ENTROPY GENERATION DUE TO EXTERNAL FLUID FLOW AND HEAT TRANSFER FROM A CYLINDER BETWEEN PARALLEL PLANES

by

Omar A. MELHEM, Ahmet Z. SAHIN*, and Bekir S. YILBAS
Mechanical Engineering Department, King Fahd University of Petroleum and Minerals,
Dhahran, Saudi Arabia

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In the present study, second law analysis is introduced for circular cylinder confined between parallel planes. An analytical approach is adopted to study the effects of blockage, Reynolds and Prandtl numbers on the entropy generation due to the laminar flow and heat transfer. Four different fluids are considered in the present analysis for comparison purposes. Heat transfer for the cylinder at an isothermal boundary condition is incorporated. In general, the entropy generation rate decreases as the blockage ratio decreases. In addition, the entropy generation rate increases with increasing Reynolds and Prandtl numbers. At a fixed Reynolds number, the effect of blockage becomes more noticeable for higher Prandtl number fluid. Similarly, for the same fluid, the effect of blockage becomes more noticeable as the Reynolds number increases.

Key words: *heat, cylinder, parallel planes, entropy*

Introduction

A cylinder placed in flows restricted by walls is a common configuration for many applications such as cross-flow heat exchangers, shrouded heat sinks, and electric heating elements in boilers. Such flows are accompanied by entropy generation due to fluid friction and heat transfer thus limiting the efficiency of the system. Therefore, it becomes extremely important to examine the effects of various parameters on entropy generation in such flow systems in order to reduce irreversibility and suggest more efficient system design. Poulikakos and Johnson [1] studied entropy generation for combined convective heat and mass transfer in external flows from a flat plate and from a cylinder in cross-flow. They obtained a general expression that accounts for irreversibility due to the presence of heat transfer across a finite temperature difference, mass transfer across a finite difference in the chemical potential of a species, and flow friction. They showed that the entropy-generation expression can be minimized and be used in defining the optimum design for the configurations of interest. Fowler and Bejan [2] showed how to correlate the optimal sizes of bodies with specified external forced convection heat transfer, when the objective is to minimize the total rate of entropy generation for a cylinder in cross-flow and for other configurations. They found that the optimal size increased monotonically with the heat transfer duty parameter. Abu-Hijleh [3] numerically evaluated the entropy generation rate due to laminar mixed heat convection from an isothermal heated cylin-

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*Corresponding author, e-mail: azsahin@kfupm.edu.sa

der in an air cross-flow. The parameters considered were the Reynolds number, buoyancy parameter, and cylinder diameter. The author studied overall entropy generation rate as a function of cylinder diameter and buoyancy parameter at selective values of Reynolds number and found that large cylinder diameters resulted in less entropy generation rate. Demirel and Kahraman [4] analyzed the convective heat transfer in an annular packed bed by applying the first and second laws of thermodynamics. They found that there existed equipartition of the entropy generation in the packed annulus along the space compared with that of the empty annulus. Thus the configuration leading to less dissipation and lost energy could be recommended, producing thermodynamically optimum design with minimum lost energy. Sadeghipour and Razi [5] studied laminar natural convection from an isothermal horizontal cylinder confined between vertical walls. They found that there is an optimum distance between the walls for which heat transfer is maximum. Naphon [6] presented theoretical and experimental results of the second law analysis on the heat transfer and flow of a horizontal concentric tube heat exchanger. The author discussed the effects of the inlet conditions of both working fluids flowing through the heat exchanger on the heat transfer characteristics, entropy generation, and exergy loss. He showed that the hot water and the cold water mass flow rates had significant effect on the enhancement of entropy generation. He also indicated that for a given hot water mass flow rate at constant inlet cold water temperature, the entropy generation increased relatively steadily with increasing inlet hot water temperature. Taufiq *et al*. [7] presented analytical second law analysis for the optimal geometry of fin array by forced convection. Their analysis involved the achievement of a balance between the entropy generation due to heat transfer and entropy generation due to fluid friction. They found that the increase in cross-flow fluid velocity would enhance the heat transfer rate that would reduce the heat transfer irreversibility. They also determined the optimum thickness for fin array based on entropy generation minimization. Haseli *et al*. [8] evaluated the optimum cooling water temperature during condensation of saturated water vapor within a shell and tube condenser, through minimization of exergy destruction. They found that as the upstream steam mass flow rate increased, the optimal inlet cooling water temperature and exergy efficiency decreased, whereas optimal exergy destruction increased. Their results were higher for optimum values at a higher condensation temperature compared to those at a lower condensation temperature. Ibrahim and Moawed [9] conducted an experimental investigation of the heat transfer characteristics and entropy generation of electrically heated elliptic tubes with longitudinal fins separately installed in a square tunnel. Their results revealed that the fin position on the elliptic tube had an effect on the results of heat transfer coefficient, friction factor, and irreversibility ratio. They also found that the irreversibility ratio of the elliptic tube without fins was greater than that with fins. Nwosu [10] employed the minimization of entropy generation method in sizing the pin fin. His results indicated that high efficiency of the optimized fin improved the heat absorption and dissipation potential of a solar air heater. Khan *et al*. [11] investigated the blockage effects on fluid flow and heat transfer from a circular cylinder confined between parallel planes. They obtained closed-form solutions for the fluid flow and heat transfer from the cylinder with blockage ratio, and Reynolds and Prandtl numbers as parameters. They showed that the blockage ratio controls the fluid flow and the heat transfer from the cylinder. The authors did not consider the blockage ratio effects on the entropy generation. Entropy generation in channels has been the subject of many recent publications due to its importance in fluid flow end heat exchanger applications [12-15].

In the present study, a circular cylinder confined between parallel planes is considered. An analytical approach is adopted to study the effects of blockage, Reynolds and Prandtl numbers on the entropy generation rate due to fluid friction and heat transfer.

Analysis

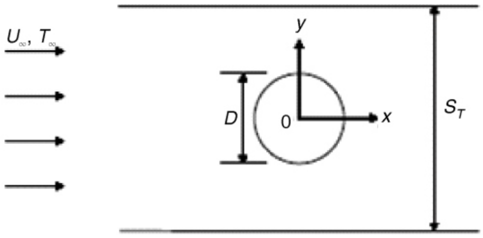

Figure 1. Physical model and co-ordinate system [11]

A cylinder placed in flows restricted by walls is a configuration that has received considerable consideration in the literature because of the wide range of applications. Such flows are accompanied by entropy generation due to fluid friction and heat transfer thus limiting the efficiency of the system. Therefore, it becomes necessary to examine the effects of various parameters on entropy generation in such flows to reduce irreversibility and suggest more efficient system design.

A uniform flow of a Newtonian fluid (Pr   0.71) past a stationary circular cylinder of diameter, D, confined between parallel planes is considered as shown in fig. 1. The approaching velocity of the fluid is U_∞ and the temperature is T_∞. The surface temperature of the cylinder wall is, T_w, for the isothermal boundary condition.

The flow is assumed to be laminar, steady, and 2-D. The results are presented for different fluids: air, water, ethylene glycol, and engine oil. The thermophysical properties used in the analysis are summarized in tab. 1. Constant properties can be assumed for the range of temperatures used in this study.

Table 1. Themophysical properties and parameters used [16]

Thermo-physical parameter	Air	Water	Ethylene glycol	Engine oil
Inlet fluid temperature [K]	300	300	300	300
Wall temperature [K]	350	350	350	350
Reference temperature [K]	325	325	325	325
Prandtl number	0.704	3.42	64.25	1585
K [Wm^{-1}K^{-1}]	28.2e-03	645e-03	259e-03	142e-03
μ [Nsm^{-2}]	196.4e-07	528e-06	6.6e-03	11.2e-02
ρ [kgm^{-3}]	1.0782	987.2	1092.8	868.8
c_p [Jkg^{-1}K^{-1}]	1008	4182	2014	2527

The main objective of this study is to investigate analytically the effects of blockage ratio, $b = D/S_\text{T}$, on the entropy generation due to external fluid flow and heat transfer from a cylinder under isothermal boundary condition. Khan *et al.* [11] performed an analytical study to investigate the effects of blockage on fluid flow and heat transfer from a circular cylinder confined between parallel planes. The authors obtained a correlation for total drag coefficient:

$$C_D \quad \frac{45.72 \quad 39.9\exp(\quad 0.95b^{3.44})}{\sqrt{\mathrm{Re}_D}} \quad 6.1 \quad 4.95\exp(\quad 0.76b^{2.63}) \quad \frac{1.49 \quad 0.23\exp(\quad 5.81b^{2.15})}{\mathrm{Re}_D} \tag{1}$$

The authors also obtained a correlation for heat transfer from the cylinder under the isothermal boundary condition:

$$\frac{\mathrm{Nu}_D}{\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}} \quad 0.843 \quad 0.25\exp(\quad 2.65b^{2.5}) \tag{2}$$

The previous correlations will be used in addition to an analytical expression of total entropy generation to perform a second law analysis for a flow across a circular cylinder confined between parallel planes. The total rate of entropy generation associated with external flow across a body at uniform temperature is given by Bejan [17]:

$$\dot{S}_{\mathrm{gen}} \quad \frac{T_{\mathrm{w}} \quad T_\infty}{T_\infty}^{2} \quad \frac{\mathrm{Nu}_D k_{\mathrm{f}}}{D} \quad A \quad \frac{1}{T_4} F_D U_\infty \tag{3}$$

where k_{f} is the thermal conductivity of the fluid, A – the surface area of the cylinder in cross-flow, and F_D is the total drag force.

The total drag force F_{D} is given by:

$$F_{\mathrm{D}} \quad A_{\mathrm{p}} C_{\mathrm{D}} \quad \frac{1}{2}\rho U_\infty^2 \tag{4}$$

where A_{p} is the projected area of the cylinder.

Defining a dimensionless entropy generation rate as:

$$\psi \quad \frac{\dot{S}_{\mathrm{gen}} D}{k_{\mathrm{f}} A}$$

eq. (3) can be written as:

$$\Psi \quad (\tau \quad 1)^2 (\mathrm{Nu}_D) \quad \frac{A_{\mathrm{p}}}{2A} \mathrm{Re}_D \,\mathrm{Pr}\, C_{\mathrm{D}} \xi \tag{5}$$

where

$$\tau \quad \frac{T_{\mathrm{w}}}{T_\infty}$$

and is fixed for given inlet fluid temperature and fixed surface temperature and

$$\xi \quad \frac{U_\infty^2}{c_p T_\infty}$$

is a parameter that indicates the significance of kinetic energy of incoming flow relative to its enthalpy. The ξ clearly dependence on Reynolds number and the fluid into consideration. Table 2 summarizes the values of ξ at different Reynolds numbers for the different fluids adopted in this study. The values tabulated are for a 1cm diameter cylinder.

Table 2. Values of ξ for the different fluids and Reynolds numbers

Dimensionless parameter ξ	Air	Water	Ethylene glycol	Engine oil
ξ for $\mathrm{Re}_D = 100$	1.10e-07	2.28e-11	6.04e-09	2.19e-06
ξ for $\mathrm{Re}_D = 1000$	1.10e-5	2.28e-09	6.04e-07	2.19e-04
ξ for $\mathrm{Re}_D = 3000$	9.88e-05	2.05e-08	5.43e-06	1.97e-03

Results and discussion

Four fluids with wide range of Prandtl number variation, tab. 1, are considered to study the effect of the blockage ration, *b*, on the entropy generation for external flow past a stationary circular cylinder between parallel planes.

Figure 2 shows the variation of the entropy generation rate with the blockage ratio *b* for two Reynolds numbers. It is to be noted that logarithmic scale is used along the y-axis for the dimensionless entropy generation rate due to the variation in Prandtl numbers thus leading to large variation in the entropy generation between the various fluids incorporated in this study. The entropy generation decreases with *b*. This is because both the drag and the Nusselt number decrease, figs. 3 and 4, as the *b* decrease, thus leading to more available work and less entropy generation. Furthermore, at a fixed Reynolds number, the effect of the blockage becomes more noticeable for the fluids with higher Prandtl numbers. This can be associated with the behavior of Nusselt number as observed in fig. 3. That is, the slope of the Nusselt number variation with *b* increases as Prandtl number increases. Nusselt number is almost constant as *b* varies for the lowest Prandtl number fluid. It is clear from the figures that at a fixed Reynolds number the entropy generation increases as the Prandtl number increases. This can be associated directly with the increase in the amount of heat transfer represented by the increase in Nusselt number as Prandtl number increases. This can be inferred from eq. (2) and is depicted clearly in fig. 3.

The drag force does not depend directly on Prandtl number. However, the drag force is associated with the fluid type through its dependence on density, ρ, and viscosity, μ, (eq. 4). It can be indirectly deduced from the aforementioned and from the presented data that for low Prandtl numbers drag controls the entropy generation and is the dominant factor leading to loss of available work. On the other hand, heat transfer controls the entropy generation for high Prandtl numbers and is the dominant factor leading to loss of available work. This can be understood if the definition of Prandtl number as the ratio of momentum diffusivity to thermal diffusivity is recalled.

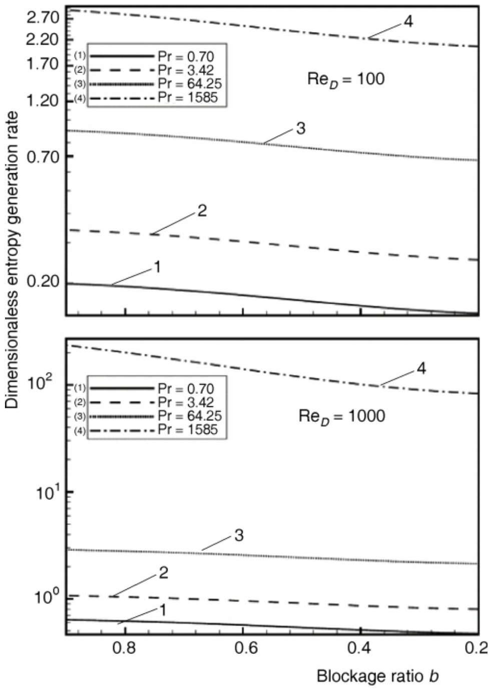

Figure 2. Effect of blockage ratio and Prandtl number on dimensionless entropy generation

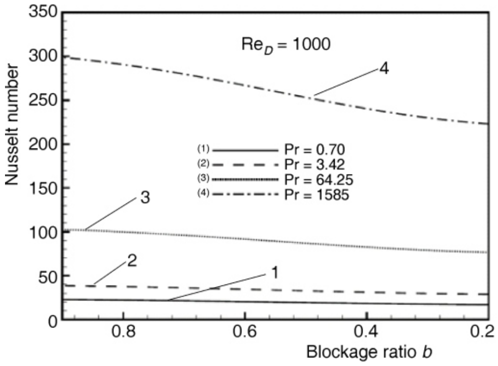

Figure 3. Effect of blockage ratio and Prandtl number on Nusselt number

Figures 5(a) and 5(b) show the variation of the entropy generation with the blockage ratio for four Prandtl numbers and three Reynolds numbers. It should be noted that a logarithmic

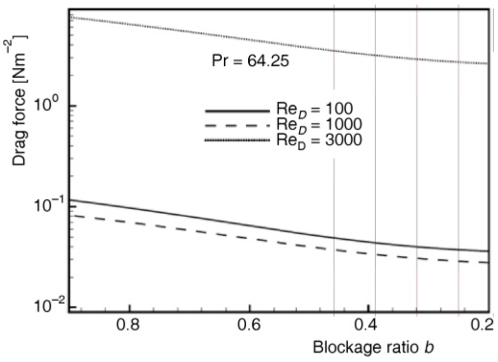

Figure 4. Effect of blockage ratio and Reynolds number on drag force

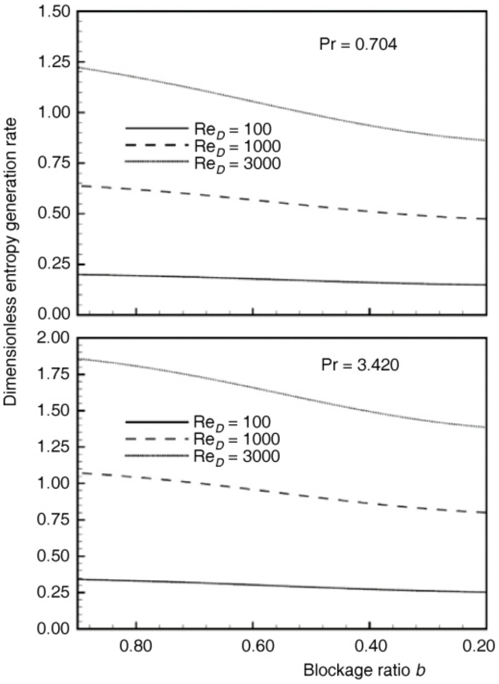

Figure 5(a). Effect of blockage ratio and Reynolds number on dimensionless entropy generation

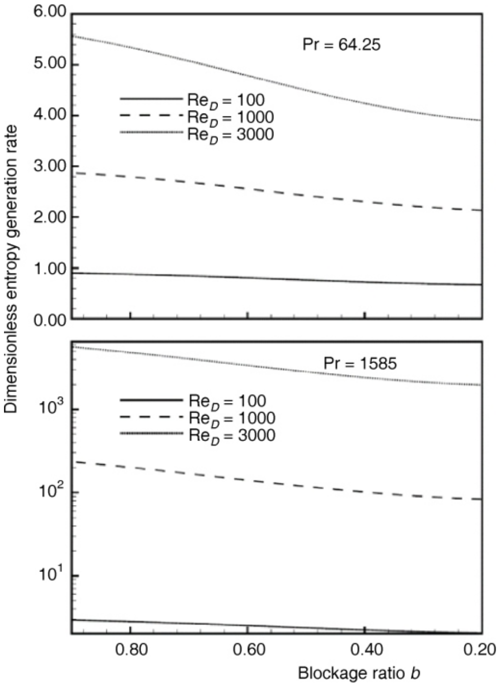

Figure 5(b). Effect of blockage ratio and Reynolds number on dimensionless entropy generation

scale is used for the dimensionless entropy generation rate in the bottom half of fig. 5(b). The entropy generation decreases with the blockage ratio. Furthermore, at a fixed Prandtl number, the effect of the blockage becomes more noticeable at higher Reynolds numbers. This has to be attributed to both the behavior of heat transfer and drag effects as Reynolds number varies. It can be shown that Nusselt number and consequently the heat transfer rate increase as Reynolds number increases [11]. For the Reynolds numbers considered in this work, the drag force decreases as the Reynolds number increases from 100 to 1000 as depicted in fig. 4. Drag force then significantly increases when Reynolds number equals 3000. This increase can be associated with turbulence effects as transition to turbulence is expected to occur at this value of Reynolds number. It is clear from figs. 5(a) and 5(b) that at a fixed Prandtl number the entropy generation increases as the Reynolds number increases. For the highest Prandtl number (Pr = = 1585) entropy generation increases with Reynolds number. However, the curves showing the variation of the dimensionless entropy generation with the blockage ratio are almost parallel.

This again can be associated with the fact that at high Prandtl numbers heat transfer is the main mechanism contributing to the loss available work because of the relative ease by which the fluid flow occurs when compared to the ease by each heat transfer across a finite temperature difference occurs.

Conclusions

Second law analysis is applied to circular cylinder confined between parallel planes. An analytical approach has been adopted to study the effects of blockage, Reynolds, and Prandtl numbers on the entropy generation due to drag caused by laminar fluid flow and heat transfer from the cylinder at an isothermal boundary condition. In general, the entropy generation decreases as the blockage ratio decreases. Moreover, the entropy generation increases with increasing Reynolds and Prandtl numbers. At a fixed Reynolds number, the effect of the blockage becomes more noticeable for the higher Prandtl number fluid. Similarly, for the same fluid the effect of the blockage becomes more noticeable as the Reynolds number increases. Entropy generation or exergy loss is directly related to ease by which fluid flow and heat transfer occur. This has lead to the conclusion that at low Prandtl numbers drag controls the entropy generation and is the dominant factor leading to loss of available work because of the increase in fluid friction associated with the low momentum diffusivity and the relative ease by which heat transfer occurs across the finite temperature difference. On the other hand, heat transfer controls the entropy generation for high Prandtl numbers and is the dominant factor leading to loss of available work because of the relative difficulty by which heat transfer occurs, which can be attributed to the relatively low thermal diffusivity.

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Nomenclature

A – surface area of the cylinder, equivalent to LD, [m^2]
A_p – projected area of the cylinder, equivalent to LD, [m^2]
b – blockage ratio, equivalent to D/S_T, [–]
c_p – specific heat of fluid, [Jkg^{-1}K^{-1}]
C_D – total drag coefficient, [–]
D – cylinder diameter, [m]
F_D – drag force, [Nm^{-2}]
k – thermal conductivity, [Wm^{-1}K^{-1}]
L – length of cylinder, [m]
Nu_D – average Nusselt number based on diameter of cylinder, [–]
Pr – Prandtl number, [–]
Re_D – Reynolds number, [–]
S_gen – entropy generation, [WK^{-1}]
S_T – vertical distance between parallel planes, [m]
T – temperature, [K]
U – velocity, [ms^{-1}]

Greek symbols

μ – absolute viscosity of fluid, [kgm^{-1}s^{-1}]
ξ – dimensionless parameter ($=U^2_\infty/c_pT_\infty$), [–]
ρ – density of fluid, [kgm^{-3}]
τ – dimensionless temperature ($=T_\mathrm{w}/T_\infty$), [–]
Ψ – dimensionless entropy generation, [–]

Subscripts

w – wall
∞ – freestream conditions

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