

## AN INSPECTION TO THE HYPERBOLIC HEAT CONDUCTION PROBLEM IN PROCESSED MEAT

by

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*This paper analyzes a hyperbolic heat conduction problem in processed meat with the non-homogenous initial temperature. This problem is related to an experimental study for the exploration of thermal wave behavior in biological tissue. Because the fundamental solution of the hyperbolic heat conduction model is difficult to be obtained, a modified numerical scheme is extended to solve the problem. The present results deviate from that in the literature and depict that the reliability of the experimentally measured properties presented in the literature is doubtful.*

Key words: bioheat transfer; thermal wave, Laplace transform

### Introduction

The classical Fourier's law, implying an infinitely fast propagation of thermal signal, is always employed to study the thermal behavior for the majority of practical applications. From the viewpoint of heat transfer, the velocity of heat transfer should be limited in non-homogenous materials such biological tissues. Thus a modified flux model for the transfer processes with a finite speed wave was suggested [1-5]. Since the relevant researchers [6-10] paid attention on the finite velocity of heat propagation, the non-Fourier modes of bioheat transfer were developed [11-13]. The experimental study [9] inspired the investigation of thermal wave behavior in biological tissues and has become the important experimental support for the theoretical studies of bioheat transfer.

In order to observe the thermal wave behavior of heat transfer in processed meat and to evidence that the hyperbolic heat conduction model can accurately represent the heat conduction process in such biological tissue, Mitra *et al.* [9] brought the samples at different temperatures in contact with each other, and heat transfer was created within the samples. The thermal characteristic time of processed meat was determined by measuring the thermal diffusivity and the penetration time and is of the order of 15 seconds. This thermal characteristic time has been widely used in the relevant studies. To show the reliability of the experimentally measured properties, Mitra *et al.* [9] calculated the analytical results with those property values and made a comparison with the measurement temperature data.

The fundamental solution of the hyperbolic heat conduction model is difficult to be obtained and the major difficulty encountered in the solution of the hyperbolic heat conduction

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problems is in the sharp discontinuities [14, 15]. To inspect the results in the literature [9], this paper analyzes the hyperbolic heat conduction problem relevant to the experiment III [9]. A modified numerical scheme based on the Laplace transform is used to overcome such mathematical difficulties and analyze the present problem. The efficiency of the present numerical scheme for solving such problems has been evidenced [16, 17]. The comparison of the present results with the results in the literature [9] is made, and the deviation between them is found and discussed.

### Problem description

The relationship between the heat flux vector and the thermal disturbance in the thermal wave model is described [1-5]:

$$\bar{q} + \tau \frac{\partial \bar{q}}{\partial t} = -k \nabla T \quad (1)$$

where  $\tau$  is the relaxation time and can be approximated by  $\tau = \alpha/V^2$ . The  $k$ ,  $t$ ,  $\alpha$ , and  $V$  denote thermal conductivity, time, thermal diffusivity, and heat propagation velocity, respectively. The time derivative term of heat flux in eq. (1) mathematically described the effect of the relaxation time (or the finite propagation effect).

To consider the finite heat propagation effect, the thermal wave model of bioheat transfer (TWMBT) is proposed:

$$\begin{aligned} \nabla \cdot (k \nabla T) + w_b c_b (T_b - T) + q_m + q_r + \tau \left( -w_b c_b \frac{\partial T}{\partial t} + \frac{\partial q_m}{\partial t} + \frac{\partial q_r}{\partial t} \right) = \\ = \rho c \left( \tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} \right) \end{aligned} \quad (2)$$

where  $\rho$ ,  $c$ , and  $T$  denote density, specific heat, and temperature of tissue, respectively. The  $c_b$  is the specific heat,  $w_b$  – the perfusion rate of blood,  $q_m$  – the metabolic heat generation,  $q_r$  – the heat source for spatial heating, and  $T_b$  – the arterial temperature regarded as a constant. Equation (2) is a hyperbolic equation and is more mathematically complex than the traditional Pennes' bioheat equation. As  $\tau = 0$ , eq. (2) become the Pennes' bioheat equation.

### The case relevant to the experiment III [9]

In order show the thermal wave superposition, Mitra *et al.* [9] connected three processed meat samples at the different temperatures together for their experiment III, as shown in fig. 1. The temperatures of those three samples are  $T_{ir} = 24.1$  °C,  $T_{im} = 14.3$  °C, and  $T_{ic} = 6.2$  °C. The temperature variation at the location where is at a distance of 3.2 mm and 7.2 mm from the interfaces  $x = x_{im}$  and  $x = x_{mc}$ , respectively, was measured. The circumference of the samples was well insulated, and the problem has become 1-D hyperbolic heat conduction problem for  $w_b = 0$ ,  $q_m = 0$ , and  $q_r = 0$ . Therefore, eq. (2) is rewritten:

$$\alpha \frac{\partial^2 T}{\partial x^2} = \tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} \quad (3)$$

The studied problem has the initial conditions:

$$T(x, 0) = T_i \quad \text{and} \quad \frac{\partial T(x, 0)}{\partial t} = 0 \quad (4)$$

where  $T_i$  is  $T_{ir}$  for the room sample, is  $T_{im}$  for the medium sample, and is  $T_{ic}$  for the cold sample.

The boundary conditions at the interfaces of two adjacent layers are obtained from the assumption that temperature and heat flux are continuous. The boundary conditions at the interfaces can be mathematically described:

$$T_r = T_m \quad \text{and} \quad \frac{dT_r}{dx} = \frac{dT_m}{dx} \quad \text{for} \quad x = x_{rm} \quad (5)$$

$$T_m = T_c \quad \text{and} \quad \frac{dT_m}{dx} = \frac{dT_c}{dx} \quad \text{for} \quad x = x_{mc} \quad (6)$$

#### A modified case

For convenience of comparison and the purpose of the paper, it is done to eliminate the thermal wave subtraction and strengthen the thermal response within the medium sample. Therefore, the previous case is modified with removing the cold sample from fig. 1, and it becomes the second case analyzed in this paper. The surface of the medium sample originally contacting with the cold sample is insulated, as shown in fig. 2, and the boundary condition is:

$$q(0,t) = 0 \quad \text{or} \quad \frac{dT_m(0,t)}{dx} = 0 \quad (7)$$

#### Analysis method

First, the Laplace transform method is employed to map the transient problem into steady one. Equation (3) can be written in the transform domain for the initial conditions equation (4):

$$\frac{d^2 \tilde{T}}{dx^2} - \lambda^2 \tilde{T} = -f \quad (8)$$

where

$$\lambda^2 = \frac{\tau s^2 + s}{\alpha} \quad (9)$$

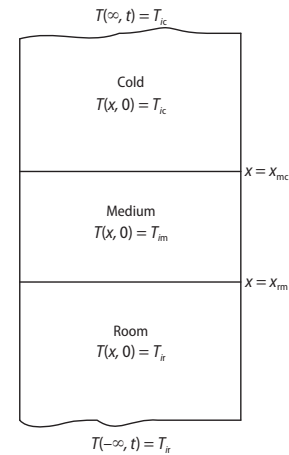
$$f = \frac{\tau s + 1}{\alpha} T_i \quad (10)$$

The function  $T$  is written as  $\tilde{T}$  in the Laplace domain and  $s$  is the Laplace transform parameter with respect to  $t$ .

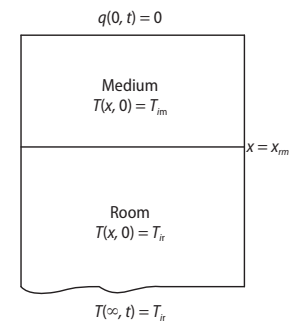
The present problem is numerically analyzed with a modified discretization scheme. For the details of the present numerical scheme, please refer [16].

#### Results and discussion

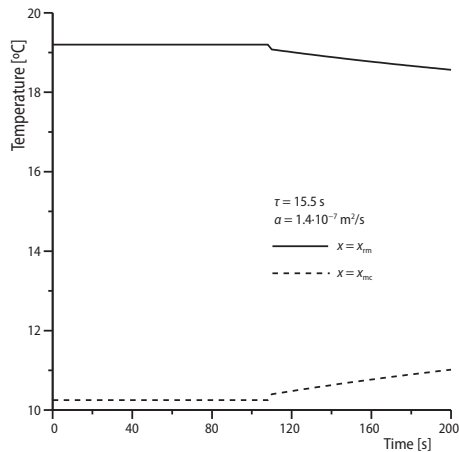
Mitra *et al.* [9] ascertained experimentally the thermophysics parameters and had thermal diffusivity  $\alpha = 1.4 \cdot 10^{-7} \pm 0.12 \cdot 10^{-7} \text{ m}^2/\text{s}$  and relaxation time  $\tau = 15.5 \pm 2.1$  seconds. For convenience of comparison and discussion, these values are used in the present paper.



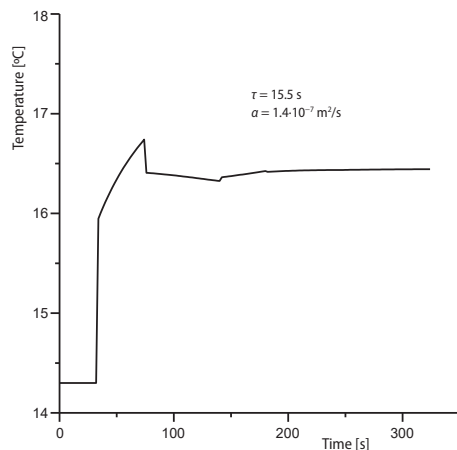
**Figure 1. Schematic diagram relevant to experiment III in the literature [9]**



**Figure 2. Modified schematic diagram without the cold sample**



**Figure 3. Variation of temperature at the interfaces**



**Figure 4. Temperature variation at the measurement location with  $\alpha = 1.4 \cdot 10^{-7} \text{ m}^2/\text{s}$  and  $\tau = 15.5$  seconds**

Mitra *et al.* [9] concludes that at the instant of contact, the temperatures at the interfaces  $x = x_{rm}$  and  $x = x_{mc}$  will stay at the intermediate temperatures  $(T_{ir} + T_{im})/2$  and  $(T_{im} + T_{ic})/2$ , respectively, until the wave generated by the other interface reaches it. Figure 3 presents the temperatures at the interfaces  $x = x_{rm}$  and  $x = x_{mc}$  stay constant at 19.2 °C and 10.25 °C, respectively, until they are disturbed by the reflected thermal waves. It depicts the efficiency of the present numerical scheme for solving such a problem.

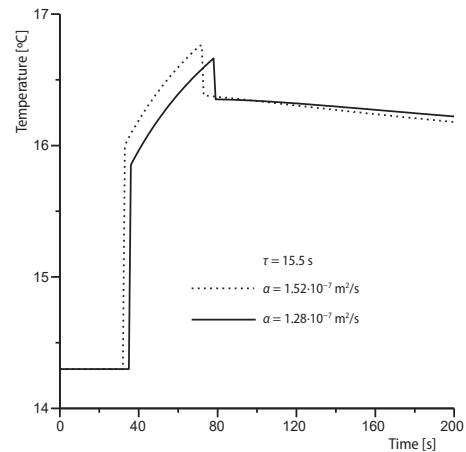
Figure 4 presents the temperature variation at the measurement location with  $\alpha = 1.4 \cdot 10^{-7} \text{ m}^2/\text{s}$  and  $\tau = 15.5$  seconds. As  $t = 34$  seconds, the thermal wave front has reached the measurement location and created a temperature jump up to about 15.9 °C. This result is the same as depicted in fig. 4 of [9]. However, the peak value of the present results is just over 16.5 °C at  $t = 76$  seconds and obviously deviates from that is up to 17.4 °C at  $t = 76$  seconds. As presented in [16], the thermal wave would induce the reflection-and-transmission phenomenon at the interfaces. Therefore, at the measurement location, the additional temperature jumps are created by the reflected thermal wave front at  $t \approx 150$  and  $t \approx 180$  seconds, but they are not observed in fig. 4 of [9]. The previous results motivate the authors to do the further exploration.

First, the effect of the experimentally measured properties is considered. Table 1 in [9] shows the uncertainty  $\pm 0.12 \cdot 10^{-7} \text{ m}^2/\text{s}$  for the thermal diffusivity and  $\pm 2.1$  seconds for the relaxation time. In order to investigate the uncertainty effect of thermal diffusivity on the analytical solution, fig. 5, presents the

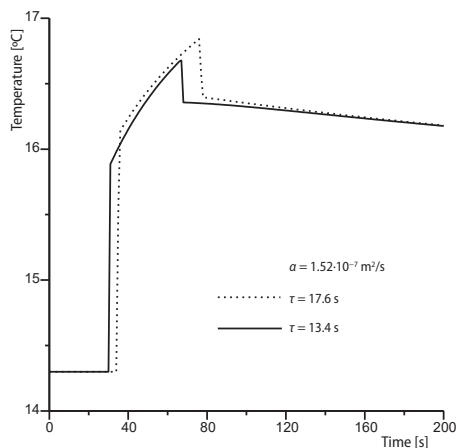
calculated results of temperature variation at the measurement location with  $\tau = 15.5$  seconds for  $\alpha = 1.52 \cdot 10^{-7} \text{ m}^2/\text{s}$  and  $\alpha = 1.28 \cdot 10^{-7} \text{ m}^2/\text{s}$ . It is found that the peak value of temperature proportionally depends on the thermal diffusivity. In other words, the peak temperature is higher with  $\alpha = 1.52 \cdot 10^{-7} \text{ m}^2/\text{s}$ . However, it still does not over 17.0 °C.

It is possible that the uncertainty effect of the relaxation time will affect the analytical results. Therefore, the paper calculates the temperature variation at the measurement location with  $\alpha = 1.52 \cdot 10^{-7} \text{ m}^2/\text{s}$  for  $\tau = 17.6$  seconds and  $\tau = 13.4$  seconds and presents in fig. 6. Figure 6 depicts that the peak value of temperature is also proportional to the value of relaxation time. It is known from the previous results that the maximum peak value of the case relevant to the experiment should appear at  $\alpha = 1.52 \cdot 10^{-7} \text{ m}^2/\text{s}$  and  $\tau = 17.6$  seconds. However, it is very clear that the value with  $\alpha = 1.52 \cdot 10^{-7} \text{ m}^2/\text{s}$  and  $\tau = 17.6$  seconds is less than 17.0 °C. It does not meet the result, reaching 17.4 °C at  $t = 76$  seconds, and implies that the present problem is worthy to be inspected.

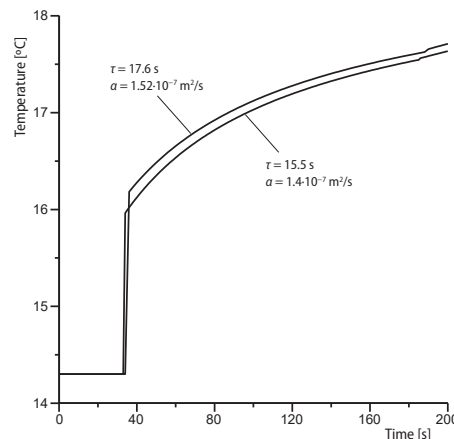
For attempting to raise the temperature at the measurement location, the cold sample in fig. 1 is removed and the surface of the medium sample originally contacting with the cold sample is insulated, as shown in fig. 2. As a result, heat energy from the room sample would completely be reserved in the medium sample. This paper calculates the temperature variation at the measurement location with  $\alpha = 1.4 \cdot 10^{-7} \text{ m}^2/\text{s}$  and  $\tau = 15.5$  seconds and  $\alpha = 1.52 \cdot 10^{-7} \text{ m}^2/\text{s}$  and  $\tau = 17.6$  seconds. The results are presented in fig. 7. It is seen that the temperature at the measurement location can not reach  $17.4 \text{ }^\circ\text{C}$  at  $t = 76$  seconds even without the effect of wave subtraction. Figure 7 also shows that time has been over 150 seconds as the temperature is up to  $17.4 \text{ }^\circ\text{C}$ . Such result further implies that the conclusions in [9] are needed to be explored and discussed in advance.



**Figure 5.** Effect of thermal diffusivity on the temperature variation at the measurement location with  $\tau = 15.5$  seconds



**Figure 6.** Effect of relaxation time on the temperature variation at the measurement location with  $\alpha = 1.52 \cdot 10^{-7} \text{ m}^2/\text{s}$



**Figure 7.** Temperature variations at the measurement location without the cold sample

## Conclusion

The hyperbolic heat conduction problem in processed meat relevant to the experiment study has been solved with a modified numerical scheme, which is an efficient method for solving such problems. The present results are calculated in accordance with the experimentally measured properties presented in the literature, thermal diffusivity  $\alpha = 1.4 \cdot 10^{-7} \pm 0.12 \cdot 10^{-7} \text{ m}^2/\text{s}$  and relaxation time  $\tau = 15.5 \pm 2.1$  seconds. It is obvious that the analytical results can not have the peak temperature  $17.4 \text{ }^\circ\text{C}$  at  $t = 76$  seconds under the circumstances. In other words, the experimentally measured properties presented in the literature can not offer the theoretical solution, which agrees with the experimental data, based on the thermal model. It implies that the reliability of the experimentally measured properties in the literature is doubtful or the model for heat transfer in biological tissue should be further explored.

## Acknowledgment

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## Nomenclature

$c$	– specific heat of tissue, [ $\text{Jkg}^{-1}\text{K}^{-1}$ ]
$c_b$	– specific heat of blood, [ $\text{Jkg}^{-1}\text{K}^{-1}$ ]
$f$	– parameter defined in eq. (10)
$k$	– thermal conductivity, [ $\text{Wm}^{-1}\text{K}^{-1}$ ]
$q_m$	– metabolic heat generation, [ $\text{Wm}^{-3}$ ]
$q_r$	– spatial heating source, [ $\text{Wm}^{-3}$ ]
$s$	– Laplace transform parameter, [–]
$t$	– time, [s]
$T$	– temperature of tissue, [ $^{\circ}\text{C}$ ]
$T_i$	– initial temperature of tissue, [ $^{\circ}\text{C}$ ]
$V$	– heat propagation velocity
$w_b$	– perfusion rate of blood, [ $\text{m}^3\text{s}^{-1}\text{m}^{-3}$ ]

## Greek symbols

$\alpha$	– thermal diffusivity, [ $\text{m}^2\text{s}^{-1}$ ]
$\lambda$	– parameter defined in eq. (9)
$\rho$	– density, [ $\text{kgm}^{-3}$ ]
$\tau$	– relaxation time, [s]

## Subscripts

c	– cool sample
m	– medium sample
r	– room sample

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