THE NEW ANALYTICAL STUDY FOR BOUNDARY-LAYER SLIP FLOW AND HEAT TRANSFER OF NANOFLUID OVER A STRETCHING SHEET

by

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In this article, the semi-analytical numerical technique known as the homotopy analysis method is employed to derive solutions for partial slip effects on the heat transfer of nanofluids over a stretching sheet. An accurate analytical solution is presented which depends on the Prandtl number, slip factor, Lewis number, Brownian motion number, and thermophoresis number. The variation of the reduced Nusselt and reduced Sherwood numbers with Brownian motion number, and thermophoresis number for various values Prandtl number, slip factor, Lewis number is presented in tabular and graphical forms. The results of the present article show the flow velocity and the surface shear stress on the stretching sheet and also reduced Nusselt number and Sherwood number are strongly influenced by the slip parameter. It is found that hydrodynamic boundary-layer decreases and thermal boundary-layer increases with slip parameter. Comparison of the present analysis is made with the previously existing literature and an appreciable agreement in the values is observed for the limiting case.

Key words: nanofluid, velocity slip, stretching sheet, auxiliary parameter, homotopy analysis method solution

Introduction

The enhancement in thermal conductivity of conventional fluids through suspensions of solid particles is a recent development in engineering technology aimed at increasing the coefficient of heat transfer. The thermal conductivity of solid metals is higher than the base fluids, so the suspended particles are able to increase the thermal conductivity and heat transfer performance. Choi and Eastman [1] were most likely the first researchers to combine a mixture of nanoparticles and base fluid, which they subsequently termed a nanofluid, and it was adopted by many researchers. The materials with sizes of nanometers possess sole physical and chemical properties. They can flow smoothly through micro-channels without clogging them because they are small enough to behave similarly to liquid molecules [2].

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Nanofluids offer many diverse advantages in industrial application such as microelectronics, fuel cell, nuclear reactors, biomedicine, and transportation, many of which have been reviewed by Wong and Leon [3]. Hwang et al. [4] measured the thermal conductivities of various nanofluids and verified that the volume fraction of suspended particle is the effective parameter in enhancing the thermal conductivity. Several numerical studies related to regular fluid/nanofluids have been reported in the literature [5-16].

Recently, the Cheng-Minkowycz model for natural convective boundary-layer flow in a porous medium saturated by a nanofluid is investigated by Nield and Kuznetsov [17]. For natural convective boundary-layer nanofluid flow over a vertical surface is reported by Kuznetsov and Nield [18]. Khan and Pop [19] numerically studied the laminar nanofluid transport phenomena from a stretching sheet. Bachok et al. [20] have analyzed steady boundary-layer nanofluid flow over a moving semi-infinite flat plate. Rana and Bhargava [21] used finite element and finite difference methods to study nanofluid heat and mass transfer from a non-linearly stretching sheet. Buongiorno [22] investigated the steady buoyancy-driven dissipative magnetoconvective flow from a vertical non-linear stretching sheet. Abel et al. [23] have produced the exact solutions at various values of the physical parameters for the model studied by Khan and Pop [19]. In all previous studies, the researchers ignored the slip effects at the wall. Majumder et al. [24] have shown that the nanofluidic flow generally exhibits partial slip against the solid surface, which can be characterized by slip-length (around 3.4-68 mm for different liquids). Wang [25] reported that the partial slip between the fluid and the moving stretching surface, Sahoo and Do [26] examined the slip effects on third grade fluid over stretching sheet. Noghrehabadi et al. [27] investigated the effect of partial slip boundary condition on nanofluid flow and heat transfer over a stretching sheet. While Aly et al. [28] utilized Chebyshev pseudo-spectral differentiation matrix to obtain the exact solution for the nanofluid flow model. Rashidi et al [29] and Rashidi and Erfani [30] investigated slip condition of MHD flow in different physical situations. Taking into account the significance of the partial slip previously mentioned, in the present work we derive semi-analytical solutions for boundary-layer flow and heat transfer of nanofluid over stretching sheet with constant wall temperature using homotopy analysis method (HAM) which was proposed by Liao [31]. Numerous researchers have successfully applied HAM on many physical problems in science and engineering [32-41].

The considered problem studied first by Noghrehabadi et al. [27] and exerted the similarity solution.

Flow analysis and mathematical formulation

Consider a 2-D, steady and incompressible viscous flow of a nanofluid past over a stretching surface. The velocity of the surface is considered to be linear, i.e. $U_w(x) = cx$. Here, $c$ is constant and $x$ is co-ordinate measured along the surface. The nanofluid flows at $y = 0$, where $y$ is the co-ordinate normal to the surface. We assumed that the wall temperature, $T_w$, and the nanoparticle fraction, $\varphi_w$, are constant at stretching surface. When $y$ tends to infinity, the ambient values of temperature and nanoparticle fraction are denoted by $T_\infty$ and $\varphi_\infty$, respectively. The co-ordinate system and the flow model are shown in fig. 1. For nanofluids, in Cartesian co-ordinates system of $x$ and $y$, the governing steady conservation of momentum, thermal energy, and nanoparticles equations including the dynamic effects of nanoparticles can be written [19]:

![Figure 1. Schematic diagram of the problem](image-url)
The boundary conditions for the velocity components, temperature, and nanoparticle fraction are defined:

\[ u = U_s - U, \quad v = 0, \quad T = T_s, \quad \phi = \phi_s \quad \text{for} \quad y = 0 \]  
\[ u = 0, \quad v = 0, \quad T = T_s, \quad \phi = \phi_s \quad \text{for} \quad y = \infty \]  

Here, \( u \) and \( v \) are the velocity components along the axes \( x \) and \( y \), respectively, \( p \) – the fluid pressure, \( \alpha \) – the thermal diffusivity, \( \nu \) – the kinematic viscosity, \( \rho_r \) – the density of the base fluid, \( U_s \) – the velocity slip at the wall, \( D_B \) – the Brownian diffusion coefficient, and \( D_T \) – the thermophoresis diffusion coefficient. The \( \tau = (\rho c)/(\rho c_f) \) is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid with \( \rho \) being the density, \( c \) – the volumetric volume coefficient, \( \rho_p \) – the density of the particles, and \( \phi \) – rescaled nanoparticle volume fraction.

Noghrehabadi et al. [27] introduced the following transformations:

\[ \psi = \sqrt{c \nu x f}(\eta), \quad \eta = \frac{c}{\nu}, \quad \theta(\eta) = \frac{T - T_s}{T_w - T_s}, \quad \phi(\eta) = \frac{\phi - \phi_s}{\phi_e - \phi_s} \]  

where \( \psi \) represent the stream function and is defined as \( u = \partial \psi / \partial y \), \( v = - \partial \psi / \partial x \), so that eq. (1) satisfied identically. We have taken into account that the pressure in the outer (inviscid) flow is \( p = p_0 \) (constant). Hence, the flow occurs only due to the stretching of the sheet, therefore, the pressure gradient can be neglected. The governing equations are reduced by using eq. (8) as:

\[ f'' + f f' - f'^2 = 0 \]  
\[ \frac{1}{Pr} \theta' + f \theta' + Nb \phi' \theta' + Nt \theta'^2 = 0 \]  
\[ \phi'' + \frac{Nt}{Nb} \theta' + Le f \phi' = 0 \]
Here, by using boundary-layer approximations and introducing Navier’s condition the velocity on the surface can be written:

\[ u - U_w = N \rho v \frac{\partial u}{\partial y} = U \]  

(12)

where \( \rho \) is the density and \( N \) is the slip constant. Using similarity transformations eq. (12) reduces to:

\[ f'(0) - 1 = \lambda f^*(0) \]  

(13)

where \( \lambda = N \rho \sqrt{v} \) is the dimensionless slip factor. Equations (9)-(11) can solved analytically subject to the transformed boundary conditions:

\[ f(0) = 0, \quad f'(0) = 1 + \lambda f^*(0), \quad f'(\infty) = 0, \]
\[ \theta(0) = 1, \quad \theta(\infty) = 0, \quad \phi(0) = 1, \quad \phi(\infty) = 0 \]  

(14)

where primes denote differentiation with respect to \( \eta \) and the four parameters are defined:

\[ \Pr = \frac{\nu}{\alpha}, \quad \Le = \frac{\nu}{D_b}, \quad \Nb = \frac{(\rho c_p)_b D_b (\phi_w - \phi_0)}{(\rho c)_w v}, \quad \Nt = \frac{(\rho c)_w D_b (T_w - T_0)}{(\rho c)_w T_\infty} \]  

(15)

Here, \( \Pr, \Le, \Nb \) and \( \Nt \) denote the Prandtl number, Lewis number, Brownian motion parameter, and thermophoresis parameter, respectively. It is important to note that this boundary value problem reduces to the classical problem of flow and heat and mass transfer due to a stretching surface in a viscous fluid when \( \Nb \) and \( \Nt \) are zero in eqs. (10) and (11). (The boundary value problem for \( \phi \) then becomes ill-posed and is of no physical significance).

In the continuum modeling of fluidic transport, nanofluid usually exhibits partial slip against the solid surface, which can be characterized by the so-called slip length (around 3.4 to 68 mm for different liquids). Most nanofluids examined to date have large values for the Lewis number, \( \Le > 1 \) [24]. For water nanofluids at room temperature with nanoparticles of 1 to 100 nm diameters, the Brownian diffusion coefficient, \( D_b \), ranges from \( 4 \times 10^{-4} \) to \( 4 \times 10^{-12} \) m²/s [24]. Furthermore, the ratio of Brownian diffusivity coefficient to thermophoresis coefficient for particles with diameters of 1 to 100 nm can be varied in the ranges of 2 to 0.02 for Al, and from 2 to 20 for Cu nanoparticles. Rana and Bhargava [21] studied \( \Nb \) and \( \Nt \) in the range of 0.1 to 0.5 and Lewis number in the range of 1 to 25 for the nanofluid boundary-layer over the stretching sheets. Hence, the variation of dimensionless parameters of nanofluids in the present study is considered to vary in the mentioned range.

The quantities of practical interest, in this study, are the Nusselt number and the Sherwood number which are defined:

\[ \Nu = \frac{-x q_w}{k (T_w - T_0)}, \quad \Sh = \frac{x q_w}{D_b (\phi_w - \phi_0)} \]  

(16)

where \( q_w \) and \( q_w \) are the wall heat and mass fluxes, respectively. Using eq. (8), we obtain:

\[ \Re^{1/2} \Nu = -\theta'(0), \quad \Re^{1/2} \Sh = -\phi'(0) \]  

(17)

where \( \Re = U_s(x)/v \) is the local Reynolds number based on the stretching velocity \( U_s(x) \). It is interesting to note that Wang [25] obtained an exact solution for eq. (9) subject to the boundary conditions (14).
The HAM solution

The dimensionless velocity $f(\eta)$, temperature $\theta(\eta)$, and concentration $\phi(\eta)$ profiles can be expressed by the set of base functions:

$$\{\eta^k \exp(-n\eta) | k \geq 0, n \geq 0\}$$ (18)

in the form of following series:

$$f(\eta) = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} a_{n,k} \eta^k \exp(-n\eta),$$

$$\theta(\eta) = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} b_{n,k} \eta^k \exp(-n\eta),$$

$$\phi(\eta) = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} c_{n,k} \eta^k \exp(-n\eta)$$ (19)

where $a_{n,k}$, $b_{n,k}$, and $c_{n,k}$ are the coefficients. We choose the initial guesses $f_0(\eta)$, $\theta_0(\eta)$, and $\phi_0(\eta)$ based on boundary condition (14) and the Liao [31] rule of solution expression:

$$f_0(\eta) = \frac{1}{1 + \lambda} (1 - e^{-\eta}), \quad \theta_0(\eta) = e^{-\eta}, \quad \phi_0(\eta) = e^{-\eta}$$ (20)

The linear operators $L_1$, $L_2$, and $L_3$ in the following way:

$$L_1(f) = \frac{d^3 f}{d\eta^3} + \frac{d^2 f}{d\eta^2}, \quad L_2(\theta) = \frac{d^3 \theta}{d\eta^3} + \frac{d\theta}{d\eta}, \quad L_3(\phi) = \frac{d^3 \phi}{d\eta^3} + \frac{d\phi}{d\eta}$$ (21)

The operators $L_1$, $L_2$, and $L_3$ have the following properties:

$$L_1(C_1 + C_2 \eta + C_3 e^{-\eta}) = 0, \quad L_2(C_4 + C_5 \eta + C_6 e^{-\eta}) = 0, \quad L_3(C_7 + C_8 e^{-\eta}) = 0$$ (22)

where $C_i$ ($i = 1-5$) are arbitrary constants. Let $q \in [0,1]$ represents an embedding parameter and $h \neq 0$ be the auxiliary parameter to adjust the convergence rate of the perturbation series. We construct the following 0th order deformation of the problem:

$$(1-q)L_1 \left[ \hat{f}(\eta;q) - f_0(\eta) \right] = qh_1 N_1 \left[ \hat{f}(\eta;q) \right]$$ (23)

$$(1-q)L_2 \left[ \hat{\theta}(\eta;q) - \theta_0(\eta) \right] = qh_2 N_2 \left[ \hat{f}(\eta;q), \hat{\theta}(\eta;q) \right]$$ (24)

$$(1-q)L_3 \left[ \hat{\phi}(\eta;q) - \phi_0(\eta) \right] = qh_3 N_3 \left[ \hat{f}(\eta;q), \hat{\phi}(\eta;q) \right]$$ (25)

subject to the conditions:

$$\hat{f}(0;q) = 0, \quad \hat{f}'(0;q) = 1 + \lambda \hat{f}'(0;q), \quad \hat{f}'(\infty;q) = 0$$ (26)

$$\hat{\theta}(0;q) = 1, \quad \hat{\theta}(\infty;q) = 0, \quad \hat{\phi}(0;q) = 1, \quad \hat{\phi}(\infty;q) = 0$$ (27)

where the non-linear operators are defined:
\[
\frac{N_1}{\eta} = \frac{\partial^3 \tilde{f}(\eta; q)}{\partial \eta^3} + \tilde{f}(\eta; q) \frac{\partial^2 \tilde{f}(\eta; q)}{\partial \eta^2} - \left[ \frac{\partial \tilde{f}(\eta; q)}{\partial \eta} \right]^2
\]  
\[\text{(28)}\]

\[
N_2 = \frac{1}{\Pr} \frac{\partial^2 \tilde{\theta}(\eta; q)}{\partial \eta^2} + \tilde{f}(\eta; q) \frac{\partial \tilde{\theta}(\eta; q)}{\partial \eta} + Nb \frac{\partial \tilde{\phi}(\eta; q)}{\partial \eta} \frac{\partial \tilde{\theta}(\eta; q)}{\partial \eta} + Nt \frac{\partial^3 \tilde{\theta}(\eta; q)}{\partial \eta^3}
\]  
\[\text{(29)}\]

\[
N_3 = \frac{\partial^2 \tilde{\phi}(\eta; q)}{\partial \eta^2} + \frac{Nt}{Nb} \frac{\partial^2 \tilde{\theta}(\eta; q)}{\partial \eta^2} + \frac{Le}{\Pr} \tilde{f}(\eta; q) \frac{\partial \tilde{\phi}(\eta; q)}{\partial \eta}
\]  
\[\text{(30)}\]

For \(q = 0\) and \(q = 1\) we have:
\[
\tilde{f}(q; \eta; 0) = f_o(q; \eta), \quad \tilde{f}(q; \eta; 1) = f(q; \eta)
\]  
\[\text{(31)}\]

\[
\tilde{\theta}(q; \eta; 0) = \theta_o(q; \eta), \quad \tilde{\theta}(q; \eta; 1) = \theta(q; \eta)
\]  
\[\text{(32)}\]

\[
\tilde{\phi}(q; \eta; 0) = \phi_o(q; \eta), \quad \tilde{\phi}(q; \eta; 1) = \phi(q; \eta)
\]  
\[\text{(33)}\]

Defining:
\[
f_m(\eta) = \frac{1}{m!} \frac{\partial^m \tilde{f}(\eta; q)}{\eta^m} \bigg|_{q=0}, \quad \theta_m(\eta) = \frac{1}{m!} \frac{\partial^m \tilde{\theta}(\eta; q)}{\eta^m} \bigg|_{q=0}, \quad \phi_m(\eta) = \frac{1}{m!} \frac{\partial^m \tilde{\phi}(\eta; q)}{\eta^m} \bigg|_{q=0}
\]  
\[\text{(34)}\]

and expanding \(\tilde{f}(q; \eta), \tilde{\theta}(q; \eta), \text{ and } \tilde{\phi}(q; \eta)\) by means of Taylor’s theorem with respect to \(q\), we obtain:
\[
\tilde{f}(q; \eta) = f_o(\eta) + \sum_{m=1}^{\infty} f_m(\eta) q^m
\]  
\[\text{(35)}\]

\[
\tilde{\theta}(q; \eta) = \theta_o(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) q^m
\]  
\[\text{(36)}\]

\[
\tilde{\phi}(q; \eta) = \phi_o(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta) q^m
\]  
\[\text{(37)}\]

The auxiliary parameters are properly chosen so that series eqs. (35)-(37) converge at \(q = 1\) and thus:
\[
f(\eta) = f_o(\eta) + \sum_{m=1}^{\infty} f_m(\eta)
\]  
\[\text{(38)}\]

\[
\theta(\eta) = \theta_o(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta)
\]  
\[\text{(39)}\]

\[
\phi(\eta) = \phi_o(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta)
\]  
\[\text{(40)}\]

The resulting problems at the m\textsuperscript{th} order deformation are:
\[ L_1 \left[ f_m'(\eta) - \mathcal{X}_m f_{m-1}'(\eta) \right] = h_f R_1^m(\eta) \]
\[ L_2 \left[ \theta_m'(\eta) - \mathcal{X}_m \theta_{m-1}'(\eta) \right] = h_{\theta} R_2^m(\eta) \]
\[ L_3 \left[ \phi_m'(\eta) - \mathcal{X}_m \phi_{m-1}'(\eta) \right] = h_{\phi} R_3^m(\eta) \]
\[ f_m(0) = 0, \quad f'_m(0) = \lambda f''_m(0), \quad f''_m(\infty) = 0, \quad \theta_m(0) = 0, \quad \theta_m(\infty) = 0, \quad \phi_m(0) = 0, \quad \phi_m(\infty) = 0 \]

\[ R_m^f(\eta) = f''_{m-1} + \sum_{k=0}^{m-1} (f_k f''_{m-1-k} - f'_k f''_{m-1-k}) \]
\[ R_m^\theta(\eta) = \frac{1}{Pr} \theta''_{m-1} + \sum_{k=0}^{m-1} f_k \theta''_{m-1-k} + Nb \sum_{k=0}^{m-1} \phi_k' \theta''_{m-1-k} + Nt \sum_{k=0}^{m-1} \theta_k' \phi''_{m-1-k} \]
\[ R_m^\phi(\eta) = \phi''_{m-1} + \left( \frac{Nt}{Nb} \right) \phi''_{m-1} + Le \sum_{k=0}^{m-1} f_k' \phi''_{m-1-k} \]

\[ \mathcal{X}_m = \begin{cases} 0, & m \leq 1, \\ 1, & m > 1 \end{cases} \]

The general solutions of eqs. (41)-(43) can be written:

\[ f_m(\eta) = f''_m(\eta) + C_1 + C_2 \eta + C_3 e^{-\eta} \]
\[ \theta_m(\eta) = \theta''_m(\eta) + C_4 + C_5 e^{-\eta} \]
\[ \phi_m(\eta) = \phi''_m(\eta) + C_6 + C_7 e^{-\eta} \]

where \( f''_m(\eta), \theta''_m(\eta), \) and \( \phi''_m(\eta) \) are the particular solutions and the constants and should be determined by the boundary condition eq. (44).

**Convergence of homotopy solution**

Liao [31] has mentioned that the convergence rate of approximation of the HAM solution is strongly dependent upon the values of non-zero auxiliary parameter \( h \). As the eqs. (49)-(51) involve \( h, h_{\theta}, \) and \( h_{\phi} \), so we can adjust the convergence of our HAM solution. To compute the range of admissible values of \( h, h_{\theta}, \) and \( h_{\phi} \), we display the \( h \)-curves of the function \( f''(0), \theta'(0), \) and \( \phi'(0) \) for 14th orders of approximations. Figure 2 depicts that for the range of admissible values of \( h \)-curves are \(-0.85 \leq h \leq -0.3, -0.75 \leq h \leq -0.25, \) and \(-0.7 \leq h \leq -0.2). Figure 2 shows the \( h \)-curves for the dimensionless velocity, temperature, and concentration profiles. Convergence of the series solution up to 40th order of approx-
Table 1. Convergence of HAM solution for different order of approximations at $Pr = 1, Nt = Nb = 0.5, Le = 2$, $\lambda = 0.2$, and $h_\eta = h_y = -0.5$

<table>
<thead>
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<th>Order of approximations</th>
<th>$f''(0)$</th>
<th>$-\theta'(0)$</th>
<th>$-\phi'(0)$</th>
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<tr>
<td>40</td>
<td>-0.77470</td>
<td>0.32217</td>
<td>0.70004</td>
</tr>
</tbody>
</table>

The non-linear ordinary differential eqs. (9)-(11) together with boundary condition (14) are solved analytically using HAM. The HAM computations have been carried out for various range of slip boundary condition and for different values of parameters such as Prandtl number, the Lewis number, the Brownian motion parameter, and the thermophoresis parameter. The effects of the emerging parameters on dimensionless velocity, temperature, the rate of heat and mass transfer, and the nanoparticle volume fraction are investigated and presented graphically in figs. 3-10. As a test of accuracy of the HAM solution, comparison has been done between the results of HAM and previously published study [15, 22-24] for $-f''(0)$, $-\theta'(0)$, and $-\phi'(0)$ in tabs. 2-4 which confirm that the present simulation is accurate.

Figure 3 exhibits the effect of slip parameter on the velocity and shear stress. An examination of fig. 3 reveals that with the increase in velocity slip parameter, $\lambda$, the fluid velocity decreases because in the case of slip condition, the effect of the pull of the stretching sheet is

![Figure 3](image-url)

**Results and discussions**

The non-linear ordinary differential eqs. (9)-(11) together with boundary condition (14) are solved analytically using HAM. The HAM computations have been carried out for various range of slip boundary condition and for different values of parameters such as Prandtl number, the Lewis number, the Brownian motion parameter, and the thermophoresis parameter. The effects of the emerging parameters on dimensionless velocity, temperature, the rate of heat and mass transfer, and the nanoparticle volume fraction are investigated and presented graphically in figs. 3-10. As a test of accuracy of the HAM solution, comparison has been done between the results of HAM and previously published study [15, 22-24] for $-f''(0)$, $-\theta'(0)$, and $-\phi'(0)$ in tabs. 2-4 which confirm that the present simulation is accurate.

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only partially transmitted to the fluid as shown in fig. 3. It is further observe that maximum velocity occur for $\lambda = 0$. The velocity curves show that the rate of transport decreases with the increasing distance, $\eta$, normal to the sheet. In addition, it is noticed that the magnitude of shear stress decreases with the slip parameter.

Figures 4 and 5 illustrate the dimensionless temperature and concentration profiles for various values of the slip parameter. The temperature and nanoparticle concentration are clearly observed to be significantly enhanced with increasing slip parameter values. Figure 6 exhibits the effect of the Prandtl number on the dimensionless temperature. It is observed that the thermal boundary-layer significantly decrease with Prandtl number. This is because the decrease in Prandtl number leads to the increase in the thermal conductivity of the fluid so that heat is able to diffuse away from the heated surface more quickly. Hence, for the smaller values of Prandtl number the thermal boundary-layer is thicker. Figure 7 shows the effect of the Lewis number on the concentration profile for selected parameters. The concentration boundary-layer thickness reduces when the Lewis number values intensify.

The variation of the Nusselt number with slip boundary condition, the thermophoresis parameter

<table>
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<tr>
<th>$\lambda$</th>
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<th>$-\phi'(0)$</th>
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and Prandtl number is presented in fig. 8. It is observed that the dimensionless heat transfer rate is higher for variation of slip factor, $\lambda$, in smaller values of the thermophoresis parameter, $Nt$, and this change decreases with the increase of $Nt$. It is also seen that in the absence of slip parameter the heat transfer rate is maximum. In addition, it is noticed that the Nusselt number increase with the Prandtl number because higher Prandtl number corresponds to thinner thermal boundary-layer. In figs. 9 and 10 the variation of reduced Sherwood number with slip boundary condition, Prandtl number, the thermophoresis parameter, and Lewis number is observed. It is noticed from fig. 9 that the dimensionless mass transfer rates with increase in thermophoresis parameter is decreased for small Prandtl numbers and increased for larger values of Prandtl number. In both situations, increase in slip factor results in decrease in Sherwood number. It is observed from fig. 10 that the increase of Lewis number increases the dimensionless mass transfer rates.

Conclusions

In this paper, HAM is applied to obtain the approximate solutions of heat transfer of a nanofluid fluid past a stretching sheet with slip boundary condition. The validity of our
solutions is verified by the numerical published data. The analytical solution presented which depends on the slip parameter, Prandtl number, Brownian motion number, Lewis number, and thermophoresis number. The variation of reduced Nusselt and Sherwood numbers with $Nt$ and $\lambda$ for various values of Prandtl and Lewis number is presented in both tabular and graphical forms. The main observations of the present study are as follows:

- The reduced Nusselt and Sherwood numbers are the decreasing functions of slip parameter.
- Increasing Prandtl number increase the reduced Sherwood number.
- The reduced Nusselt number is the increasing function of $Nt$ and Lewis number.

### Nomenclature

- $c$ – volumetric volume coefficient
- $D_B$ – Brownian diffusion coefficient
- $D_T$ – thermophoresis diffusion coefficient
- $f$ – similarity function for velocity
- $k$ – thermal conductivity, $[\text{W m}^{-1}\text{K}^{-1}]$
- $Le$ – Lewis number
- $Nb$ – Brownian motion parameter
- $N$ – slip constant
- $Nu$ – local Nusselt number
- $Pr$ – Prandtl number
- $p$ – constant pressure
- $q_w$ – wall heat flux, $[\text{W m}^{-2}]$
- $Re_x$ – local Reynolds number
- $Sh$ – Sherwood number
- $T$ – temperature, $[\text{K}]$
- $T_w$ – surface temperature, $[\text{K}]$
- $T_\infty$ – ambient temperature, $[\text{K}]$
- $U_s$ – velocity slip, $[\text{m s}^{-1}]$
- $U_e$ – stretching velocity, $[\text{m s}^{-1}]$
- $u$ – velocity in x-direction, $[\text{m s}^{-1}]$
- $v$ – velocity in y-direction, $[\text{m s}^{-1}]$
- $x$ – co-ordinate along the plate, $[\text{m}]$
- $y$ – co-ordinate normal to the sheet, $[\text{m}]$

### Greek symbols

- $\alpha$ – thermal diffusivity, $[\text{m}^2\text{s}^{-1}]$
- $\eta$ – similarity independent variable
- $\theta$ – similarity function for temperature
- $\lambda$ – slip parameter
- $\nu$ – kinematic viscosity, $[\text{kg m}^{-1}\text{s}^{-1}]$
- $\rho$ – fluid density, $[\text{kg m}^{-3}]$
- $\rho_f$ – density of the base fluid, $[\text{kg m}^{-3}]$
- $\rho_p$ – density of the particles, $[\text{kg m}^{-3}]$
- $\phi_m$ – nanoparticle volume fraction
- $\psi$ – stream function

### References


