instance, in the second case ( $\beta = 0.5 \text{ m}^{-1}$ ,  $\omega = 0.95$ ) the relative error of the obtained  $\beta$  and  $\omega$  are 38.954% and 24.867%, respectively, which are intolerable. The SFL algorithm can be applied to estimate  $\beta$  and  $\omega$  accurately, but it is very time consuming (tab. 2). Therefore, it is suggested that the results can be used to reduce the searching space of  $\beta$  and  $\omega$ . With this idea, a technique was proposed to accelerate the inverse process by reducing the searching space of the radiative parameters. First,  $\beta$  and  $\omega$  are calculated by eqs. (14) and (15), which were denoted by  $\beta_0$  and  $\omega_0$ . In the SFL, the searching space can be set as  $[0.5\beta_0, 1.5\beta_0]$  and  $[0.5\omega_0, 1.5\omega_0]$ . As we can see, the SFL combined with relations shows its superiority in retrieving  $\beta$  and  $\omega$ . It is faster than the SFL with the same accuracy.

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Term	Case $(\beta, \omega)$	Time [s]	Estimated result		Relative error [%]	
			$\beta$ [m <sup>-1</sup> ]	ω	β	ω
	(1.0, 0.35)	-	0.7821	0.4634	21.792	32.389
Relation	(0.5, 0.95)	-	0.6948	0.7138	38.954	24.867
	(0.75, 0.5)	-	0.7443	0.5235	0.754	4.709
	(1.0, 0.35)	14790	1.0000	0.3500	0.004	0.004
SFL	(0.5, 0.95)	27932	0.5000	0.9499	0.007	0.010
	(0.75, 0.5)	16712	0.7501	0.4999	0.017	0.015
	(1.0, 0.35)	12192	1.0000	0.3500	0.003	0.003
SFL-relation	(0.5, 0.95)	12485	0.5001	0.9499	0.015	0.012
	(0.75, 0.5)	12996	0.7499	0.5001	0.014	0.011

Table 2. Estimated results of  $\beta$  and  $\omega$  using SFL

#### Conclusions

The transient radiative transfer in 1-D homogeneous media with ultra-short Gaussian pulse laser irradiated was investigation in the present work. The similarity criteria of 1-D homogeneous absorbing and scattering but non-emitting media exposed to a Gaussian pulse laser was analyzed. Furthermore, time ct rather than dimensionless time  $\beta ct$  was recommended to describe the time-resolved transmittance and reflectance signals in the inverse problems. The sensitivity of the reflectance to  $\beta$  and  $\omega$  was analyzed. On this basis, the SFL algorithm was introduced to the transient radiative problems. It demonstrates that the extinction coefficient and scattering albedo can be retrieved accurately even with noisy data. Moreover, to reduce the computational time, the correlation between the maximum hemispherical reflectance  $R_{max}$ ,  $\omega$ ,  $\beta ct_p$ , and  $\beta L$ , were utilized in the inverse procedure. The searching space of  $\beta$  and  $\omega$  were determined according to the results obtained by the correlations. All the results show that by introducing the correlation into the inverse process, the computation time is reduced significantly with the same accuracy.

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#### Nomenclature

- ā - vector of estimated properties, [-]
- F - objective function, [-]
- Heaviside function, [-] Η
- thickness of the media, [m] L
- N, - number of grids, [-]
- the number of polar angle, [-]  $N_{\theta}$
- $r_1, r_2$  random numbers, [–]
- time or generation, [s / -] t
- incident pulse width, [s]
- $\dot{\Delta}t$  time step, [s]
- x x-co-ordinate value, [-]

#### Greek symbols

- $\beta$  extinction coefficient, [m<sup>-1</sup>]
- $\varepsilon$  tolerance for objective function, [–]
- $\Phi$  scattering phase function, [–]
- $\kappa_{\rm a}$  absorption coefficient, [m<sup>-1</sup>]
- $\rho_{\rm mea}^{a}$  measured reflectance signals, [-]  $\rho_{\rm est}$  estimated reflectance signals, [-]  $\rho_{\rm est}^{---}$  estimated reflectance signal  $\sigma_{\rm s}^{----}$  scattering coefficient, [m<sup>-1</sup>]
- optical thickness, [-] τ
- $\omega$  single scattering albedo, [–]
- $\Omega'$  solid angle in the direction  $\mu'$ , [sr]

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