EXPERIMENTAL CALIBRATION OF THE MATHEMATICAL MODEL OF AIR TORQUE POSITION DAMPERS WITH NON-CASCADING BLADES

by

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> Original scientific paper DOI:10.2298/TSCI150131123B

This paper is focused on the mathematical model of the air torque position dampers. The mathematical model establishes a link between the velocity of air in front of the damper, position of the damper blade, and the moment acting on the blade caused by the air flow. This research aims to experimentally verify the mathematical model for the damper type with non-cascading blades. Four different types of dampers with non-cascading blades were considered: single blade dampers, dampers with two cross-blades, dampers with two parallel blades, and dampers with two blades of which one is a fixed blade in the horizontal position. The case of a damper with a straight pipeline positioned in front of and behind the damper was taken in consideration. Calibration and verification of the mathematical model was conducted experimentally. The experiment was conducted on the laboratory facility for testing dampers used for regulation of the air flow rate in heating, ventilation and air conditioning systems. The design and set-up of the laboratory facility, as well as construction, adjustment and calibration of the laboratory damper are presented in this paper. The mathematical model was calibrated by using one set of data, while the verification of the mathematical model was conducted by using the second set of data. The mathematical model was successfully validated and it can be used for accurate measurement of the air velocity on dampers with non-cascading blades under different operating conditions.

Key words: air torque position damper, heating ventilation air conditioning, air velocity

Introduction

The pursuit of method for simple measurement of the air flow motivated the researchers to examine the possibility of using a damper as a measuring instrument. Dampers are used in heating ventilation air conditioning systems (HVAC) to control the air flow rate. As a contribution to these efforts, the verified mathematical model of the air torque position (ATP) damper is hereby presented. This mathematical model associates the velocity of the air in front of the damper with the blade angle of attack, α , and the moment, M, of the air acting on the blade. The relationship between the given parameters can be used for the ATP damper for the purpose of

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indirect measurement of the air velocity, $v \text{ [ms}^{-1}\text{]}$, by conducting measurements of the blade angle of attack, α [°], and the moment, M [Nm], of the air acting on the blade.

In search of a way to predict the moment characteristic of the butterfly valve, scientists originated the idea of the ATP damper. In early studies, it was performed theoretically [1-4]. Sarpkaya [5, 6] was the first author to experimentally verify the mathematical model of the prediction of the butterfly valve moment characteristic. He developed and verified the mathematical model of the butterfly valve with a thin blade, under the assumption that the air flow is irrotational and incompressible. Hasennpflug [7] corrected the Sarpkaya's mathematical model by using the potential flow theory. Morris and Dutton [8] developed a mathematical model of the moment characteristic of the butterfly valve that takes into account the compressibility of the fluid.

Federspiel [9] was the first author who used the accumulated knowledge and experience collected in the field of prediction of the butterfly valve moment characteristic for the purpose of development of ATP dampers. He expanded the mathematical models developed by Sarpkaya [6] and Hasennpflug [7] in a way to become applicable for dampers with more blades and in cases when the axis of rotation of the blades moves in longitudinal and transverse direction. During the development of the mathematical model, Federspiel took into consideration the case of irrotational and incompressible air flow around one blade of the damper, fig. 1. He showed that mathematical modeling with the incompressible flow is quite adequate for ATP dampers installed in HVAC systems.

He developed the mathematical model by using basic equations of fluid mechanics: the equation of continuity, the Bernoulli equation, and the momentum equation. Dampers can be installed in HVAC systems in three positions: with the straight pipeline section before the damper placed at the end of the pipeline, with the straight pipeline section after the damper placed at the pipeline entrance, and with the straight pipeline section placed both before and after the damper.



For all three positions of the damper in HVAC systems, Federspiel [9] established the same correlation between the velocity of air in front of the damper, the blade angle of attack and the moment of air stream acting on the blade:

$$v\left|v\right| = G^{2}(\alpha) \frac{2M}{\rho A_{\rm u} D_{\rm h}} \tag{1}$$

where $v \text{ [ms}^{-1}\text{]}$ is the velocity, M [Nm] – the torque; $\rho \text{ [kgm}^{-3}\text{]}$ – the air density, G [-] – the correlation function, $D_{h} \text{ [m]}$ – the hydraulic diameter, and $A_{u} \text{ [m}^{2}\text{]}$ – the cross-section area up-

perstream of the blade. The density of air, ρ , is in the denominator of the mathematical model eq. (1). By placing sensors to measure the pressure and the temperature of the air stream directly in front of the ATP damper blade, the density of air ρ can be calculated from the Ideal gas law. In this way, the ATP damper can be used to measure the air velocity under different operating conditions. The correlation function eq. (2) reads [9]:

$$G(\alpha) = \left(\frac{D_{\rm h}}{\frac{y}{C_{Q,a}^2} + \frac{x}{C_{Q,1}^2 \, \text{tg}\alpha}}\right)^{1/2}$$
(2)

This way, the obtained mathematical model for the single blade ATP damper becomes semi-empirical, because the correlation function given by the eq. (2) is semi-empirical. The correlation function can be calculated only with the knowledge of the longitudinal flow coefficient $C_{Q,a}$ [-] and the lateral flow coefficient $C_{Q,l}$ [-] which can only be obtained through an experiment.

Flow coefficients in the longitudal and the lateral direction are defined by [9]:

$$C_{Q,a} = \frac{C_{c,a}A_{a,a}}{A_{u} - C_{c,a}A_{a,a}}$$
(3)

$$C_{Q,l} = \frac{C_{c,l}A_{a,l}}{A_{u} - C_{c,l}A_{a,l}}$$
(4)

where $C_{\rm c}$ [–] is the contraction coefficient and $A_{\rm a}$ [m²] is the apparent area.

The contraction coefficient is defined as the ratio between the open area and the apparent open area:

$$C_{\rm c} = \frac{A_{\rm c}}{A_{\rm a}} \tag{5}$$

The contraction coefficient is a function of the blade angle of attack and can be determined analytically. Hassenpflug [7] used the theory of conformal mapping, fig. 2, with the assymption that in the case of a fully open damper there is no resistance.

In the longitudinal direction, resistance shall be taken into account with the coefficient of contraction less than one, when the damper is fully opened. In the case of the fully opened damper, the coefficient of contraction can be calculated from the Federspiel's equation [9]:

$$C_{\rm c}^2 (C_{\rm D} A_{\rm f} A_{\rm a}^2 - A_{\rm u} A_{\rm a}^2) + C_{\rm c} (2A_{\rm u}^2 A_{\rm a}) - A_{\rm u}^3 = 0$$
(6)

where the experimentaly determined $C_{\rm D}$ [-] is the drag coefficient and $A_{\rm f}$ [m²] – the frontal area of the damper blade, when it is open. The coefficient of contraction calculated for the fully opened damper is used for the proportional scaling of the coefficient of contraction presented in fig. 2. The coefficient of contraction remains 0.611 in the case of the closed damper, but it should be calculated in accordance with the previous equation in the case of the fully opened damper.

The coefficient of contraction in the lateral direction is as presented in fig. 2 and it is not modified by the drag coefficient, C_D . The contraction coefficient in the lateral direction in the case of a fully opened damper is equal to 1.



Figure 2. The contraction coefficient vs. the angle of attack

Figure 3. Types of ATP dampers with non-cascading blades

Federspiel [10] conducted the verification of the mathematical model, eq. (1), with the damper with following characteristics: square cross section of 0.61×0.61 m, placement at the entrance of the pipeline with the straight section of the pipeline positioned after the damper, four oppositely guided straight blades (cascading). He verified the mathematical model which can be used for accurate measurement of the air velocity at different operating conditions. The difference between the measured and the model velocity was $\pm 10\%$ of the measured velocity or $\pm 5\%$ of the full scale. In this way, Federspiel established the mathematical model of the ATP damper with a potentially universal character.

To regulate the air flow rate in HVAC systems, dampers with non-cascading blades (number of blades is less than three) are very often used in the pipeline (straight sections of the pipeline behind and in front of dampers). The aim of this paper, is to verify the proposed mathematical model, eq. (1), for previously described ATP dampers. The scientific contribution of the paper shall be to establish a mathematical model that can be used for air velocity measurement, since this type of dampers is very common in HVAC systems.

Four possible types of dampers with non-cascading blades have been hereby taken into consideration, fig. 3: dampers with a single blade (A), dampers with two cross-blades (B), dampers with two blades guided in parallel (C), and with two blades of which one is used for measuring and the other remains fixed in horizontal position (D).

Material and method

Verification of the mathematical model, eq. (1) was performed experimentally by means of two independent series of measurements. The measurement range of the flow velocity was 0-10 m/s, while the measurement range of the blade angle of attack was 0-90°. The mathematical model was calibrated, correlation function, $G(\alpha)$, was experimentally determined, by means of one set of measurements, and the verification of the mathematical model was performed by using the second set of measurements.

In the Laboratory of Fluid Mechanics, at the Faculty of Technical Sciences, University of Novi Sad, a laboratory facility for testing ATP dampers was set-up, in keeping with recommendations provided by the standard [11] for testing dampers for the control of the air flow rate in HVAC systems, fig. 4.



Figure 4. Schematic diagram of the laboratory facility for testing ATP dampers

The air flow through the laboratory facility is supplied by the fan and its electric motor. The air flow rate is regulated by the control unit. From the fan, air enters the diffuser which converts the kinetic energy into the pressure energy of the air stream and partially breaks the eddies that occur behind the fan. The air from the diffuser flows into the laminarization chamber with a sieve to stabilize the air stream. The chamber and the sieve break the eddies and supply the uniform air stream to the straight section for the flow velocity measurement. Accurate measurement of the velocity of air directly in front of the blade of the ATP damper is not possible, because of the presence of the blade in the air stream. As a result, the air velocity measurement (1) and the section placed directly in front of the ATP damper blade (2), fig. 4.

The air velocity, $v_1 \text{ [ms}^{-1]}$, was measured with the hot wire anemometer manufactured by *Testo*, model 425 (accuracy $\pm 0.03\% + 5\%$ of the measured velocity), on the basis of the one-point method, in keeping with recommendations provided by the standard [12]. For the purpose of positioning of the hot wire anemometer, a trolley with an adjustable tripod was used. Beside the air velocity in the straight section, the air stream temperature, t_1 [°C], was measured with the mercury thermometer manufactured by *Tlos* (measuring range 0-50 °C, resolution 0.1 °C), and the gauge, p_{m1} [Pa], was measured with a manometer manufactured by *Testo*, model 525 (measuring range 0-2500 Pa and accuracy $\pm 0.1\%$). Directly in front of the blade, the air temperature, t_2 [°C], was measured with a mercury thermometer manufactured by *Precision* (measuring range 0-52 °C, resolution 0.05 °C) and the gauge, p_{m2} [Pa], was measured with a manometer manufactured by *Testo*, model 521 (measuring range 0-1000 Pa and accuracy of $\pm 0.1\%$ of full scale). The atmospheric pressure, p_a [Pa], was measured by a digital barometer manufactured by *PCE Instruments*, model THB 38 (with uncertainty of $\pm 1.5\%$ of the measuring range up to 1000 mbar and $\pm 2\%$ of the measuring range above 1000 mbar). The air density in the duct in sections (1) and (2) was determined from the Ideal gas law:

$$\rho_1 = \frac{p_{m1} + p_a}{\mathbf{R}T_1} \tag{7}$$

$$\rho_2 = \frac{p_{m2} + p_a}{\mathbf{R}T_2} \tag{8}$$

where R $[Jkg^{-1}K^{-1}]$ is the gas constant; the section (1) is a circular cross-section of inner diameter D = 235 mm, and section (2) is a square cross-section b = 250 mm. From the equality of mass flow rate in sections (1) and (2), the average air velocity in the section (2) directly in front of the blade of the ATP damper can be calculated:

$$v_2 = \frac{\rho_1 v_1 A_1}{\rho_2 A_2} \tag{9}$$

Flange joints were sealed with rubber seals. According to the recommendations provided by the standards [11] the straight section of the pipeline before the damper was 3 m long, while the straight section after the damper was 2 m long. The following procedure of measuring operating parameters of the ATP damper was performed: The blade angle of attack α was adjusted on the ATP damper. The moment was examined with the control weight of known mass. The laboratory facility was turned on, the fan unit was set to supply the desired air flow velocity v_1 , by operating the control unit. After reaching the air flow stationary conditions, the air temperature t_1 , the gauge pressure p_{m1} , the air velocity v_1 , the air temperature t_2 , the air gauge pressure p_{m2} , the atmospheric pressure p_a , and the moment M_{mer} were measured.

Set-up and calibration of the laboratory ATP damper

In order to facilitate accurate measurement of the operating parameters and therefore conduct the successful calibration and verification of the mathematical model, a functional laboratory ATP damper was assembled, set-up, and calibrated. The laboratory ATP damper has the optional blade replacement feature and changeable blade driving set-up, devised for the purpose of making all four types of dampers with non-cascading blades, fig. 3.

The ATP damper with a square cross-section of side length b = 250 mm was tested. The damper blade is made of galvanized steel. Blade lengths were L = 248 and 124 mm, blade widths were B = 248 mm, while thickness of blades was W = 0.75 mm. Along the blades, at a distance of 31 mm on both sides of its longitudinal axis, there were two reinforcements, 10 mm wide and 2 mm thick. The distances between the center of axle and the damper blade, normal to and along the damper blade, were $\delta = 20$ mm and $\Delta = 105$ mm.

The moment M, of the air acting on the damper blade was measured with a moment meter. The moment meter is constructed with a lever arm of length l = 100 mm, mass, m [kg], weighing cell manufactured by *HBM*, model PW4MC3 (accuracy Class C3 and 0.5 g minimum resolution) and scaling electronics manufactured by *HBM*, model WE2110 (classes 6000 d and the measuring range 0-3.5 mV/V) to display the measured moment, M_{mer} , fig. 5. The blade was rigidly attached to the shaft, the shaft was bolted rigidly to the lever, and the lever was rigidly connected with a spherical joint to the weighing cell. In this way, the moment, M, of the air acting on the blade was transferred to the moment meter.

The idea was to transfer the moment of the air stream, as much as possible, to the moment meter, in order to secure more accurate measurement. However, in the process of transfer of the moment to the meter, a certain part of the moment was being spent by the overcoming parasite moment (the moment being spent on the deflection of the blade, the moment of friction in bearings, the resistance moment of the transfer mechanism). Hence, the measured moment M_{mer} has a lower value than the moment M of the air stream acting on the blade (eq. 10):

$$M_{mer} = m g l \tag{10}$$

where $g [ms^{-2}]$ is the gravitational constant.

The axis of rotation was displaced from the axis of the blade, in order to obtain a higher moment of the air stream acting on the blade. According to Kirchoff [13], the position

of the center of the pressure, D, changes with the blade angle of attack α . The pressure will be higher on that end of a blade which first encounters the air stream. For this reason, the rotary axis, O, was shifted from the axis of the blade towards the opposite end of the blade which first encounters the air stream. For the purpose of making a comparison of results, all four discussed types of ATP dampers had the same position of the axis of rotation. In order to achieve this, the blade pitch of the damper type C with blades driven in parallel was significantly shortened. For this reason, on the ATP damper type C, angles of attack were only measured within the zone of the open damper.

The calibration of the moment meter was performed with weights of known mass, fig. 5. Since the weighing cell for mass measurement has linear characteristics (dependence of electric power and mass), the resulting characteristic of the moment meter was also linear, fig. 6.



Figure 5. Schematic diagram of the ATP damper moment meter



With their weight, the blade (1) and its supporter (2) produce the dead moment of the blade, M_d [Nm], which was tarred on weighing cell electronics as useless [14], fig. 7:

$$M_{\rm d} = g\delta \left[m_1 \left(1 - \frac{\Delta}{\delta} \operatorname{ctg} \alpha \right) + \frac{m_2}{2} \right] \sin \alpha \tag{11}$$

Figure 8 presents the dependence of the dead moment in terms of the blade angle of attack α for four discussed types of ATP dampers. Unsurprisingly, the dead moment was lower for damper types with two blades: B, C, and D, compared to type A, due to the smaller mass of the measuring blade. Although the dead moment was tarred as useless on weighing



Figure 7. The dead moment of the ATP damper blade



Figure 8. Dead moment in terms of blade angle of attack

electronics, the construction of the ATP damper blade should be such to diminish the dead moment, as much as possible. Low dead moment is needed for two reasons. The first reason is that a large dead moment (in case of a dead moment greater than the maximum moment that can be measured by the moment meter under the influence of the air stream) can permanently deform the strain gauge for cell mass measurement. Another reason is that a greater dead moment creates a greater load of bearing and of the whole mechanism of the moment transfer, which also increases the proportion of parasitic moments.

Reduction of the weight of the blade and its supporter also reduces the dead moment. However, deflection of the blade increases. In this way, the proportion of parasitic moments increases, since a part of the air stream moment is being spent on the deflection of the blade, instead of being transferred to the moment meter.

Thin galvanized sheet used for the construction of the blade created a light-weight blade and therefore, the dead moment acting on the measuring blade was diminished. At the same time, reinforcements significantly reduced the deflection of the blade under the influence of the air stream. In this way, two opposing requirements were met: the dead moment of the blade remained as small as possible, as well as the deflection of the blade.

The moment correction factor was determined by the weight of known mass, m, which was hung on the shaft of the moment meter, fig. 5. In this way, the real value of the moment acting on the meter M was obtained. The moment minus the value of parasitic moments, M_{mer} , was measured at the weighing cell. The correction factor of the moment, γ [-], that takes into account the parasitic moments of the ATP damper is defined as the ratio of the moment of the air acting on the blade M and the measured moment M_{mer} :

$$\gamma = \frac{M}{M_{mer}} \tag{12}$$



In order to measure the damper blade angle of attack, a rotary potentiometer manufactured by Dada Electronics, model Tyco 10 (measuring range 0-10 k Ω) and a digital mul-

only

timeter were used. The rotary potentiometer was calibrated in accordance with the protractor that monitors the position of the blade with the rotation axis displaced from the axis blade, fig.10. The blade position was defined by the angle, α , (position of the blade defined in relation to the horizontal direction).

70

60

The blade angle of attack α was used to measure the position of the blade. During the calibration of the rotary potentiometer, a 30° shift was made and the field of non-linear

1.06

1.055

1.05

1.045

1.04

1.035

1.03

1.025

1.020

0.015

►A

⊷В

-D

10 20 30 40 50

dampers type A, B, and D

Figure 9. Moment correction factors of ATP

Correction factor of torque [–]

characteristic of the potentiometer was abandoned. In this way, the approximate linear relationship between the blade angle of attack and the electric resistance of the rotary potentiometer was created, fig 11.



Figure 10. The ATP damper blade angle of attack

Figure 11. Typical calibration curve of the rotary potentiometer

Results and discussion

The first series of measurements was conducted in order to determine the correlation function eq. (2), *i. e.* to calibrate the mathematical model eq. (1). The correlation function eq. (2) was determined experimentally. Figures 12-15 present the dependence of the squared correlation function, $G^2(\alpha)$, in terms of the blade angle of attack. It can be observed that the correlation function depends solely on the angle of attack. Due to the shortened blade pitch and measurement of the angles of attack only in the zone of the opened ATP damper, the correlation functions of the damper type C become slightly different from those of dampers type A, B, and D.





Figure 13. The correlation function of the ATP damper type B

With the second set of data, the previously calibrated mathematical model was verified. Velocities from the second set of measurements and the velocity obtained from the calibrated mathematical model were compared. Figures 16-19 are showing the difference between the measured, v_{mer} , and the model velocity, v_{mod} , for four types of the discussed ATP dampers. With few points excluded, it can be observed that this difference remains within the limits of ±10%. It is interesting to see in which positions of the blade angle of attack this difference extends beyond the boundaries of ±10%.

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Figure 14. The correlation function of the ATP damper type C



Figure 16. Dependence between the measured and the model velocity for the damper type A



Figure 18. Dependence between the measured and the model velocity of the damper type C



Figure 15. The correlation function of the ATP damper type D



Figure 17. Dependence between the measured and the model velocity of the damper type B



Figure 19. Dependence between the measured and model velocity of the damper type D





Figure 20. The difference between the measured and the model velocity for the ATP damper type A



Figure 22. The difference between the measured and the model velocity for the ATP damper type C



Figure 21. The difference between the measured and the model velocity for the ATP damper type B



Figure 23. The difference between the measured and the model velocity for the ATP damper type D

Namely, for $\alpha = 0^{\circ}$, there is a considerable difference between the measured and the model velocity Δv . Precisely for this angle of attack, the difference between the measured and the model velocity Δv is outside of the $\pm 10^{\circ}$ scope. This is a horizontal position of the blade, when the ATP damper is totally opened. The moment of air acting on the blade is very small in this position. Due to the limited sensitivity of the measuring device, in this position of the blade, a considerably higher moment is produced by the air stream acting on the blade carrier, when compared to the moment of the air stream acting on the blade itself.

Conclusions

In order to prove the universal applicability of the existing mathematical model on all variants of ATP dampers, the model was verified for the case of a square cross section damper with non-cascading blades and the straight duct section positioned in front and behind it. Four possible types of ATP dampers with non-cascading blades were considered: a single blade damper, dampers with two cross driven blades, dampers with two blades driven in parallel, and with two blades of which one is the measuring blade and the other is fixed in the horizontal position.

Identical tendency of results for all four considered types of ATP dampers was noted. The difference between the measured air velocity and the air velocity of the model was in the range of $\pm 10\%$. The mathematical model for ATP dampers with non-cascading blades and with the straight duct section positioned in front of and behind the blades was successfully verified. The presented case of the ATP damper with the verified mathematical model can be used successfully for accurate measurement of the air velocity under different operating conditions.

For the purpose of validation of the universal applicability of the mathematical model, verification of the mathematical model for dampers with different cross-section shapes of blades, different numbers of blades and types of blade driving is required in future research. In addition, influence of the position of dampers in the system, the damper hysteresis, local resistance in front of and behind the damper, on the applicability of the mathematical model, should be explored as well.

Acknowledgments

This paper is a result of the research conducted within the project TR31058, 2011-2015, which was supported by the Ministry of Education, Science and Technology, Republic of Serbia.

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Paper submitted: January 31, 2015 Paper revised: August 22, 2015 Paper accepted: August 22, 2015

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