An Efficient Spectral Solution for Unsteady Boundary Layer Flow and Heat Transfer Due to a Stretching Sheet

F. Mohammadi*[1], M.M. Rashidi1[2,3]

[1]Department of Mathematics, Hormozgan University, P. O. Box 3995, Bandarabbas, Iran,

[2]Shanghai Key Lab of Vehicle Aerodynamics and Vehicle Thermal Management Systems, Tongji University, 4800 Cao An Rd., Jiading, Shanghai 201804, China

[3]ENN-Tongji Clean Energy Institute of advanced studies, Shanghai, China

e-mail:[1] f.mohammadi62@hotmail.com, [2] mm_rashidi@tongji.edu.cn

Abstract

In this paper, an efficient Spectral Collocation method based on the shifted Legendre polynomials is applied to study the unsteady boundary-layer flow and heat transfer due to a stretching sheet. A similarity transformation is used to reduce the governing unsteady boundary layer equations to a system of nonlinear ordinary differential equations. Then, the shifted Legendre polynomials and their operational matrix of derivative are used for producing an analytical aproximate solution of this system of nonlinear ordinary differential equations. The main advantage of the proposed method is that the need for guessing and correcting the initial values during the solution procedure is eliminated and a stable solution with good accuracy can be obtained by using the given boundary conditions in the problem. A very good agreement is observed between the obtained results by the proposed Spectral Collocation method and those of previously published ones.

Keywords: Nanofluid; Entropy generation; Spectral Collocation method; MHD flow; Shifted Legendre polynomials; Rotating porous disk

1 Introduction

Since the time of Fourier, orthogonal functions and polynomials have been used in the analytic study of differential equations and their applications for numerical solution of ordinary differential equations refer, at least, to the time of Lanczos (1938). It is well known that the eigenfunctions of certain singular Sturm-Liouville problems such as Legendre or Chebyshev orthogonal polynomials allow the approximation of functions $C^{\infty}[a,b]$ where truncation error approaches zero faster than any negative power of the number of basic functions used in the approximation, as that number (order of truncation N) tends to infinity. This phenomenon is usually referred to as spectral accuracy [1–3].

The spectral collocation method has been applied for numerical solution of different kind of differential and integral equations. For example, it has been used for deriving approximate solution of Burgers-type equation [4], stochastic Burgers equation [5], Navier-Stokes equations [6], two-point boundary value problem in modelling viscoelastic flows [7], Poisson equation in polar and cylindrical coordinates [8], Volterra integral equations [9,10], compressible flow, two-dimensional and axisymmetric boundary layer problems [11], hypersonic boundary layer stability [12], Helmholtz and variable coefficient equations in a disk [13] and Burgers-Huxley equation [14].

The Legendre polynomials [1] are well known family of orthogonal polynomials on the interval $[0, 1]$ of the real line. These polynomials present very good properties in the approximation of functions. Therefore, Legendre polynomials appear frequently in several fields of Mathematics, Physics and Engineering. Spectral methods based on Legendre polynomials as basis functions for solving numerically differential equations have been used by many authors, (see for example [15–18]).

The study of heat transfer over stretching surface has received much attention in several industrial and engineering processes such as such as Aerodynamics, extrusion of plastic sheets, the boundary-layer along a liquid film, condensation process of metallic plate in a cooling bath and glass blowing, polymer industries, paper production, metal spinning, drawing plastic films and artificial fibres. In all such applications the final product depends on the rate of cooling and boundary layer flow near the stretching surface [19–28]. Crane [19] studied the boundary-layer flow due to a stretching surface in an ambient fluid and applied a similarity transformation for the steady boundary-layer flow by stretching of a sheet when its velocity varying linearly with the distance from a fixed point. Furthermore, Carragher and Crane [20] considered the influence of heat transfer on the flow over a stretching surface in the case when the temperature difference between the surface and the ambient fluid is proportional to a power of distance from the fixed point. The temperature field in the flow over a stretching surface when a uniform heat flux is exerted to the surface, was investigated by Dutta [21], Grubka and Bobba [22]. Elbashbeshy [23] studied the steady heat transfer over a stretching surface with a variable surface heat flux and uniform heat flux subjected to injection and suction. Elbashbeshy and Bazid [24] have presented similarity solutions of the boundary layer equations, which describe the unsteady flow and heat transfer over a stretching sheet. Moreover, Sharidan et al. [25] investigated the unsteady boundary layer flow and heat transfer due to stretching sheet for the especial distribution of the stretching velocity and surface temperature. In [30] the problem of unsteady boundary-layer and heat transfer due to stretching sheet have been studied by using the quasi-linearization technique. The aim of the present study is to find an analytical approximate solution for unsteady boundary-layer and heat transfer due to stretching sheet using the Legendre Spectral Collocation method.

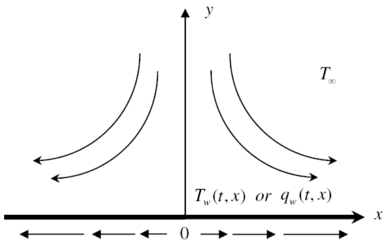

Figure 1: Geometry of the problem and coordinate system.

2 Flow analysis and mathematical formulation

Let us consider the unsteady flow and heat transfer of a viscous and incompressible fluid past a semi-infinite stretching sheet in the region $y > 0$, as shown in Fig. 1 . Keeping the origin fixed, two equal and opposite forces are suddenly applied along the x-axis. These forces stretch the sheet and the flow is generated. The wall temperature $T_w(x,t)$ of the sheet is suddenly raised from T_∞ to $T_w(t,x) > T$ or there is suddenly imposed a heat flux $qw(t,x)$ at the wall [29]. Under these assumptions, the basic unsteady boundary layer equations governing the flow and heat transfer due to the stretching sheet are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \nu\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + \nu\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

subject to the following boundary conditions

$$\begin{array}{l} u = u_w(0,x), \quad v = 0, \quad as \quad y = 0, \quad u \to 0, \quad T \to T_\infty, \quad as \quad y \to \infty \\ T = T_w(x,t)\ (VWT)\ or\ \ \frac{\partial T}{\partial y} = -\frac{q_w(x,t)}{k}\ (VHF) \end{array} \tag{4}$$

where t is the time, u and v are the velocity components along the x- and y-axes respectively, T is the temperature, α is the thermal diffusivity, ν is the kinematic viscosity and k is the thermal conductivity. Now we assume that the velocity of the sheet $u_w(t,x)$ the sheet temperature $T_w(t,x)$ and the heat flux $q_w(t,x)$ are defined

$$u_w(t,x) = \frac{cx}{1-\gamma t}, \quad T_w(t,x) = T_\infty + \frac{c}{2\nu x^2(1-\gamma t)^{3/2}}, \quad q_w(t,x) = \frac{q_{w0}}{2x^2}\left(\frac{c}{\nu}\right)\frac{1}{(1-\gamma t)^2}, \tag{5}$$

where c is the stretching rate being a positive constant, γ is a positive constant, which measures the unsteadiness and q_{w0} is a characteristic heat transfer quantity [25]. By introducing the following similarity transforms [25]

$$\eta = \sqrt{\frac{c}{\nu(1-\gamma t)}}y, \qquad \psi = \sqrt{\frac{c\nu}{(1-\gamma t)}}xf(\eta), \tag{6}$$

$$T = T_\infty + \frac{c}{2\nu x^2(1-\gamma t)^{3/2}}\theta(\eta),\ (VWT),\quad T = T_\infty + \frac{q_{w0}}{k}\frac{c}{2\nu x^2(1-\gamma t)^{3/2}}\theta(\eta),\ (VHF)$$

where ψ is the stream function and is defined as $u = \frac{\partial\psi}{\partial y}$ and $v = \frac{\partial\psi}{\partial x}$, the governing equations (1)-(3) are reduced to the following ordinary differential equations

$$f''' + ff'' - f'^2 - A\left(f' + \frac{\eta}{2}f''\right) = 0 \tag{7}$$

$$\frac{1}{\mathrm{Pr}}\theta'' + f\theta' + 2f'\theta - \frac{A}{2}\left(3\theta + \eta\theta'\right) = 0 \tag{8}$$

subject to the boundary conditions

$$\begin{array}{l} f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = 0, \\ \theta(0) = 1\ \ (VWT), \quad \theta'(0) = -1\ \ (VHF), \quad \theta(\infty) = 0 \end{array} \tag{9}$$

where Pr is the Prandtl number, $A = \frac{\gamma}{c}$ is a non-dimensional constant which measures the flow and heat transfer unsteadiness and primes denote the differentiation with respect to the similarity variable [25].

2.1 Skin friction coefficient and Nusselt number

The skin friction coefficient C_f and the local Nusselt number Nu_x are the important physical quantities in this problem and are defined as

$$C_f = \frac{\tau_w}{\rho u_w^2}, \qquad Nu_x = \frac{xq_w}{k\left(T - T_w\right)}, \tag{10}$$

where the skin friction τ_w and the heat transfer from the sheet q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \qquad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{11}$$

and μ is the dynamic viscosity. By using equations (6), it is obvious to get

$$C_f Re_x^{\frac{1}{2}} = f''(0), \qquad \frac{Nu_x}{Re_x^{\frac{1}{2}}} = -\theta'(0)\ \ (VWT), \qquad \frac{Nu_x}{Re_x^{\frac{1}{2}}} = \frac{1}{\theta(0)}\ \ (VHF). \tag{12}$$

where $Re_x = \frac{u_w x}{\nu}$ is the local Reynolds number.

3 Shifted Legendre polynomials and their properties

The well known Legendre polynomials are defined on the interval and can be determined with the aid of the following recurrence formulae [1]

$$L_{m+1}(t) = \frac{2m+1}{m+1}\, t\, L_m(t) - \frac{m}{m+1}\, L_{m-1}(t), \qquad m = 1,\ 2,\ 3,\ ..., \tag{13}$$

where $L_0(t) = 1,\ \ L_1(t) = t$. In order to use Legendre polynomials on the interval $[0, 1]$ we define the so-called shifted Legendre polynomials by introducing the change of variable $t = 2x - 1$. The orthogonality condition for these polynomials is

$$\int_0^1 P_m(x) P_n(x)\, dx = \begin{cases} \frac{1}{2m+1} & for\ m = n, \\ 0 & for\ m \neq n. \end{cases} \tag{14}$$

A function $f(t)$ definedover $[0, 1]$ may be expanded in the terms of shifted Legendre polynomials as

$$f(t) = \sum_{k=0}^{\infty} c_k P_k(t), \tag{15}$$

where $c_k = (f(t), P_k(t))$, in which $(.,.)$ denotes the inner product. If theinfinite series in Eq. (15) is truncated, then it can be written as

$$f(t) = \sum_{k=0}^{N} c_k P_k(t) = C^T \Phi(t), \tag{16}$$

where C and $\Phi(t)$ are $(N + 1)$ vectors given by

$$C^T = [c_1, c_2, ..., c_N]\,, \qquad \Phi(t) = [P_1(t), P_2(t), ..., P_N(t)]\,. \tag{17}$$

In the next theorem we derived a relation between shifted Legendre polynomials and their derivatives that is very important for deriving the operational matrix of derivative for shifted Legendre polynomials.

Theorem 1. *[1] Let $\Psi(t)$ be the Legendre polynomial vector defined as*

$$\Psi(t) = [P_0(t), P_1(t), ..., P_N(t)], \tag{18}$$

the derivative of this vector can be expressed by

$$\frac{d\Psi(t)}{dt} = D\Psi(t), \tag{19}$$

which D is $(N + 1) \times (N + 1)$ matrix and its (i, j)-th element is defined as below

$$D_{i,j} = \begin{cases} 2(2j-1) & j = 1, ..., i-1 \ and\, (i+j)\, odd, \\ 0 & o.w. \end{cases} \tag{20}$$

4 Method of solution

Consider the coupled nonlinear differential equations (8)-(9) subject to boundary conditions (10). By using change of variable

$$t = \frac{\eta}{\eta_\infty}, \quad g(t) = f(t\eta_\infty), \quad \vartheta(t) = \theta(t\eta_\infty), \tag{21}$$

we have the following nonlinear differential systems in the interval $[0, 1]$,

$$g''' + \eta_\infty g g'' - \eta_\infty \left(g'\right)^2 - A\left(\eta_\infty^2 g' + \frac{\eta_\infty^2 t}{2} g''\right) = 0, \tag{22}$$

$$\frac{\vartheta''}{\Pr} + \eta_\infty g\vartheta' + 2\eta_\infty \vartheta'\vartheta - \frac{A}{2}\left(3\eta_\infty^2 \vartheta + \eta_\infty^2 t\vartheta'\right) = 0, \tag{23}$$

the boundary conditions become

$$\begin{aligned} &g(0) = 0, \;\; g'(0) = \eta_\infty, \;\; g'(1) = 0 \\ &\vartheta(0) = 1 \;\; (VWT), \quad \vartheta'(0) = -\eta_\infty \;\; (VHF), \quad \theta(\infty) = 0. \end{aligned} \tag{24}$$

Now we expand the unknown function $f(t)$ and $\theta(t)$ by the shifted Legendre polynomial into interval $[0, 1]$ as

$$g(t) \simeq C_1^T \Phi(t), \qquad \vartheta(t) \simeq C_2^T \Phi(t), \tag{25}$$

where C_1 and C_2 are the unknown shifted Legendre polynomial coefficient vectors defined in (17). By using the operational matrix derived in (19) we get

$$f'(t) \simeq C_1^T D\Phi(t), \qquad g'(0) \simeq C_1^T D^2\Phi(t), \qquad g'(t) \simeq C_1^T D^3\Phi(t), \tag{26}$$

$$\vartheta'(t) \simeq C_2^T D\Phi(t), \qquad \vartheta'(0) \simeq C_2^T D^2\Phi(t), \tag{27}$$

substituting Eqs. (26) and (27) into (22) and (23), we have

$$\begin{aligned} &C_1^T D^3\Phi(t) + \eta_\infty \left(C_1^T\Phi(t)\right)\left(C_1^T D^2\Phi(t)\right) - \eta_\infty \left(C_1^T D\Phi(t)\right)^2 \\ &-A\left(\eta_\infty^2 \left(C_1^T D\Phi(t)\right) + \frac{\eta_\infty t}{2}\left(C_1^T D^2\Phi(t)\right)\right) \simeq 0, \end{aligned} \tag{28}$$

$$\frac{C_2^T D^2\Phi(t)}{\Pr} + \eta_\infty \left(C_1^T\Phi(t)\right)\left(C_2^T D\Phi(t)\right) + 2\eta_\infty \left(C_1^T D\Phi(t)\right)\left(C_2^T\Phi(t)\right)$$

$$-\frac{A}{2}\left[3\eta_\infty^2\left(C_2^T\Phi(t)\right)+\eta_\infty^2 t\left(C_2^T D\Phi(t)\right)\right]\simeq 0. \tag{29}$$

Moreover, boundary conditions (24) result

$$\begin{array}{l} C_1^T\Phi(0)\simeq 0, \quad C_1^T D\Phi(0)\simeq \eta_\infty, \quad C_1^T D\Phi(1)\simeq 0 \\ C_2^T\Phi(0)\simeq 1 \;\; (VWT), \quad C_2^T\Phi(0)\simeq 1 \;\; (VHF), \quad C_2^T\Phi(1)\simeq 0 \end{array} \tag{30}$$

to find the approximate solution of the nonlinear system (22-23), we use the typical collocation method and collocate Eqs. (28) at $(M-2)$ different points and Eq. (29) at $(M-1)$ different points in the interval $[0,1]$. For choosing suitable collocation points, we use the first roots of shifted Legendre $P_{M+1}(t)$. These equations together with equations in (30) (VWT or VHF case) generate 2(M + 1) nonlinear equations. The well-known Newton-Raphson have been used for approximate solution of derived nonlinear systems. After finding the solution of this nonlinear systems we obtain unknown vectors C_1 and $C2$. By substituting these vectors in Eq. (25) the solution functions $g(t)$ and $\vartheta(t)$ can be approximated. Now, the change of variable in Eq. (21) resuls approximation of functions $f(\eta)$ and $\theta(\eta)$.

5 Numerical results

In this section, The nonlinear differential Eqs. (8)-(9) subject to the boundary conditions (10) have been solved analytically by using the Legendre collocation method presented in Section 4. All numerical results are derived by using Maple 17 with 20 digits precision. In order to verify the results of this study, the results have been compared with previously published numerical ones of Ref. [25]. Table 1-2 show the values of heat transfer $-\theta'(0)$ and skin friction coefficient$-f''(0)$, for various values of parameter A and Pr for the VWT case. Numerical values of heat transfer $\theta(0)$ for various values of A and Pr for the VHF case are shown in Table 3. From the tables, it is possible to see that a very good agreement between our obtained results and previously published ones is exists. One can observe that with increasing the coefficient related t flow and heat transfer unsteadiness, Nu number and value of heat transfer will be decreased, and with increasing Pr number, heat transfer increases. Figs. 2-3 show the profile $\theta(\eta)$ for VHF and VWT cases obtained for several values of A and Pr. As these temperature graphs indicate that when A increases, thickness of momentum and thermal boundary layer decreases. The profile $f(\eta)$ for VHF and VWT cases obtained for several values of A and $Pr = 0.01$ are plotted in Fig. 4. The velocity profile $f'(\eta)$ for VHF and VWT cases derived for several values of A and $Pr = 0.01$ are presented in Fig. 5. The velocity graphs decreases monotonically as unsteadiness parameter increases. From the graph, we can also observe that the velocity boundary layer thickness decreases as the distance from the sheet increases.

Table 1: Numerical values of heat transfer $-\theta'(0)$ for various values of A and Pr (VWT case).

	$A = 0.8$			$A = 1.2$			$A = 2$		
Pr	0.01	0.1	1	0.01	0.1	1	0.01	0.1	1
Ref. [25]	0.092274	0.229433	0.471190	0.114053	0.311720	0.788173	0.1503170	0.438750	1.243741
Present	0.113848	0.229978	0.471188	0.130002	0.311814	0.788171	0.1594415	0.438753	1.243739

Table 2: Numerical values of heat transfer $-f''(0)$ for various values of A and Pr (VWT case).

	$A = 0.8$			$A = 1.2$			$A = 2$		
Pr	0.01	0.1	1	0.01	0.1	1	0.01	0.1	1
Ref. [25]	1.261042	1.261042	1.261042	1.377722	1.377722	1.377722	1.587362	1.587362	1.587362
Present	1.261042	1.261042	1.261042	1.377722	1.377722	1.377722	1.587366	1.587366	1.587366

Table 3: Numerical values of heat transfer $\theta(0)$ for various values of A and Pr (VHF case).

	$A = 0.8$			$A = 1.2$			$A = 2$		
Pr	0.01	0.1	1	0.01	0.1	1	0.01	0.1	1
Ref. [25]	10.837316	4.358565	2.122870	8.767842	3.208012	1.268756	–	–	–
Present	10.837516	4.348228	2.122295	8.769116	3.207038	1.268759	6.271892	2.279183	0.804026

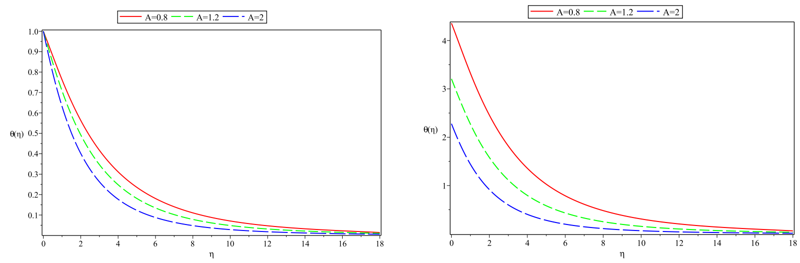

Figure 2: The profile $\theta(\eta)$ for VHF (Left) and VWT (Right) obtained for several values of A and $Pr = 0.1$.

6 Conclusion

An efficient Legendre Specral Collocation method are introduced for approximate solution of the coupled nonlinear ordinary differential equations derived from similarity transform- for unsteady boundary-layer flow and heat transfer due to a stretching sheet. In the proposed method, the requirement of guessing the initial condition $f'(0), f''(0)$ and $\theta'(0)$ in order to start the solution which is required in the conventional shooting methods is dismissed. The results show that the non-dimensional velocity profiles are compressed and

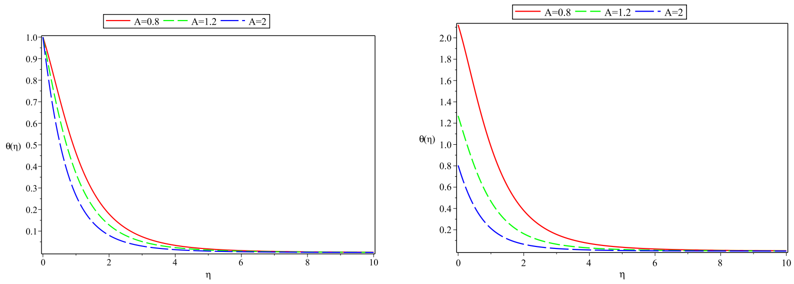

Figure 3: The profile $\theta(\eta)$ for VHF (Left) and VWT (Right) obtained for several values of A and $Pr = 1$.

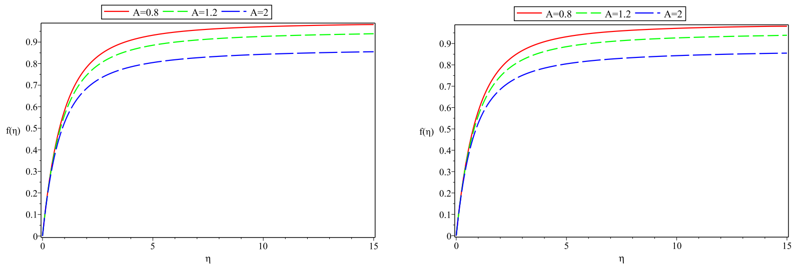

Figure 4: The profile $f(\eta)$ for VHF (Left) and VWT (Right) obtained for several values of A and $Pr = 0.01$.

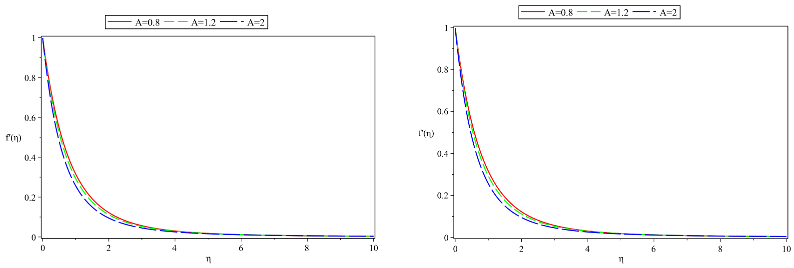

Figure 5: The velocity profile $f'(\eta)$ for VHF (Left) and VWT (Right) obtained for several values of A and $Pr = 0.01$.

suppressed toward the sheet with increasing values of the unsteady parameter A. Temperature profiles, on the other hand, become fuller and the surface heat flux increases and the wall temperature considerably decreases with the increase of A. Also, the surface heat transfer increases with increasing Pr causing a decrease in the thermal boundary layer thickness.

Nomenclature

A	Dimensionless measure of the unsteadiness	u_w	Velocity of the moving sheet
c	Stretching rate	α	Thermal diffusivity of the fluid
C_f	Skin friction coefficient	γ	Positive constant
k	Thermal conductivity	η	Similarity variable
Nu_x	Local Nusselt number	θ	Non-dimensional temperature
Pr	Prandtl number	ν	Kinematic viscosity
Re_x	Local Reynolds number	ρ	Density
q_w	Heat flux at the surface of the sheet	τ_w	Skin friction
q_{w0}	Characteristic wall heat flux	ψ	Stream function
T_w	Fluid temperature	L_m	Legendre polynomial
T_∞	Surface temperature	P_m	Shifted Legendre polynomial
T	Ambient temperature	Ψ	Shifted Legendre polynomial vector
u, v	Velocity components along x and y axes	C_1, C_2	Shifted Legendre polynomial coeffcient
x, y	Cartesian coordinates along the sheet	D	Operational matrix of derivative

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