# THERMOMAGNETIC CONVECTION OF A MAGNETIC NANOFLUID INFLUENCED BY A MAGNETIC FIELD

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We present a numerical study of thermomagnetic convection in a differentially heated cavity. The magnetic nanofluid (ferrofluid) is subjected to a uniform magnetic gradient oriented at an angle,  $\varphi$ , with respect to the thermal gradient. The motivation for this work stems largely from a desire to extent preexisting works focused on horizontal and vertical orientations  $\varphi = 0^{\circ}$ , 90°, 180°, and 270°. Our main goal is to get data on the flow and heat transfer for any orientation in the entire range 0-360°. The generalized problem lends itself to the investigation of orientations that give maximum heat transfer. It is found that, (1) at a given magneto-gravitational coupling number, N, orientations 0°, 90°, and 270°, for which magnetization gradient is unstable, are not the optimum ones, (2) for  $0 < N \le 1$ , heat transfer reaches a maximum between 270° and 360°, (3) for N > 1, a second maximum occur between 0° and 90° owing to reverse flow phenomenon, (4) at strong magnetic gradients, the two heat transfer peaks take the same value, and (5) optimization parameter,  $\omega$ , reflecting the strongest magnetic effect, grows with N. Unlike the gravity, magnetic gradient may supply various strengths and spatial configurations, which makes thermomagnetic convection more controllable. Also, the magnetic mechanism is a viable alternative for the gravity one in microgravity, where thermo-gravitational convection ceases to be efficient.

Key words: thermomagnetic convection, ferrofluid, maximum heat transfer, magnetization stratification, magnetic gradient, optimum orientation

#### Introduction

Theory permits the possibility of liquid ferromagnetism, but the known ferromagnetic solids lose their strong magnetism and become paramagnetic above what is known as the Curie point, which is well below the melting point. The liquid ferromagnetism exists in the form of so-called magnetic nanofluids known also as ferrofluid (FF). The FF are colloidal suspensions of mono-domain ferromagnetic particles in a base liquid. The first stable FF were produced in the mid-60's by the technique invented by Papell [1]. Nanoparticles are coated with a non-magnetic surfactant layer which, together with the Brownian motion, inhibits their aggregation due to the ubiquitous magnetic and Van der Walls forces. The FF synthesis is a veritable task [2, 3]. Liquidity and strong magnetism explain the keen interest taken in FF whose research field is multi-disciplinary. Chemists study their synthesis and physicists propose theories explaining their proprieties. Engineers study their applicability and use them in technical prod-

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ucts. Biologists and physicians study their biomedical potential. The FF were first used as fuel in weightlessness [1]. Nowadays, they serve in a wide range of various areas such as sealing, damping, and biomedicine [3, 4]. Unlike in classical MHD, where Lorentz force can be nonzero even in a uniform magnetic field, the body force in ferrohydrodynamics (FHD) requires no currents but material magnetization and non-uniform field. At the basis of thermomagnetic convection (TMC) mechanism lays the interplay of a field with FF pyromagnetic properties. The FF are smart materials with promising potential for thermal engineering. They would have great advantages over ordinary liquids in devices where heat transfer is the limiting factor such as high-power transformers. The replacing the cooling oil of the transformer by a FF based on this oil can take advantage of the preexisting leakage magnetic fields to improve and maximize heat removal [5]. Besides the macroscopic applications, FF may be a viable option to reinforce heat removal in challenging miniature systems where thermogravitational convection (TGC) can not be harnessed [6]. Finlayson [7] was the first to propose a modification of the classical Benard experiment to probe the way in which a uniform field affects the instability threshold in a FF. Schwab et al. [8] implemented the experiment suggested and confirmed the pioneer predictions [7]. Generalizations and extensions of the theoretical model [7] may be found in [9-16] to name a few. The TMC in cavities has been studied both numerically and experimentally in magnetic fields created and controlled by various sources. Several authors studied the effect of a uniform field on heat transfer [17-23], while others assumed a non-uniform field [5, 6, 24-32]. This paper deals with TMC in differentially heated cavity placed in a uniform magnetic field gradient. Survey of works [24-28] has shown that studied are only the magnetic gradient horizontal and vertical orientations. To the best of our knowledge, no complete study was carried out on the magnetic gradient orientation effects. To remedy this lack of information, we assume a magnetic gradient arbitrarily oriented with respect to the thermal gradient and gravity.

# **Physical model**

The working FF is confined in rectangular cavity made of non-magnetic materials to not distort the external non-uniform magnetic field,  $\vec{H}_{ex}$ . The right hot and left cold walls of height, *L*, are kept at constant temperatures,  $T_h$  and  $T_c$ , while the top and bottom walls of width, *W*, are isolated. The uniform magnetic gradient,  $\nabla H_{ex}$ , is oriented at an angle  $\varphi$  with respect to the thermal gradient  $\nabla T$ , fig. 1.



Figure 1. Physical system

#### Ferrohydrodynamic model

The pioneer FHD model [33] treats the FF as non-conducting isotropic mono-component and mono-phase continuum with instantaneous magnetic relaxation, *i. e.*, equilibrium magnetization is reached in times smaller than the characteristic convective time. This quasi-stationary macroscopic model holds for well-stabilized Newtonian FF at low and moderate con-

centrations. Boussinesq approximation enables to write the coupled laminar thermomechanics equations describing the physical model:

$$\vec{\nabla} \cdot \vec{\mathbf{v}} = \mathbf{0} \tag{1}$$

$$\rho_0 \left( \frac{\partial}{\partial t} + \vec{\mathbf{v}} \cdot \vec{\nabla} \right) \vec{\mathbf{v}} = -\vec{\nabla} P + \rho \vec{\mathbf{g}} + \eta \nabla^2 \vec{\mathbf{v}} + \mu_0 \left( \vec{\mathbf{M}} \cdot \vec{\nabla} \right) \vec{\mathbf{H}}$$
(2)

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$$\rho_0 C \left( \frac{\partial}{\partial t} + \vec{\mathbf{v}} \cdot \vec{\nabla} \right) T = \lambda \nabla^2 T - \mu_0 T \left( \frac{\partial M}{\partial T} \right)_{\mathrm{H}} \left( \frac{\partial}{\partial t} + \vec{\mathbf{v}} \cdot \vec{\nabla} \right) H$$
(3)

The last term in eq. (2) stand for the Kelvin magnetic force acting on FF. The last term of eq. (3) reflects FF temperature change due to the magnetocaloric effect which is associated both with the change in field intensity H with time and FF motion along the magnetic field gradient lines. This adiabatic effect proves to be negligible far below the Curie temperature,  $T_{\rm C}$ , and maximum near it.

With no conduction and displacement currents, Maxwell equations governing the total field  $\vec{H}$  are:

$$\vec{\nabla} \cdot (\vec{M} + \vec{H}) = 0$$
 and  $\vec{\nabla} \times \vec{H} = \vec{0}$  (4a,b)

The FHD model [33] consider magnetically *soft* particles whose anisotropy energy  $K_aV_f$  is much lower than their thermal energy  $k_BT$ , *i. e.*, the magnetic moment is not frozen into the body particle so that the field does not affect the particle orientation. Thus, FF is free from magnetoviscous effect (MVE) [34]. With magnetization vector aligned with the field, the expression of the Kelvin body force becomes:

$$\vec{\mathbf{M}} = \left(\frac{M}{H}\right)\vec{\mathbf{H}} \text{ and } \vec{\mathbf{F}} = \mu_0 M \vec{\nabla} H$$
 (5a,b)

Nanoscaled sizes and low concentrations of particles weaken their magnetic interaction and thus their structuration in chains and clusters even in the absence of the field. Thus, Langevin's theory is adapted for mono-disperse identical particles to get the superparamagnetic magnetization law [35]

$$M = n_{\rm p} m_{\rm p} \left( \coth \xi - \frac{1}{\xi} \right) \tag{6}$$

where  $\xi = \mu_0 m_p H (k_B T)^{-1}$  is the Langevin argument,  $n_p = \varepsilon / V_p$  the particle concentration ( $\varepsilon$  is the solid volume fraction and  $V_p$  the particle hydrodynamic volume, respectively), and  $m_p = M_f V_f$  the particle magnetic moment ( $M_f$  is the particle domain magnetization and  $V_f$  the particle volume ferromagnetic portion).

In narrow ranges of H and T, the density and magnetization equations of state are linearized about the reference field  $H_r$  and the walls average temperature  $T_0 = (T_h + T_c)/2$ :

$$\rho = \rho_0 + \rho_0 \beta_\rho (T - T_0) \text{ and } M = M (H_r, T_0) + \chi (H - H_r) - K (T - T_0)$$
(7a,b)

where  $\chi = (\partial M \partial H)_T$  is the magnetic susceptibility and  $K = -(\partial M \partial T)_H$  the pyromagnetic coefficient which, at magnetic saturation  $M_{\text{sat}} = n_p m_p$  ( $\xi \gg 1$ ) and weak magnetic fields ( $\xi \ll 1$ ), is given by [36]:

$$K = M_{\text{sat}}(\beta_{\text{m}} + \beta_{\rho}) \quad \text{at} \quad \xi \ll 1 \quad \text{and} \quad K = \frac{\xi M_{\text{sat}}}{3} \left(\frac{1}{T} + 2\beta_{\text{m}} + \beta_{\rho}\right) \quad \text{at} \quad \xi \ll 1$$
(8a,b)

where  $\beta_{\rho} = -(1/\rho)(\partial \rho/\partial T)$  and  $\beta_m = -(1/m_p)(\partial m_p/\partial T)$  which is very weak far below the Curie point.

In zero approximation, the field perturbation may be ignored, *i. e.*, the field is assumed prescribed  $\vec{H} \approx \vec{H}_{ex}$  and Maxwell equations are mathematically isolated from others equations.

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Condition for zero approximation to be applied is  $\nabla H_{\text{ex}} \gg \nabla H_{\text{in}}$  where  $\nabla H_{\text{in}} \sim K \nabla T$  is the induced inner magnetic gradient by the non-isothermal FF. Thus, far below  $T_{\text{C}}$ , the steady-dimensionless equations are cast in the form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \Pr\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \Pr\left(\operatorname{Rm}\cos\varphi\right)\Theta$$
(10)

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \Pr\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \Pr\left(\operatorname{Rg} - \operatorname{Rm}\sin\varphi\right)\Theta$$
(11)

$$u\frac{\partial\Theta}{\partial x} + v\frac{\partial\Theta}{\partial y} = \frac{\partial^2\Theta}{\partial x^2} + \frac{\partial^2\Theta}{\partial y^2}$$
(12)

Dimensionless temperature is  $\Theta = (T - T_0)/(T_h - T_c)$ . Boundary conditions consist of no-slip on all walls (u = v = 0), zero heat flux across the top and bottom walls ( $\partial \Theta / \partial y = 0$ ), and isothermal right and left walls ( $\Theta = \pm 0.5$ ). Control parameters are: the aspect ratio Ar = L/W,  $Pr = C\eta/\lambda$ , gravitational and magnetic Rayleigh numbers  $Rg = \beta_\rho g \nabla T W^4/(va)$  and  $Rm = \mu_0 K \nabla H \nabla T W^4/(a\eta)$ , respectively. Magnetogravitational coupling number N = Rm/Rg = $= \mu_0 K \nabla H / (\rho_0 \beta_\rho g)$  is the ratio of thermomagnetic force (TMF) to thermogravitational force (TGF). Besides being dependent on the field and *T*-distribution, TMC is affected by the pyromagnetic coefficient. Thus, to strengthening it one can use either solids with higher *K* or higher  $\varepsilon$ . So, thermo-sensitive FF, whose magnetization strongly depend on temperature, are needed. Only near  $T_c$  a change in *T* cause a high change in *M*. Usual ferromagnets have high  $T_c$ , *e. g.* 1331 °C for cobalt. So, considering the base liquid boiling point, FF with  $T_c$  close to the devices operating range are needed. Table 1 show high *K* and low  $T_c$  for Mn-Zn and Fe-Zn ferrites, and various FF used in different studies.

Table 1. Pyromagnetic coefficient of thermo-sensitive FF and various	FF	used
in different studies on TMC		

Reference	Carrier liquid	Magnetic material	ε [%]	$T_{\rm C}$ [°C]	K [Am <sup>-1</sup> K <sup>-1</sup> ]
	Diester	$Mn_{0.5}Zn_{0.5}-Fe_2O_4$	8.91	67	5125
Parekh et al. [37]	Diester	Fe <sub>0.5</sub> Zn <sub>0.5</sub> -Fe <sub>2</sub> O <sub>4</sub>	4.94	91	3125
	Diester	Fe <sub>0.3</sub> Zn <sub>0.7</sub> -Fe <sub>2</sub> O <sub>4</sub>	6.99	74	2128
Li et al. [38]	Kerosene	Mn Zn–Fe <sub>2</sub> O <sub>4</sub>	4.50	80	1050
Sustov [39]	Kerosene	Fe <sub>3</sub> O <sub>4</sub>	—	—	100
Engler et al. [40]	Synthetic ester	Fe <sub>3</sub> O <sub>4</sub>	6.30	—	35.3
Tangthieng et al. and Jue [5, 30]	HC oil	Fe <sub>3</sub> O <sub>4</sub>	1.60	—	30
Schwab et al. and Stiles and Kagan [8, 9]	HC	-	_	—	27.3
Sawada et al. [26]	Water	Fe <sub>3</sub> O <sub>4</sub>	_	—	19.11

To estimate the heat flux at the hot wall, the local and mean Nusselt numbers are calculated:

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$$\operatorname{Nu} = \frac{h W}{\lambda} = \frac{\partial \Theta}{\partial x} \bigg|_{x=1} \qquad \overline{\operatorname{Nu}} = \frac{\int_{0}^{Ar} \operatorname{Nu}(y) \, \mathrm{d}y}{Ar}$$
(13)

# Association phenomena and transport properties of FF

A typical FF contains on the order of 10<sup>23</sup> particles per cubic meter, and collisions between them are frequent. Particles clustering may be regarded as a step in the direction of sedimentation which may destroy the FF. In strong fields, particles tend to form chains parallel to the field with a mean particles number  $n_{\infty} = [1 - 0.66\varepsilon \gamma^{-2}e^{2\gamma}]^{-1}$  [41] (where  $\gamma$  is the coupling parameter). Like any fluid, hydrothermal processes in FF are affected by changes in its thermophysical properties. The highly stable Newtonian i-butanol based FF show no MVE while the methyl-ethyl-ketone based FF show a strongly non-Newtonian behavior owing to a significant increase of its effective viscosity,  $\eta$ , caused by aggregation initiated by an incomplete surfactant covering of particles [3]. Stability analysis and experiment show that the MVE delay the TMC onset [15, 40]. Higher TMC threshold in sample with coarser particles is due to chain-like structures causing a strong MVE reinforcing the stabilizing viscous forces which in turn hinder TMC [40]. In the theoretical model describing the MVE [42], which agree with experiment [43], increment of  $\eta$ for the co-toluene FF in a field parallel to the flow is higher than that for a normal field. However, in [44], MVE for a normal field is stronger than the very weak one in a parallel field. These data agree well with experimental data for the Fe-water FF [45]. Non-magnetic nanofluids exhibit effective thermal conductivity,  $\lambda$ , higher than that of carrier liquid, which is benefit for thermal applications. In [46] a new cooling system is proposed where the use of the Al<sub>2</sub>-water nanofluid may limit thermal stresses in wind turbines. Some works focused on FF thanks to their special properties. It is found that for a kerosene-base FF,  $\lambda$  grows with the strength of a field parallel to heat flux while a normal field has no effect [47]. The measured data for the Fe-water FF [45] agree with data of [47]. High increase of  $\lambda$  in a parallel field is also found for Fe<sub>3</sub>O<sub>4</sub> in kerosene, hexadecane or oil [48, 49]. In [45, 48-50], it is claimed that aggregates and their aspect ratio plays a vital role in the grows of anisotropic  $\lambda$ . In [51],  $\lambda$  grows with H for the parallel alignment while for the normal case, it has an inverse relationship to H. Experiment fit well with prediction [52]. As outlined in [53], besides the apparent paradox about the field direction effect on FF thermal and rheological properties, these studies were conducted in the stagnant state. Thus, the question is can aggregates be build in magnetic fields and affect properties of flowing FF?

#### Numerical method and solution procedure

Finite volumes method [54] is used to discretize eqs. (9)-(12). Pressure velocity coupling is handled by the SIMPLER algorithm. Under-relaxed algebraic equations are solved with a tri-diagonal matrix algorithm line-by-line solver. A residual shows how perfectly these equations are satisfied. Numerical tool was checked for accuracy against benchmark data [55] for TGC of air (Pr = 0.71) and Ar = 1. We get  $\overline{Nu} = 1.118 \text{ vs.} 1.118$ , 2.244 vs. 2.243, 4.535 vs. 4.519, and 8.866 vs. 8.800 for Rg = 10<sup>3</sup>, 10<sup>4</sup>, 10<sup>5</sup>, and 10<sup>6</sup>, respectively. Good accord between both data highlights adequacy of numerical tool. For TMC, grid dependency tests are performed at Rg = 10<sup>4</sup>, Pr = 7 and Ar = 1 on the grids 61 × 61 and 231 × 231 for four magnetic gradient orientations:  $\varphi = 0^{\circ}, 45^{\circ}, 225^{\circ}$ , and 315° with two values of N tested for each  $\varphi$ . The Nu and deviations based on the finest grid are listed in tab. 2. It is seen that data obtained with both grids are very close with a good relative error. Thus, as for pure TGC, the grid 61 × 61 is chosen as a trade-off between calculus time and accuracy for all computations, except for higher Ar and high N where it is refined.

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Table 2. Grid-dependence of TMC data at various magnetic gradient orientations,  $Rg = 10^4$ , Pr = 7, and Ar = 1

φ [°]	Ν	Grid size	Nu	Deviation [%]
0 (45) (225) (315)	1	61 × 61	2.823 (2.241) (2.244) (3.034)	-0.5 (-1.3) (0.6) (-0.1)
		231 × 231	2.828 (2.254) (2.238) (3.035)	-
	25	61 × 61	4.869 (6.257) (2.596) (6.431)	-3.4 (4.3) (5.3) (4.8)
		231 × 231	4.903 (6.214) (2.543) (6.383)	-

#### **Results and discussion**

# Horizontal and vertical orientations $\varphi = 0^{\circ}$ , 180°, 90°, and 270° of the magnetic gradient

Berkovsky et al. [24] conducted experimental and numerical studies of TMC in a layer (Ar = 27) of kerosene-based FF (Pr = 36) prepared by magnetite (Fe<sub>2</sub>O<sub>4</sub>) particles dispersion ( $\varepsilon = 1.2\%$ ) with acid oleic admixed as a stabilizer agent. The FF is placed in a horizontal magnetic gradient  $\nabla H = 3.1 \cdot 10^5 \text{ A/m}^2$ . Computations showed that at Ar > 5, increasing Ar result in slight decrease of  $\overline{Nu}$ , e. g. at Rm = 10<sup>6</sup>, and Rg = 3.6·10<sup>5</sup>,  $\overline{Nu}$  for Ar = 10 exceeds the measured one by less than 10%. Possibility of extrapolation suggested them to study numerically the magnetic effect. Qualitative accord is seen between data [24], fig. 2, and our results, fig. 3, for Ar = 5. When  $\nabla H$  and  $\nabla T$  are parallel ( $\varphi = 0^{\circ}$ ), the magnetization gradient is unstable  $(\nabla M \uparrow \downarrow \nabla H \downarrow \downarrow \nabla T)$ . The cold layers (higher *M*) are sucked into zones of higher *H* displacing the hot layers (lower M). Thus, TMC enhance TGC. Horizontal lines of curves 1-2-3 indicate a weak magnetic effect as compared to the gravity one. The TGC enhancement takes place when TMF is competitive to TGF ( $Rm \approx Rg$ ). Higher the Rm more will be TMF leading to higher Nu. At strong  $\nabla H$ , the three curves overlap each other indicating a same  $\overline{\mathrm{Nu}}$  due to a weak gravity effect, e. g. at Rg (or Ra) =  $10^3$ , is compensated by a strong magnetic effect, e. g.  $N = 5 \cdot 10^3$  at Rm = 5.10<sup>6</sup>, as compared to cases Rg = 10<sup>4</sup> (N = 5.10<sup>2</sup>) and Rg = 10<sup>5</sup> (N = 50). If  $\nabla H$  and  $\nabla T$  are adverse ( $\varphi = 180^\circ$ ), the magnetization gradient is stable ( $\vec{\nabla}M \uparrow \vec{\nabla}H \uparrow \downarrow \vec{\nabla}T$ ), *i. e.*, TMC works against TGC. Horizontal lines of curves 1'-2'-3' reflect a weak damping effect which becomes significant at Rm  $\approx$  Rg. Then, Nu reduce with Rm until unity. Cold layers are so fixed by the TMF so that heat transfer is by conduction ( $\overline{Nu} = 1$ ). The required Rm to suppress the flow grows with Rg. The TMC is not associated with gravity. Thus, FF greatest interest is when it is used in microgravity conditions, where cooling by TGC ceases to be efficient (e. g. orbital stations). The Nu vs. Rm is plotted in fig. 4 for  $\varphi = 0^{\circ}$  and Rg = 0. Horizontal line for pure conduction



Figure 2. An effect of gradient magnetic field on heat transfer for Ar = 5 [24]



Figure 3. An effect of gradient magnetic field on heat transfer for Ar = 5, present work

 $(\overline{\text{Nu}} = 1)$  breaks up when TMC comes into play as soon as TMF will overcome the stabilizing effects of viscosity and thermal diffusion. Beyond the TMC threshold (~2700), fig. 4,  $\overline{\text{Nu}}$  grows with Rm. We can speak of magnetic Rayleigh-Benard system with horizontal magnetic gravity  $gm_x = +Ng$ . In elegant experiment [56], TMC is studied in microgravity using sounding rockets and at the drop tower.

When  $\nabla H$  augments gravity, the magnetization gradient is unstable ( $\varphi = 270^{\circ}$  $\nabla H \downarrow \downarrow \vec{g} \perp \nabla M \uparrow \downarrow \nabla T$ ). Thus, TMF acts in the same direction than TGF with an apparent gravity  $ga = g + gm_y$  where  $gm_y = +Ng$  is a pseudo vertical magnetic gravity. Horizontal line in fig. 5 implies a dominant role for TGC. At N > 0.1, the magnetic effect grows with N. When  $\nabla H$  opposes gravity,  $\nabla M$  is also unstable ( $\varphi = 90^{\circ} \nabla H \uparrow \downarrow \vec{g}$ ) but TMF opposes TGF. At N < 1, TGF is dominant since  $gm_y = -Ng$  (ga > 0). Horizontal line reflects a weak damping magnetic effect which start to grow beyond N = 0.1. At N = 1, TMF and TGF are ideally turned off (ga = 0,  $\overline{Nu} = 1$ ) as shown by vertical isotherms in fig. 6. For N > 1, TMF is strong enough to





Figure 4. Ferroconvective instability in microgravity  $\varphi = 0^\circ (\overline{\nabla} H \uparrow \uparrow \overline{\nabla} T) (\varphi = 0^\circ), Ar = 1,$ Pr = 36, and Rg = 0

Figure 5. Heat transfer vs. magnetic gradient strength  $\varphi = 90^\circ$  ( $\nabla H \uparrow \downarrow g$ ), and  $\varphi = 270^\circ$  ( $\nabla H \downarrow \downarrow g$ ), Ar = 1, Pr = 7, and  $Rg = 10^4$ 



Figure 6. Velocity vector field and temperature iso-lines illustrating the reverse flow for  $\varphi = 90^{\circ}$ , Ar = 1, Pr = 7, and  $Rg = 10^{4}$ 

reverse the flow (ga < 0), *i. e.*, the warm FF spreads downward and  $\overline{\text{Nu}}$  grows as |ga| grows with *N*. Reverse flow is illustrated by the isotherms traced for N = 0.9, which are upside down compared to those of N = 1.1. The velocity vector field evidences clarify this phenomenon. Flow is counter-clockwise at N = 0.9 and clock-wise at N = 1.1. Figure 5 shows a symmetry about the line N = 1, where  $\overline{\text{Nu}}$  is the same for  $0 \le N_1 < 1$  and  $1 < N_2 \le 2$  with  $N_1 + N_2 = 2$ . This occurs because the FF feels the same net force, *i. e.*,  $ga(N_1) = |ga(N_2)|$ . This symmetry has been observed in [25] for  $ga = 9.8 \text{ m/s}^2$  ( $N_1 = 0$ ) and  $ga = -10 \text{ m/s}^2$  ( $N_2 \approx 2$  corresponding to  $\nabla H = 5.37 \cdot 10^5 \text{ A/m}^2$ ). Also,  $\overline{\text{Nu}}$  for  $\varphi = 90^\circ$  at  $N_1 > 2$  equals  $\overline{\text{Nu}}$  for  $\varphi = 270^\circ$  at  $N_2$ , such as  $N_1 - N_2 = 2$ , because the FF feels the same net force  $[|ga(N_1)| = ga(N_2)]$ . At N > 1, the disparity between Nu at  $\varphi = 90^{\circ}$  and Nu at  $\varphi = 270^{\circ}$  reduces until zero at high N where they overlap each other due to a weak gravity effect. Table 3 displays a comparison with prediction [28] and experiment [26] where three FF are tested, W-40 (Fe<sub>3</sub>O<sub>4</sub>-water), TS-40W (Mn-Zn-water), and HC-50 (kerosene-base). Comparison is for W-40 since more data are given about its properties (Rg = 37560, Pr = 44.3). Excellent accord between the 2-D data is evident. For pure TGC

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φ[°]	$ \begin{array}{c} \nabla H \times 10^5 \\ [\mathrm{Am}^{-2}] \end{array} $	ga/g	N	2-D [28]	2-D (present work)	3-D [28]	Experiment [26]
-	0	1	0	3.50	3.49	3.37	5.0
270	1.6	2.1	1.1	4.41	4.41	4.38	5.1
	4.5	4.0	3.0	5.34	5.34	5.47	5.3
	8.8	6.9	5.9	6.29	6.29	6.59	5.9
90	1.6	-0.1	1.1	1.63	1.62	1.47	2.1
	4.5	-2.0	3.0	4.34	4.34	4.30	3.9
	8.8	-49	5.9	5.69	5.69	5.87	5.6

Table 3. Comparison with experiment [26] and simulation [28] for  $\varphi = 90^{\circ}$  and  $\varphi = 270^{\circ}$ , Ar = 1, Rg = 37560, and Pr = 44.3

(N = 0), where the FF behaves as the base liquid since M = 0, prediction is in poor accord with experiment while 2-D data are adequate for low Rg laminar flow and the literature values average 3.75 [28]. For instance, Lankhorst [57] captured data for both water (Pr = 8.11) and air in one correlation:  $\overline{\text{Nu}} = [0.31 \text{Pr}^{0.035} \text{Rg}^{0.25}]^a$ ,  $(a = 1 - 15.62 \text{Rg}^{-0.431})$  for  $600 < \text{Rg} \le 10^6$ . The power 0.035 reflects the Prandtl number weak effect, also found here and in [28]. His 2-D and 3-D numerical data differ by only 0.04%. Also, 2-D data agree well with those of [55]. His correlation gives Nu = 2.202 vs. 2.243, 4.526 vs. 4.519, and 8.834 vs. 8.800 at  $Rg = 10^4$ ,  $10^5$ , and  $10^6$ . For Rg = 37560 and Pr = 44.3, it gives Nu = 3.77. Excellent fit is achieved between simulation and his own experiments. Moreover, his data agree well with measurements of other investigators. To compare with experiment, it is required to known many of W-40 properties accurately. Density,  $\rho$ , and kinematic viscosity, v, are given in [26]. As the thermal expansion coefficient, that of water is adopted:  $\beta_{\rho} = 2.6 \cdot 10^{-4} \text{ K}^{-1}$  [58]. Unfortunately, as pointed out [58], because FF is a mixture with complex physicochemical properties, it was very difficult to obtain the exact values of specific heat, C, and  $\lambda$ , which are estimated from literature:  $C = 3.0 \cdot 10^3$  J/kgK is computed using  $C_w$  and  $C_f$  of water and Fe<sub>3</sub>O<sub>4</sub> found in [59] and particles diameter given in [60] is used to get  $\lambda$  using a formula provided in [61] for Co or Fe in toluene or HC oil. Both  $\lambda_{\rm w}$  and  $\lambda_{\rm f}$  are found in [62]. However, as aforementioned, various FF may have various models for  $\lambda$ . Besides, both FF used in [26, 58] have the same  $\rho = 1.4 \cdot 10^3 \text{ kg/m}^3$  and different v  $(28.5 \cdot 10^{-6} \text{ m}^2/\text{s} [26], \text{ and } 8.7 \cdot 10^{-6} \text{ m}^2/\text{s} [58])$ . Note that Pr = 13.53 in [58], *i. e.*,  $\eta = 39.9$  kg/ms in [26] is 3.28 times greater than  $\eta = 12.8$  kg/ms in [58]. Thus, both samples have different  $\varepsilon$ . Since FF properties are affected by  $\varepsilon$ , why both samples have the same C and  $\lambda$  but different  $\eta$ ? Besides, the surfactant layer is considered in estimating  $\lambda$  and ignored in that of C. Unfortunately, there is no companion numerical study for experiment in [26] for complete comparison. So, it is probably unreasonable to expect a precise comparison of data. Relatively good accord is found at  $\varphi = 90^{\circ}$ . Note that the reverse flow (ga < 0) for TS-40W occurs at smaller  $\nabla H$  due to its higher K [26]. At  $\varphi = 270^\circ$ , computation agree with experiment for ga/g = 4 but not for ga/g = 2.1 and 6.9. There is another unexplained result. The previous symmetry is found for  $\varphi = 90^{\circ} (ga/g = -2)$  and  $\varphi = 270^{\circ} (ga/g = 2.1) (N_1 - N_2 \approx 2 \text{ and } 7\% \text{ of difference})$  while measured data differ by 120%.

#### Generalized problem

Simulations are for a water-based FF (Pr = 7), Ar = 1 and Rg = 10<sup>4</sup>. The angle  $\varphi$  is varied from 0° to 360° in steps of 5°. Generalized apparent gravity is written as  $\vec{G} = \vec{g} + Ng\vec{e}$  ( $\vec{e}$  is unit vector in direction of  $\vec{\nabla}H$ ). Horizontal and vertical gravities are  $gm_x = +N\cos\varphi$  g and  $ga = g + gm_y$  where  $gm_y = -N\sin\varphi$  g. At  $N \le 1$ , vertical TMF never exceed TGF ( $ga \ge 0$ ), *i. e.* the flow can not be reversed as shown by the velocity vector field for N = 1 in fig. 7. Flow is that of one counter-clockwise rotating cell in the entire domain 0-360°. The  $\vec{\nabla}H$  turn causes the cell turning due to change in TMF components. As is known, heat transfer depends on the



Figure 7. Velocity vector field (streamlines) at various magnetic gradient orientations: Ar = 1, Rg = 10<sup>4</sup>, and Rm = 10<sup>4</sup> (N = 1)

flow structure. Thus, change in  $\nabla H$  direction may cause change in heat flux across the cavity. Figure. 8(a) depicts  $\overline{\text{Nu}}$  vs.  $\varphi$ . In 0-90°, TMC enhance partly TGC ( $gm_x > 0 \nabla H_x \downarrow \downarrow \nabla T gm_y < 0$  $\nabla H_{v} \uparrow \downarrow \vec{g}$ ). The decrease of  $\nabla H_{x}$  with  $\varphi$  causes a decrease of  $gm_{x}$  while ga reduces ( $|gm_{v}|$  grows) with growing  $\nabla H_{y}$ . The resulting flow intensity decrease leads to lower Nu. In 90-180°, where TMC opposes totally TGC  $(gm_x < 0 \nabla H_x \uparrow \downarrow \nabla T gm_y < 0 \nabla H_y \uparrow \downarrow \vec{g})$ , Nu declines with  $\varphi$  and passes through a minimum at  $\varphi_{\min}$  reflecting the strongest damping magnetic effect. This trend is ascribable to its dependence upon the opposite effects of increasing ga (decreasing  $|gm_y|$ ) and  $|gm_x|$ . In 90° –  $\varphi_{\min}$ , the drop of Nu with growing  $|gm_x|$  overcomes its increase with growing ga. The opposite occurs between  $\varphi_{\min}$  and 180°. At N = 1, the minimum is reached before reaching  $\varphi = 90^{\circ}$  where the flow pattern die out, fig. 7, due a to mutual neutralization of TMF and TGF (G=0). This state persist from 80° until 115°, fig. 8(a), due to the weak apparent gravity in 80-90°  $(gm_x = +0.17g \text{ and } ga = +0.02g \text{ at } \varphi = 80^\circ)$  while the net vertical driving force is not strong enough to overcome the horizontal TMF resistant effect in 90-115° ( $gm_x = -0.42g$  and ga = +0.09g for  $\varphi = 115^{\circ}$ ). At  $\varphi > 115^{\circ}$ , this effect is overwhelmed and convection sets in  $(gm_x = -0.5g_and ga = +0.13g at \varphi = 120^\circ)$ . As in 0-90°, TMC is partly cooperating in 180-270° but  $\vec{\nabla}H$  components are inverted  $(gm_x < 0 \,\vec{\nabla}H_x \uparrow \downarrow \vec{\nabla}T \, gm_y > 0 \,\vec{\nabla}H_y \downarrow \downarrow \vec{g})$ . As  $\varphi$  grows,  $\nabla H_{v}$  reduce while  $\nabla H_{v}$  increase. Hence, the flow resistance decrease with reducing  $|gm_{v}|$  and increase of its intensity with growing ga (growing  $gm_y$ ) leads to higher Nu. In 270-360°, TMC enhance totally TGC  $(\underline{g}m_x > 0 \nabla H_x \downarrow \downarrow \nabla T \ \underline{g}m_y > 0 \nabla H_y \downarrow \downarrow \mathbf{g})$ . The  $\overline{\mathrm{Nu}}$  grows with  $\varphi$  and passes through a maximum  $\overline{\text{Nu}}_{\text{max}}$  at  $\varphi_{\text{opt}}$ . Between 270° and  $\varphi_{\text{opt}}$ , the increase of  $\overline{\text{Nu}}$  as a result of increas-



Figure 8. Dependence of heat transfer rate on magnetic gradient orientation: Ar = 1, Pr = 7,  $Rg = 10^4$ : (a)  $N \le 1$ , (b) N > 1

ing  $gm_x$  prevails on its decrease as a result of reducing ga (reducing  $gm_y$ ). The opposite is true between  $\varphi_{out}$  and 360°. At N > 1, the vertical TMF equilibrates TGF at  $\varphi_e$  given by  $N \sin \varphi_e = 1$ (ga = 0). At  $\varphi > \varphi_e$ , this force is strong enough to reverse the flow as shown for N = 3, fig. 9. In 0-19°, uni-cellular flow is rotating counter-clockwise. At 20°, one cell mode vanishes and twocell mode rise. This case is similar to that studied for  $\varphi = 0^{\circ}$  in microgravity, *i. e.*, Rm, = 2.82 \cdot 10^4 lies at a supercritical state. Physically, heat supplied by the hot wall is transported towards the cold one thanks to the horizontal TMF ( $gm_x = +2.82g$ ). Flow and thermal fields are exactly those for pure TGC (N = 0) at Rg = 2.82·10<sup>4</sup> in Rayleigh-Benard system rotated of 90°. Beyond  $\varphi_e$  $(\varphi = 20.1^{\circ})$ , two cells vanishes and one clockwise cell rise. To locate  $\varphi_e$  in the range  $2 \le N \le 25$ , an increment  $\Delta \varphi = 0.1^{\circ}$  is used. A mean difference of 0.64° between computation and theory may be considered as sufficiently accurate since the lowest value is 2.29° for N = 25. In  $0-\varphi_{e}$ , TGF is dominant (ga > 0). The decrease of both  $gm_x$  and ga (grow of  $|gm_y|$ ) with  $\varphi$  leads to weaker flow intensity causing the Nu drop until reaching a first minimum at  $\varphi_e$ , fig. 8(b). Beyond  $\varphi_e$ , the flow is reversed (ga < 0). In  $\varphi_e$ -90°,  $\overline{Nu}$  grows to reach a first peak  $\overline{Nu}_{lmax}$  at  $\varphi_{lopt}$ after which it reduces. In  $\varphi_e - \varphi_{1opt}$ , the grow of Nu with growing |ga| (growing  $|gm_v|$ ) overcomes its reduce with reducing  $gm_x$ . The opposite is true in  $\varphi_{1opt}$ -90°. For 90°  $\leq \varphi < 180^{\circ} - \varphi_e$ , vertical



Figure. 9. (a) Velocity vector field, and, (b) isotherms at various magnetic gradient orientations,  $Rm = 3.10^4$  (N = 3)

TMF is still dominant (ga < 0). Reducing |ga| (reducing  $|gm_y|$ ) and growing  $|gm_x|$  with  $\varphi$  leads to lower  $\overline{\text{Nu}}$  and  $\varphi = 180^\circ - \varphi_e$  yield a second minimum  $\overline{\text{Nu}} = 1$  (ga = 0,  $gm_x < 0$ ). Except at N = 2, the flow is conduction dominated in  $180^\circ - \varphi_e < \varphi \le 180^\circ$  due to a strong horizontal TMF ( $gm_x = -3g$  at N = 3 and  $\varphi = 180^\circ$ ). As for  $N \le 1$ , a second peak  $\overline{\text{Nu}}_{2\text{max}}$  is reached at  $\varphi_{2\text{opt}}$  in 270-360°.

#### Heat transfer optimization

Orientations  $\varphi = 0^{\circ}$ , 90°, and 270° are not the optimum ones, figs. 8(a)-(b), owing to the TMF components effect on FF motion. For  $0.1 < N \leq 1$ , the heat transfer peak occurs in 310-315°, while for  $1 < N \leq 25$ , the two peaks are reached in 55-70° and 290-305°. The two optimum orientations moves towards  $\varphi = 90^{\circ}$  and  $\varphi = 270^{\circ}$  without reaching them. This also holds for higher N since varying N from 25 to 10<sup>5</sup> results in small change in  $\varphi_{1opt}$  and  $\varphi_{2opt}$ . For  $N \propto 85$ , the two peaks occurs at 75° and 285°, and take the same value due both to the week gravity effect ( $ga \approx gm_y$ ) and the symmetry of 75° and 285° about x-direction, *i. e.*, TMF has the same strength ( $gm_x = +0.26Ng, gm_y = +0.96Ng$  at

 $\varphi = 75^{\circ}$ , and  $gm_y = -0.96Ng$  at  $\varphi = 285^{\circ}$ ). The most remarkable magnetic action on heat transfer is described by a useful quantity: the scaled  $\overline{Nu}_{max}$ :

$$\omega = \frac{\mathrm{Nu}_{\mathrm{max}}}{\mathrm{Nu}_{\mathrm{o}}} \tag{14}$$



Evaluating the optimization parameter  $\omega$  at a given N require the  $\overline{Nu}_0$  for pure TGC at the same Rg. Here,  $\overline{Nu}_0 = 2.27$  for Rg = 10<sup>4</sup>. It is seen that  $\omega$  is always greater than one, indicating an enhancement of TGC, fig.10, *e. g.* at N = 25 the maximum heat flux is three times higher than that for pure TGC ( $\omega = 3$ ).

Figure 10. The TGC maximum enhancement vs. magnetic gradient intensity: Ar = 1, Pr = 7, and  $Rg = 10^4$ 

#### Summary and conclusions

We performed a numerical study of TMC in a vertical cavity. The hydro-thermal processes are found to be sensitive to the magnetic gradient strength and its synergic orientation with thermal gradient and gravity. Orientations 0°, 90°, and 270°, for which magnetization stratification is unstable, are not the optimum ones. At weak magnetic gradients, the heat transfer peak occurs in 270-360°. At higher values, a second peak occurs in 0-90° due to reverse flow phenomenon induced when vertical TMF becomes dominant over TGF. At strong magnetic gradients, the two peaks take the same value. The FF are controllable cooling agents in which flow and heat transfer may be adjusted by a proper choice of a magnetic gradient which, unlike gravity which is almost similar everywhere, may supply various strengths and orientations. In weightlessness or microgravity environment, where TGC is prohibited or not efficient, TMC mechanism remains the only one to ensure heat transport. Also, TMC requires both thermal and magnetic interactions with the surrounding. This makes using FF as heat carrier a good option to improve the cooling system efficiency of devices which already have strong magnetic fields.

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#### Nomenclature

Ar	- cavity aspect ratio (= $L/W$ ), [-]	γ	<ul> <li>coupling parameter,</li> </ul>
a	– thermal diffusivity (= $\lambda/\rho C$ ), [m <sup>2</sup> s <sup>-1</sup> ]		$[=\mu_0 M_{\rm f}^2 V_{\rm f} (24k_{\rm B}T)^{-1}], [-]$
С	– specific heat at constant pressure, [Jkg <sup>-1</sup> K <sup>-1</sup> ]	Е	<ul> <li>magnetic phase volume fraction, [-]</li> </ul>
g	<ul> <li>normal gravity acceleration, [ms<sup>-2</sup>]</li> </ul>	η	<ul> <li>dynamic viscosity, [kgm<sup>-1</sup>s<sup>-1</sup>]</li> </ul>
ga	- apparent vertical gravity (= $g + gm_{\nu}$ ), [ms <sup>-2</sup> ]	Θ	<ul> <li>dimensionless temperature, [-]</li> </ul>
$gm_x$	- apparent horizontal magnetic gravity, [ms <sup>-2</sup> ]	λ	- thermal conductivity, [Wm <sup>-1</sup> K <sup>-1</sup> ]
$gm_v$	- apparent vertical magnetic gravity, [ms <sup>-2</sup> ]	$\mu_0$	- permeability of vacuum,
$H^{\prime}$	- magnetic field strength, [Am <sup>-1</sup> ]		$(= 1.257 \cdot 10^{-6}), [NA^{-2}]$
h	- local heat transfer coefficient, [Wm <sup>-2</sup> K <sup>-1</sup> ]	V	- kinematic viscosity, $[m^2s^{-1}]$
Κ	– pyromagnetic coefficient, [Am <sup>-1</sup> K <sup>-1</sup> ]	ξ	- Langevin argument, $[= \mu_0 m_p H(k_B T)^{-1}], [-]$
$K_{a}$	<ul> <li>magnetocrystalline anisotropy</li> </ul>	ρ	- density, [kgm <sup>-3</sup> ]
u	constant, [Jm <sup>-3</sup> ]	φ	- orientation angle of magnetic gradient, [°]
$k_{\rm B}$	$-$ Boltzman constant, (= 1.3807 $\cdot$ 10 <sup>-23</sup> ), [JK <sup>-1</sup> ]	X	<ul> <li>magnetic susceptibility, [-]</li> </ul>
Ň	- ferrofluid magnetization, [Am <sup>-1</sup> ]	ω	- heat transfer optimization parameter, [-]
$M_{\rm f}$	<ul> <li>solid saturation spontaneous</li> </ul>	C 1	
	magnetization, $[Am^{-1}]$	SUD	scripts and addreviations
$m_{\rm p}$	- nanoparticules magnetic moment, [Am <sup>-2</sup> ]	С	– Curie
$N^{r}$	- coupling number (= Rm/Rg = $\mu_0 K \nabla H / \rho \beta_0 g$ ), [-]	e	– equilibrium
Nu	- local Nusselt number $(=hW/\lambda)$ , [-]	f	- ferromagnetic
$n_{\rm p}$	– number of nanoparticles per unit	max	– maximum
1	volume, [m <sup>-3</sup> ]	р	– particle
Pr	- Prandtl number (= $\eta C/\lambda$ ), [-]	opt	– optimum
Rg	- gravitational Rayleigh number	,	1
0	$(=\beta_{a} g \nabla T W^{4} / va), [-]$	Acro	onyms
Rm	– magnetic Rayleigh number	TGC	C(F) – thermogravity convection (force)
	$(=\mu_0 K \nabla H \nabla T W^4/na), [-]$	TM	C(F) – thermomagnetic convection (force)
$V_{\mathfrak{s}}$	- particle volume ferromagnetic portion. [m <sup>3</sup> ]	TGF	- thermogravitational force
V	– particle hydrodynamic volume, [m <sup>3</sup> ]	TMI	F – thermomagnetic force
P	, , ,	FF	– ferrofluid
Gree	er symbols	FHD	) – ferrohvdrodvnamic
β.	– thermal expansion coefficient, $[K^{-1}]$	MV	E – magnetoviscous effect

- thermal expansion coefficient, [K<sup>-1</sup>] ß
- $\beta_{\rm m}$ - magnetic moment thermal coefficient, [K<sup>-1</sup>]

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