AN ANALYTICAL ALGORITHM TO DETERMINE ALLOWABLE AMPACITIES OF HORIZONTALLY INSTALLED RECTANGULAR BUS BARS

by

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The main objective of this paper is to propose an algorithm for the determination of the allowable ampacities of single rectangular-section bus bars without the occurrence of correction factors. Without correction factors, the ampacity computation of the copper and aluminum bus bars is fully automatized. The analytical algorithm has been implemented in a computer program code that along with the allowable ampacity can compute the bus bar temperature and the individual heat transfer coefficient for each side of the bus bar, as well as their corresponding power losses. Natural and forced convection correlations for rectangular bus bars are applied. Effects of the solar radiation and radiation heat losses from the bus bar surface are taken into consideration as well. The finite element method has been used for the linear/non-linear steady-state thermal analysis, i. e. for validation of the analytical algorithm. All finite element method based numerical computations were carried out using the COMSOL heat transfer module.

Key words: *allowable ampacity, analytical algorithm, bus bar, empirical correlations, heat transfer*

Introduction

In electrical power substations, bus bars connect a number of incoming circuit connections to a number of out coming circuit connections. Bus bars are made of non-insulated solid, hollow and profiled copper or aluminum conductors. Conductors with circular and rectangular cross-sections or U-shaped conductors are used in electrical power substations for rated voltages up to 35 kV, whilst stranded or tubular conductors are used in electrical power substations for rated voltages equal to or higher than 110 kV. The selection of a bus bar cross-section is carried out: (a) with regard to the I_{op} current under normal operating conditions – thermal stress that the conductor can withstand continuously, (b) with regard to the conventional value of the short-time withstand current during a three-phase short-circuit – thermal stress that the conductor can withstand in a short period of time, and (c) with regard to mechanical stresses during a three-phase short-circuit.

The allowable ampacity of the bus bar, I_{cp}, is determined based on the balance between the amount of heat generated within the bus bar material (Joule's losses) and the

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amount of heat reaching the outer surface of the bus bar (solar radiation) on the one, and the amount of heat conducted away from the bus bar (heat losses due to the convection and radiation on the outer surface of the bus bar) on the other hand. When selecting a cross-section of a bus bar, its allowable ampacity, I_{cp}, must be larger than the operating current, I_{op}. According to standards, the difference between the tabulated continuously permissible temperature of the bus bar, T_{cpT}, (65 or 70 °C) and the tabulated temperature of the surrounding air, T_{aT}, (35 or 40 °C) accounts to 30 °C regardless of the material type from which the bus bar is made.

Moreover, the allowable ampacity should be distinguished from the rated current of the bus bar, I_{nom}. The I_{cp} obtained by recomputation of the rated current to actual conditions at the location foreseen for the bus bur installation, *i. e.* by means of expression: $I_{cp} = C_1 C_2 C_3 C_4 C_5 I_{nom}$, where C_1-C_5 are appropriate correction factors [1, 2].

Within the classical procedure for selecting the bus bar cross-sections, continuously permissible temperature of the bus bar, $T_{cp,}$ appears along with the tabulated continuously permissible temperature of the bus bar, T_{cpT} [1, 2]. However, the aforementioned is a deviation from the well-established rule to use only one, in this particular case, tabulated value of its continuously permissible temperature when computing the allowable ampacity of a conductor.

Therefore, this paper presents a procedure for the determination of the allowable ampacity of a bus bar using the tabulated continuously permissible temperature without the occurrence of correction factors C_1-C_5. The allowable ampacity of a bus bar depends on the shape and dimensions of its cross-section, as well as on the number of conductors and their arrangement in the *package* of one phase, distances between the packages of phase conductors, resistivity of the bus bar material, colour and physical properties of the outer surface of the bus bar (emission and absorption coefficients), ambient conditions (solar irradiance, ambient air temperature, wind velocity, v_w, and wind direction) and thermo-physical properties of air.

Factors on which the allowable ampacity of a bus bar depend are modelled by corresponding convective (either natural or forced one) and radiation boundary conditions, as well as heat sources located inside and outside of the bus bar material [3-5]. A MATLAB programme entitled BUSBAR.m, which is based on a set of empirical correlations similar to the one in [3], has been developed and used in order to compute the allowable ampacity of the bus bar, its temperature and individual heat transfer coefficient for each side of the bus bar, as well as their corresponding losses.

Governing equation

In a general case, the time-dependent heat conduction equation can be derived from the law of conservation of energy in the volume of a running meter of the bus bar. One such volume element the dimensions of which are W_b, H_b, and L_b = 1 m is shown in fig. 1. It is assumed that the bus bar material is homogenous and isotropic, as well as that there is no longitudinal heat conduction in the bus bar. Moreover, it is also assumed that the heat, Q_{tg}, [W] is generated within the given volume element and that its thermo-physical properties (thermal conductivity, k_t, specific heat, c_t, and density, ρ) are constant.

When the law of conservation of energy is applied to the considered volume element within the time interval, Δt, then the energy equation becomes:

$$\alpha S_p Q_{tsun,s} + Q_{tg}(T_s) = [2h_{cS}S_S + h_{cT}S_T + h_{cB}S_B + h_r(T_s)S_{o1}](T_s - T_a) + V\rho c_t \frac{\partial T_s}{\partial t} \tag{1}$$

where

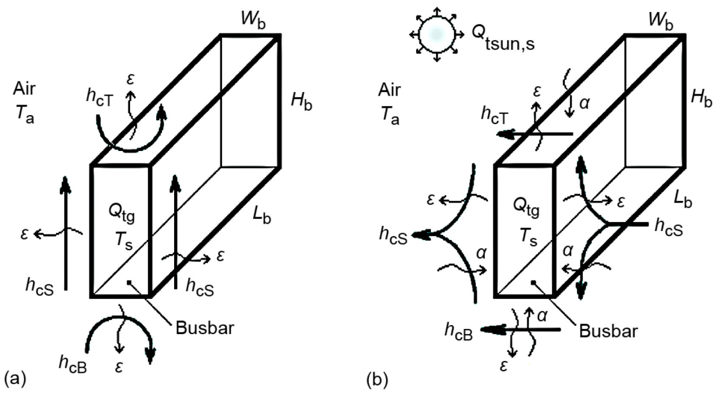

Figure 1. Heat transfer in the volume of a running meter of the bus bar for typical; (a) indoor and (b) outdoor operating conditions

$$Q_{\mathrm{tg}}(T_{\mathrm{s}}) = k_{\mathrm{s}} k_{\mathrm{p}} \rho_{\mathrm{dc}}(T_{\mathrm{s}}) I^2 \frac{V}{S^2} \tag{2}$$

$$\rho_{\mathrm{dc}}(T_{\mathrm{s}}) = \rho_{e20}[1 + \alpha_L (T_{\mathrm{s}} - 293.15)] \tag{3}$$

$$h_{\mathrm{r}}(T_{\mathrm{s}}) = \sigma_{\mathrm{SB}} \varepsilon (T_{\mathrm{s}}^2 + T_{\mathrm{a}}^2)(T_{\mathrm{s}} + T_{\mathrm{a}}) \tag{4}$$

where α is the absorption coefficient of the outer surface of the bus, S_{p} [m^2] – the projected area of a running meter of the bus bar, $Q_{\mathrm{tsun,s}}$ [Wm^{-2}] – the solar irradiance component which is perpendicular to the surface S_{p}, T_{s} [K] – the bus bar temperature, h_{cS} [Wm^{-2}K^{-1}] – the heat transfer coefficient due to the natural/forced convection between side surfaces of the bus bar and ambient air, S_{S} [m^2] – the surface area of one lateral side of the bus bar, h_{cT} [Wm^{-2}K^{-1}] – the heat transfer coefficient due to the natural/forced convection between the top surface of the bus bar and ambient air, S_{T} [m^2] – the surface area of the top side of the bus bar, h_{cB} [Wm^{-2}K^{-1}] – the heat transfer coefficient due to the natural/forced convection between the bottom surface of the bus bar and ambient air, S_{B} [m^2] – the surface area of the bottom side of the bus bar, ε – the emission coefficient of the outer surface of the bus bar, $\sigma_{\mathrm{SB}} = 5.67\ 10^{-8}$ Wm^{-2}K^{-4} – the Stefan-Boltzmann constant, S_{o1} [m^2] – the outer surface area of a running meter of the bus bar, h_{r} [Wm^{-2}K^{-1}] – the heat transfer coefficient due to the radiation heat exchange between the outer surface of the bus bar and ambient, V [m^3] – the volume of the observed bus bar element, t [seconds] – time, I [A] – the steady-state root-mean square (RMS) current of the bus bar, S [m^2] – the cross-sectional area of the bus bar. k_{s} – the extra loss coefficient for the skin effect in rectangular bus conductors, k_{p} – the extra loss coefficient for the proximity effect in rectangular bus conductors, ρ_{dc} [Ωm] – the d.c. resistivity of the bus bar material, ρ_{e20} [Ωm] – the d.c. resistivity of the bus bar material at the temperature of 293.15 K, and α_L [K^{-1}] – the temperature coefficient of the resistivity of the bus bar material.

When two phase single rectangular-section conductors are installed horizontally with a vertical major axis and with a side-by-side spacing of 0.3-1 m, the extra loss coefficient for the proximity effect k_{p} is between 0.95 and 1 [5]. For the case when these two conductors are installed with a horizontal major axis and with the same spacing, the coefficient k_{p} is between 1 and 1.05 [5]. In the case of indoor installation these side-by-side spacings are usually larger than 0.3 m. Therefore, it can be assumed that the proximity effect is negligible ($k_p \approx 1$). Furthermore, electrodynamic forces between two phase conductors are proportional to the current square. For rated currents these electrodynamic forces have small magnitudes and they are therefore not discussed.

For the steady-state heat transfer ($\partial T_s/\partial t = 0$), the bus bar temperature that is equal to T_{cpT} and a maximum amount of solar heat that can be absorbed by the outer surface of a running meter of the bus bar ($Q_{tsunM} = \alpha S_{pM} Q_{tsun,s}$), eq. (1) becomes:

$$\alpha S_{pM} Q_{tsun,s} + Q_{tg}(T_{cpT}) = [2h_{cS}S_S + h_{cT}S_T + h_{cB}S_B + h_r(T_{cpT})S_{o1}](T_{cpT} - T_a) \tag{5}$$

where S_{pM} [m^2] is the maximum projected area of a running meter of the bus bar.

Therefore, the expression for I can be expressed from eq. (5):

$$I = \frac{\{[2h_{cS}S_S + h_{cT}S_T + h_{cB}S_B + h_r(T_{cpT})S_{o1}](T_{cpT} - T_a) - \alpha S_{pM} Q_{tsun,s}\}^{1/2} S}{[k_s k_p \rho_{dc}(T_{cpT})V]^{1/2}} \tag{6}$$

where $\rho_{dc}(T_{cpT})$ and $h_r(T_{cpT})$ should be computed by means of eqs. (3) and (4).

According to eq. (1), for the steady-state heat transfer and $Q_{tsunM} = \alpha S_{pM} Q_{tsun,s}$, the unknown bus bar temperature, T_s, is:

$$T_s = \frac{\alpha S_{pM} Q_{tsun,s} + Q_{tg}(T_s)}{2h_{cS}S_S + h_{cT}S_T + h_{cB}S_B + h_r(T_s)S_{o1}} + T_a \tag{7}$$

For indoor installations, the allowable ampacities that correspond to central European conditions are determined under the following assumptions [1-3]: (a) that the ambient air is still; (b) that the surfaces of bare conductors are partly oxidized giving an emission coefficient of $\varepsilon = 0.40$ – for Cu conductors and $\varepsilon = 0.35$ – for Al conductors; or (c) that the surfaces of conductors are painted giving an emission coefficient of approximately $\varepsilon = 0.90$.

Furthermore, for outdoor installations, the allowable ampacities that correspond to central European conditions are determined under the following assumptions [1-3]: (a) that the ambient air moves slightly, *i. e.* that the wind velocity is $v_w = 0.6$ m/s; (b) that the solar irradiance is $Q_{tsun,s} = 1000$ W/m^2; (c) that the surfaces of bare conductors are normally oxidized giving an emission coefficient of $\varepsilon = 0.60$ – for Cu conductors and $\varepsilon = 0.50$ – for Al conductors, as well as an absorption coefficient of $\alpha = 0.45$ – for Cu conductors and $\alpha = 0.35$ – for Al conductors; or (d) that the surfaces of conductors are painted giving an emission coefficient of approximately $\varepsilon = 0.90$ and an absorption coefficient of $\alpha = 0.70$.

Analytical algorithm

The flowchart, shown in fig. 2, describes the algorithm of the BUSBAR.m programme. The parameters displayed in fig. 2 have the following meanings: E is the specified accuracy, J – the index of the current iteration in the inner loop, and T_s and T''_s are the programme variables reserved for values of the bus bar temperature, T_s. Other parameters appearing in fig. 2 are mentioned earlier.

In addition to the heading, declaration section, non-executable statements and ending, the analytical algorithm consists of the following blocks of executable statements:

Input: (a) Read and store input data from the Temperature.m, Density.m, Capacity.m, Viscosity.m, Conductivity.m, and Prandtl.m data files, whereas each of them has 85

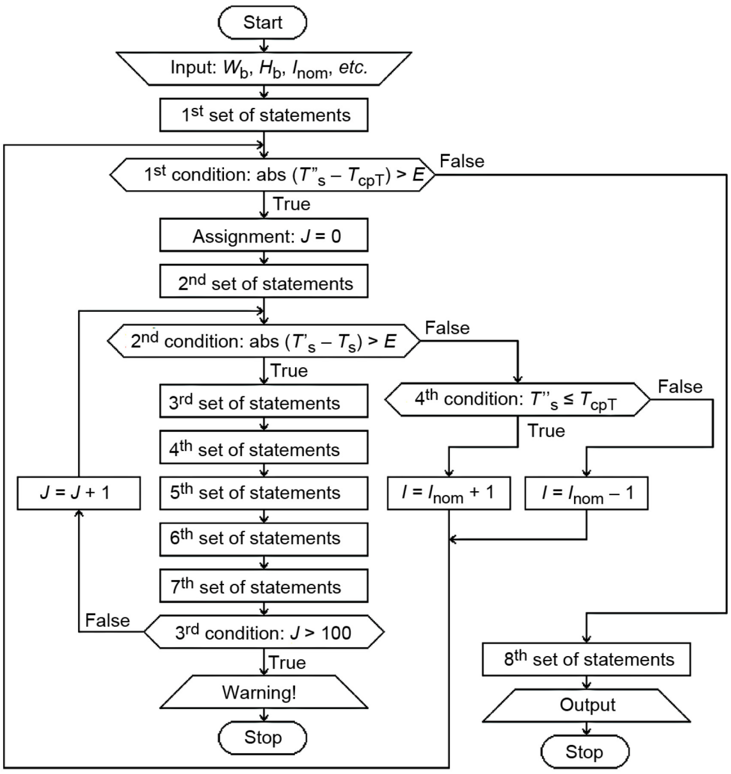

Figure 2. The flowchart illustrating the algorithm of the BUSBAR.m programme

discrete values taken from [5], *i. e.* values of temperature, density, specific heat, dynamic viscosity, thermal conductivity, and Prandtl number for air; (b) select the type of convection: 1 – for the natural or 2 – for the forced convection; (c) if the natural convection is selected, then the following should be selected: 1 – for $T_s > T_a$ or 2 – for $T_s < T_a$; (d) if the forced convection is selected, then the following should be selected: 1 – for the case when the wind direction is perpendicular to the longitudinal axis of the bus bar or 2 – for the case when the wind direction is parallel to the longitudinal axis of the bus bar; (e) enter the value for the following: W_b [m], H_b [m], $I = I_{nom}$ [A], T_{cpT} [K], ρ_{e20} [Ωm], α_L [K^{-1}], k_s [–], k_p [–], T_a [K], v_w [ms^{-1}], ε [–], α [–], and $Q_{tsun,s}$ [Wm^{-2}]; (f) for an initial estimate of T_s, for all unknown heat transfer coefficients enter the same numerical value h'_c [Wm^{-2}K^{-1}] which corresponds to the upper bound of natural/forced convection in the gases.

1st set of statements: (a) compute the value for the following: $S = W_b H_b$, $V = S1 = S$, $S_S = H_b 1 = H_b$, $S_T = W_b 1 = W_b$, $S_B = W_b 1 = W_b$, $S_{o1} = 2S_S + S_T + S_B$, (b) compute the diagonal of the bus bar cross-section D_H as: $D_H = (W_b^2 + H_b^2)^{1/2}$; (c) compute the maximum projected area of a running meter of the bus bar S_{pM} as: $S_{pM} = D_H 1$; (d) compute d.c. resistivity of the bus bar material at the temperature T_{cpT} as: $\rho_{dc}(T_{cpT}) = \rho_{e20} [1 + \alpha_L (T_{cpT} - 293.15)]$; (e) compute the value for Q_{tsunM} as: $Q_{tsunM} = \alpha S_{pM} Q_{tsun,s}$.

1st condition: start iterations in the outer loop and repeat the commands in the loop until the absolute value of ($T''_s - T_{cpT}$) becomes lower than the specified accuracy, $E = 0.2$.

Assignment: assign the value 0 to the index of the current iteration J in the inner loop.

2nd set of statements: (a) compute the volume power of heat sources located in the bus bar material $Q_{tg,v}$ as: $Q_{tg,v} = k_s k_p \rho_{dc}(T_{cpT})(I/S)^2$; (b) compute the value for Q_{tg} as: $Q_{tg} = Q_{tg,v}V$; (c) assign the value h'_c to the first variable h'_{cS} reserved for the coefficient h_{cS}; (d) assign the value h'_c to the first variable h'_{cT} reserved for the coefficient h_{cT}; (e) assign the value h'_c to the first variable h'_{cB} reserved for the coefficient h_{cB}; (f) assign the value h'_c to the first variable h'_r reserved for the coefficient h_r; (g) estimate the bus bar temperature based on the computed values for Q_{tg}, Q_{tsunM}, S_S, S_T, S_B, and S_{o1}, and the entered values for h'_c and T_a as: $T'_s = (Q_{tg} + Q_{tsunM})/[h'_c(2S_S + S_T + S_B + S_{o1})] + T_a$; (h) assign the value 0 to the variable reserved for the bus bar temperature T_s.

2nd condition: start iterations in the inner loop and repeat the commands in the loop until the absolute value of $(T'_s - T_s)$ becomes lower than the specified accuracy, $E = 0.2$.

3rd set of statements: if the value of the variable T_s differs from 0, then: (a) assign the value of the 2nd variable reserved for the same convection coefficient h''_{cS} to the variable h'_{cS}; (b) assign the value of the 2nd variable reserved for the same convection coefficient h''_{cT} to the variable h'_{cT}; (c) assign the value of the 2nd variable reserved for the same convection coefficient h''_{cB} to the variable h'_{cB}; (d) assign the value of the 2nd variable reserved for the same heat transfer coefficient due to the radiation h''_r to the variable h'_r; (e) assign the value of the variable T_s to the variable T'_s.

4th set of statements: (a) compute the film temperature, T_{film}, as [4, 5]: $T_{film} = (T'_s + T_a)/2$; (b) compute the thermal expansion coefficient of air, β, as [4, 5]: $\beta = 1/T_{film}$; (c) call to the subroutine CSCURVE, which approximates the temperature dependence of the density, specific heat, dynamic viscosity, thermal conductivity, and Prandtl number for air by means of cubic spline interpolation and arrays of discrete values from the pair of input data files: Temperature.m-Density.m, Temperature.m-Capacity.m, Temperature.m-Viscosity.m, Temperature.m-Conductivity.m, and Temperature.m-Prandtl.m, respectively; which also assigns the value of density, specific heat, dynamic viscosity, thermal conductivity, and Prandtl number for air to the following variables: ρ, c_t, μ, k_t, and Pr at T_{film}, respectively; (d) compute the kinematic viscosity of air, ν, as: $\nu = \mu/\rho$.

5th set of statements: if the natural convection heat transfer is considered, then: (a) for the lateral sides of the bus bar, the following equations are valid [6]:

$$\mathrm{Gr_S} = \frac{9.81\beta(T_s - T_a)H_b^3}{\nu^2} \tag{8}$$

$$\mathrm{Nu_S} = \left\{ 0.825 + \frac{0.387(\mathrm{Gr_S\,Pr})^{1/6}}{\left[1+\left(\frac{0.492}{\mathrm{Pr}}\right)^{9/16}\right]^{8/27}} \right\}^2 \quad \text{for } \mathrm{Gr_S Pr} \le 10^2 \tag{9}$$

$$\mathrm{Nu_S} = 0.68 + \frac{0.67(\mathrm{Gr_S\,Pr})^{1/4}}{\left[1+\left(\frac{0.492}{\mathrm{Pr}}\right)^{9/16}\right]^{4/29}} \quad \text{for } \mathrm{Gr_S Pr} > 10^2 \tag{10}$$

$$h''_{\mathrm{cS}} = \frac{h'_{\mathrm{cS}} + \mathrm{Nu_S}\dfrac{k_\mathrm{t}}{H_\mathrm{b}}}{2} \tag{11}$$

where $\mathrm{Gr_S}$ and $\mathrm{Nu_S}$ are corresponding Grashof and Nusselt numbers, respectively; (b) for the top side of the bus bar, the following equations are valid [5, 7, 8]:

$$\mathrm{Gr_T} = \frac{9.81\beta(T'_\mathrm{s} - T_\mathrm{a})W_\mathrm{b}^3}{\nu^2} \tag{12}$$

$$\mathrm{Nu_T} = 0.54(\mathrm{Gr_T Pr})^{1/4} \;\text{ for }\; T_\mathrm{s} > T_\mathrm{a} \;\text{ and }\; \mathrm{Gr_T Pr} \le 8\cdot 10^6 \tag{13}$$

$$\mathrm{Nu_T} = 0.15(\mathrm{Gr_T Pr})^{1/3} \;\text{ for }\; T_\mathrm{s} > T_\mathrm{a} \;\text{ and }\; \mathrm{Gr_T Pr} > 8\cdot 10^6 \tag{14}$$

$$\mathrm{Nu_T} = 0.27(\mathrm{Gr_T Pr})^{1/4} \;\text{ for }\; T_\mathrm{s} < T_\mathrm{a} \tag{15}$$

$$h''_{\mathrm{cT}} = \frac{h'_{\mathrm{cT}} + \mathrm{Nu_T}\dfrac{k_\mathrm{t}}{W_\mathrm{b}}}{2} \tag{16}$$

where $\mathrm{Gr_T}$ and $\mathrm{Nu_T}$ are corresponding Grashof and Nusselt numbers, respectively; (c) for the bottom side of the bus bar, the following equations are valid [5, 7, 8]:

$$\mathrm{Gr_B} = \frac{9.81\beta(T'_\mathrm{s} - T_\mathrm{a})W_\mathrm{b}^3}{\nu^2} \tag{17}$$

$$\mathrm{Nu_B} = 0.54(\mathrm{Gr_B Pr})^{1/4} \;\text{ for }\; T_\mathrm{s} < T_\mathrm{a} \;\text{ and }\; \mathrm{Gr_B Pr} \le 8\cdot 10^6 \tag{18}$$

$$\mathrm{Nu_B} = 0.15(\mathrm{Gr_B Pr})^{1/3} \;\text{ for }\; T_\mathrm{s} < T_\mathrm{a} \;\text{ and }\; \mathrm{Gr_B Pr} > 8\cdot 10^6 \tag{19}$$

$$\mathrm{Nu_B} = 0.27(\mathrm{Gr_B Pr})^{1/4} \;\text{ for }\; T_\mathrm{s} > T_\mathrm{a} \tag{20}$$

$$h''_{\mathrm{cB}} = \frac{h'_{\mathrm{cB}} + \mathrm{Nu_B}\dfrac{k_\mathrm{t}}{W_\mathrm{b}}}{2} \tag{21}$$

where $\mathrm{Gr_B}$ and $\mathrm{Nu_B}$ are corresponding Grashof and Nusselt numbers, respectively.

6^th^ set of statements: if the forced convection heat transfer is considered, then: (a) for the lateral sides of the bus bar, the following equations are valid [5, 7, 9, 10]:

$$\mathrm{Re_S} = \frac{v_\mathrm{w} H_\mathrm{b}}{\nu} \tag{22}$$

$$\mathrm{Nu_S} = 0.205\,\mathrm{Re_S^{0.731} Pr^{1/3}} \;\text{ for the wind direction } \perp \text{ to the bus bar axis} \tag{23}$$

$$\mathrm{Nu_S} = 0.664\,\mathrm{Re_S^{1/2} Pr^{1/3}} \;\text{ for the wind direction } \parallel \text{ to the bus bar axis} \tag{24}$$

$$h''_{\mathrm{cS}} = \frac{h'_{\mathrm{cS}} + \mathrm{Nu_S}\dfrac{k_\mathrm{t}}{H_\mathrm{b}}}{2} \tag{25}$$

where $\mathrm{Re_S}$ and $\mathrm{Nu_S}$ are corresponding Reynolds and Nusselt numbers, respectively; (b) for the top and bottom sides of the bus bar and both directions of the wind with regard to the longitudinal axis of the bus bar, the following equations are valid [5, 6, 10]:

$$\mathrm{Re_{TB}} = \frac{v_\mathrm{w} W_\mathrm{b}}{\nu} \tag{26}$$

$$\mathrm{Nu_{TB}} = 0.664\, \mathrm{Re_{TB}^{1/2} Pr^{1/3}} \;\text{ for }\; \mathrm{Re_{TB}} \le 5{\cdot}10^5 \tag{27}$$

$$\mathrm{Nu_{TB}} = (0.037\mathrm{Re_{TB}^{4/5}} - 871)\, \mathrm{Pr^{1/3}} \;\text{ for }\; \mathrm{Re_{TB}} > 5{\cdot}10^5 \tag{28}$$

$$h''_\mathrm{cT} = h''_\mathrm{cB} = \frac{h'_\mathrm{cTB} + \mathrm{Nu_{TB}} \dfrac{k_\mathrm{t}}{W_\mathrm{b}}}{2} \tag{29}$$

where $\mathrm{Re_{TB}}$ and $\mathrm{Nu_{TB}}$ are corresponding Reynolds and Nusselt numbers, and h'_cT and h'_cB are corresponding convection coefficients.

7th set of statements: (a) compute the heat transfer coefficient due to the radiation heat exchange between the outer surface of the bus bar and the ambient h''_r:

$$h''_\mathrm{r} = \frac{h'_\mathrm{r} + \sigma_\mathrm{SB}\varepsilon(T'^2_\mathrm{s} + T^2_\mathrm{a})(T'_\mathrm{s} + T_\mathrm{a})}{2} \tag{30}$$

(b) compute the bus bar temperature, T''_s, based on the obtained values for Q_tg, Q_tsunM, S_S, S_T, S_B, S_o1, h''_cS, h''_cT, h''_cB, and h''_r, and the entered value for T_a as: $T''_\mathrm{s} = (Q_\mathrm{tg} + Q_\mathrm{tsunM})/(2h''_\mathrm{cS} S_\mathrm{S} + h''_\mathrm{cT} S_\mathrm{T} + h''_\mathrm{cB} S_\mathrm{B} + h''_\mathrm{r} S_\mathrm{o1}) + T_\mathrm{a}$; (c) if the absolute value of the difference ($T_\mathrm{s} - T''_\mathrm{s}$) is larger than the specified accuracy $E = 0.2$, then assign the computed value T''_s to the variable T_s due to the computation in the next iteration.

3rd condition: (a) if $J > 100$, then force the exit from the inner loop and display the following message: *Warning! There is a problem with the exit condition from the inner loop or input data is incorrect!*; (b) if $J \le 100$, then increase the index of the current iteration J by 1 and go to the 2nd condition, *i. e.* to the beginning of the inner loop.

4th condition: if the absolute value of the difference ($T'_\mathrm{s} - T_\mathrm{s}$) has become lower than $E = 0.2$, then: (a) if $T''_\mathrm{s} \le T_\mathrm{cpT}$, then increase the value of the bus bar current $I = I_\mathrm{nom}$ by 1 A, *i. e.* $I = I_\mathrm{nom} + 1$; (b) if $T''_\mathrm{s} > T_\mathrm{cpT}$, then decrease the value of the bus bar current $I = I_\mathrm{nom}$ by 1 A, *i. e.* $I = I_\mathrm{nom} - 1$; (c) go to the 1st condition, *i. e.* to the beginning of the outer loop.

8th set of statements: if the absolute value of the difference ($T''_\mathrm{s} - T_\mathrm{cpT}$) has become lower than $E = 0.2$, then: (a) assign the last computed values for T''_s, h''_cS, h''_cT, h''_cB, h''_r, and I to the variables T_s, h_cS, h_cT, h_cB, h_r, and I_cp, respectively; (b) compute the power losses due to the natural or forced convection heat transfer between each of the bus bar sides and ambient air, *i. e.* $Q_\mathrm{th,sS} = h_\mathrm{cS}(T_\mathrm{s} - T_\mathrm{a})$ – for the two lateral sides, $Q_\mathrm{th,sT} = h_\mathrm{cT}(T_\mathrm{s} - T_\mathrm{a})$ – for the top side, and $Q_\mathrm{th,sB} = h_\mathrm{cB}(T_\mathrm{s} - T_\mathrm{a})$ – for the bottom side; (c) compute the power loss due to the radiation heat exchange between the outer surface of the bus bar and the ambient, *i. e.* $Q_\mathrm{tr,s} = h_\mathrm{r}(T_\mathrm{s} - T_\mathrm{a})$.

Output: display the value for the following: $Q_{\mathrm{tg},V}$ [Wm^{-3}], V [m^3], Q_tg [W], S_o1 [m^2], S_pM [m^2], Q_tsunM [Wm^{-2}] and [W], T_s [K], h_cS [Wm^{-2}K^{-1}], $Q_\mathrm{th,sS}$ [Wm^{-2}] and [W], h_cT [Wm^{-2}K^{-1}], $Q_\mathrm{th,sT}$ [Wm^{-2}] and [W], h_cB [Wm^{-2}K^{-1}], $Q_\mathrm{th,sB}$ [Wm^{-2}] and [W], h_r [Wm^{-2}K^{-1}], $Q_\mathrm{tr,s}$ [Wm^{-2}] and [W], and I_cp [A].

Numerical validation of developed algorithm

The numerical validation of the analytical algorithm for occurrences of natural and forced convection heat transfers was carried out with COMSOL, by performing a thermal finite element method (FEM) analysis of single bus bars with rectangular cross-sections from

[3]. The results of simulations performed for the case of natural convection are given in tab. 1, whilst tab. 2 shows results of simulations performed for the case of forced convection.

Table 1. Rated currents and allowable ampacities of single rectangular bus bars in the case of indoor installation[a]

W_b	H_b	60 Hz k_s at 70 °C	I_{nom} from handbook [b]	I_{cp} and T_{cp}				
				Ref. [3]	PE [c]	BUSBAR	COMSOL	PE [c]
m/in	**m**/in	–	A	A	%	**A**/°C	**A**/°C	%
Bus bars installed with a vertical major axis								
0.00635 /0.25	**0.0508** /2	1.014	545	535	–1.84	**545** /69.91	**545** /69.92	0
0.00635 /0.25	**0.1524** /6	1.092	1381	1374	–0.51	**1371** /70.09	**1371** /70.11	–0.72
0.009525 /0.375	**0.1016** /4	1.100	1191	1178	–1.09	**1186** /70.06	**1186** /70.08	–0.42
0.009525 /0.375	**0.2032** /8	1.210	2098	2108	+4.77	**2081** /70.07	**2081** /70.11	–0.81
0.0127 /0.5	**0.1016** /4	1.140	1369	1356	–0.95	**1364** /70.06	**1364** /70.08	–0.37
0.0127 /0.5	**0.2032** /8	1.259	2393	2407	+0.59	**2376** /70.09	**2376** /70.13	–0.71
Bus bars installed with a horizontal major axis								
0.0508 /2	**0.00635** /0.25	1.014	530	534	+0.76	**519** /70.01	**519** /70.03	–2.08
0.1524 /6	**0.00635** /0.25	1.092	1270	1333	+4.96	**1287** /69.91	**1287** /69.95	+1.34
0.1016 /4	**0.009525** /0.375	1.100	1130	1157	+2.39	**1125** /70.07	**1125** /70.09	–0.44
0.2032 /8	**0.009525** /0.375	1.210	1820	2025	+11.26	**1993** /69.92	**1993** /69.96	+9.5
0.1016 /4	**0.0127** /0.5	1.140	1300	1332	+2.46	**1299** /70.07	**1299** /70.09	–0.08
0.2032 /8	**0.0127** /0.5	1.259	2050	2313	+12.83	**2279** /69.90	**2279** /69.94	+11.17

[a] For aluminum alloy 6101-T61 horizontally installed bus bars, T_{aT} = 40 °C, T_{cpT} = 70 °C, v_w = 0 m/s, ε = 0.35, α = 0, and f = 60 Hz; [b] According to [11], these rated currents were obtained experimentally; [c] Percentage error: $PE = (I_{cp} - I_{nom})\,100/I_{nom}$.

Dimensions (W_b i H_b), bus bar currents (rated I_{nom} or assumed I_{ass}) and skin effect coefficients, k_s, for the considered bus bars are given in tabs. 1 and 2. The d. c. resistivity of the aluminum alloy 6101-T61 at 20 °C and its corresponding temperature coefficient are $\rho_{e20} = 2.998\ 10^{-8}\ \Omega$ m and $\alpha_L = 0.00383$ K^{-1} [3], respectively. The coefficients due to the natural and forced convection corresponding to the turbulent flow in the gases are $h'_c = 12$ W/m^{2}K and $h'_c = 300$ W/m^{2}K^1, respectively [5]. According to [3], the solar irradiance

Table 2. Assumed currents and allowable ampacities of single rectangular bus bars in the case of outdoor installation[a]

W_b	H_b	60 Hz k_s at 70 °C	I_{ass}	I_{cp} and T_{cp}					
				Wind direction ⊥ to bus bar axis			Wind direction || to bus bar axis		
				BUSBAR	COMSOL	PE [b]	BUSBAR	COMSOL	PE [b]
m/in	**m**/in	–	A	**A**/°C	**A**/°C	%	**A**/°C	**A**/°C	%
Bus bars installed with a vertical major axis									
0.00635 /0.25	**0.0508** /2	1.014	545	**845** /69.93	**845** /69.97	+ 55.05	**675** /69.96	**675** /69.99	+ 23.85
0.00635 /0.25	**0.1524** /6	1.092	1381	**1962** /69.92	**1962** /69.99	+ 42.07	**1323** /70.07	**1323** /70.15	-4.2
0.009525 /0.375	**0.1016** /4	1.100	1191	**1752** /69.92	**1752** /69.96	+ 47.1	**1278** /69.90	**1278** /69.95	+ 7.3
0.009525 /0.375	**0.2032** /8	1.210	2098	**2916** /69.90	**2916** /69.98	+ 38.99	**1894** /70.08	**1894** /70.17	-9.72
0.0127 /0.5	**0.1016** /4	1.140	1369	**2013** /69.89	**2013** /69.94	+ 47.04	**1486** /69.89	**1486** /69.94	+ 8.55
0.0127 /0.5	**0.2032** /8	1.259	2393	**3331** /69.90	**3331** /69.97	+ 39.2	**2190** /70.09	**2190** /70.16	-8.48
Bus bars installed with a horizontal major axis									
0.0508 /2	**0.00635** /0.25	1.014	530	**680** /69.92	**680** /69.95	+ 28.3	**675** /69.96	**675** /69.99	+ 27.36
0.1524 /6	**0.00635** /0.25	1.092	1270	**1324** /69.90	**1324** /69.98	+ 4.25	**1316** /69.90	**1316** /69.97	+ 3.62
0.1016 /4	**0.009525** /0.375	1.100	1130	**1306** /69.91	**1306** /69.97	+ 15.58	**1278** /69.90	**1278** /69.95	+ 13.1
0.2032 /8	**0.009525** /0.375	1.210	1820	**1917** /69.91	**1917** /70.01	+ 5.33	**1883** /69.91	**1883** /69.99	+ 3.46
0.1016 /4	**0.0127** /0.5	1.140	1300	**1538** /69.90	**1538** /69.96	+ 18.31	**1486** /69.89	**1486** /69.94	+ 14.31
0.2032 /8	**0.0127** /0.5	1.259	2050	**2241** /69.90	**2241** /70.00	+ 9.32	**2177** /69.90	**2177** /69.98	+ 6.2

[a] For aluminum alloy 6101-T61 horizontally installed bus bars, T_{aT} = 40 °C, T_{cpT} = 70 °C, v_w = 0.6 m/s, ε = 0.5, α = 0.35, $Q_{tsun,s}$ = 1000 W/m^2, and f = 60 Hz; [b] Percentage error: $PE = (I_{cp} - I_{ass})100/I_{ass}$.

which reaches the outer surface of the bus bar is $Q_{tsun,s}$ = 1000 W/m^2. The thermal conductivity k_t, which amounts to 218.5 W/m^{1}K^1 for the aluminum alloy 6101-T61, has been used for the thermal FEM analysis of the bus bars [11].

For the instance of indoor installation of single rectangular aluminum alloy 6101-T61 bus bars, the validation of the analytical algorithm has been carried out in accordance with [3], *i. e.* for the following on-site conditions: installations with the vertical and horizontal major axes, tabulated continuously permissible temperature of the bus bar T_{cpT} = 70 °C, tabulated temperature of the ambient air T_{aT} = 40 °C, wind velocity v_w = 0 m/s (natural convection), emission coefficient ε = 0.35, absorption coefficient α = 0, and frequency f = 60 Hz.

For the instance of outdoor installation of single rectangular aluminum alloy 6101-T61 bus bars, the validation of the analytical algorithm has been carried out for the following on-site conditions: installations with the vertical and horizontal major axes, tabulated continuously permissible temperature T_{cpT} = 70 °C, tabulated temperature of the ambient air T_{aT} = 40 °C, wind velocity v_w = 0.6 m/s (forced convection), emission coefficient ε = 0.5, absorption coefficient α = 0.35, solar irradiance $Q_{tsun,s}$ = 1000 W/m^2, and frequency f = 60 Hz.

Values of continuously permissible temperature of bus bars obtained for instances of natural and forced convection heat transfers by means of the BUSBAR.m and COMSOL programmes differ from the temperature T_{cpT} = 70 °C by approximately ±0.2 °C. These differences are negligible and represent a consequence of the specified accuracy E = 0.2 and generation of only integer values for the allowable ampacity I_{cp}.

It should be emphasized that the COMSOL heat transfer module uses as input data some of the output data generated by the BUSBAR.m program. For instance, the bus bar current cannot be entered directly into COMSOL, but its influence is taken via the volume power of heat sources located in the bus bar material obtained by the BUSBAR.m programme. Moreover, as input data, COMSOL also uses heat transfer coefficients due to the convection generated by means of the BUSBAR.m programme.

Application of developed algorithm

The analytical algorithm, *i. e.* the BUSBAR.m programme is simple to use and it generates the allowable ampacities of the bus bars in a rather fast manner, as well as other parameters necessary for its thermal FEM analysis in COMSOL. The given algorithm/programme is applied easily when extra losses coefficients for the skin effect in the considered bus bars are known and when ambient conditions on the installation site thereof are standardised.

Since copper and aluminum conductors with rectangular cross-sections are used exclusively in indoor installations of the rated voltage of which is up to 35 kV, the instance of outdoor installation thereof is not considered herein. Allowable ampacities of copper and aluminum conductors are computed under the previously given assumptions for central European countries. Allowable ampacities of single rectangular copper bus bars for the instance of indoor installation are given in tab. 3, whilst allowable ampacities of single rectangular aluminum bus bars for the instance of indoor installation are given in tab. 4.

Conclusions

A computer programme has been developed with the capability of computing allowable ampacities of horizontally installed single bus bars with rectangular cross-sections, as well as their temperature and other parameters that can further be used for their thermal FEM analysis in software packages such as COMSOL and ANSYS. The BUSBAR.m programme is based on the law of conservation of energy and may compute the allowable ampacities of the bus bars for any bus bar material, and any ambient conditions. The effects of the wind and solar radiation are considered only when validating the analytical algorithm due to the fact that bus bars with rectangular cross-sections are used exclusively in indoor installations in the practice. Therefore, the natural convection heat transfer and radiation heat exchange between the outer surface of the bus bars and ambient air are primarily considered herein, in addition to the heat generation in the bus bar materials. Moreover, using a cubic spline interpolation, the BUSBAR.m programme forms temperature dependences of thermo-physical properties of air and automatizes the selection of values for them. All of the aforementioned options make the developed programme immensely useful.

Table 3. Rated currents and allowable ampacities of single rectangular copper bus bars in the case of indoor installation

W_B	H_B	50 Hz k_s at 65 °C	Painted bus bars [a]				Bare bus bars [b]			
			I_{nom} [c]	I_{cp} [c]	I_{nom} [d]	I_{cp} [d]	I_{nom} [c]	I_{cp} [c]	I_{nom} [d]	I_{cp} [d]
[m]	[m]	[–]	[A]	[A]	[A]	[A]	[A]	[A]	[A]	[A]
0.005	0.012	1.000	203	224	183	220	177	198	150	193
0.010	0.012	1.000	326	351	293	349	285	307	242	305
0.005	0.020	1.000	319	338	287	329	274	295	233	284
0.010	0.020	1.005	497	515	447	507	427	446	363	437
0.005	0.030	1.000	447	476	402	460	379	410	322	392
0.010	0.030	1.010	676	709	608	694	573	610	487	592
0.005	0.040	1.005	573	608	516	587	482	520	410	495
0.010	0.040	1.017	850	897	765	873	715	766	608	739
0.005	0.050	1.006	679	739	611	711	583	629	496	597
0.010	0.050	1.025	1020	1079	918	1048	852	917	724	880
0.005	0.060	1.010	826	867	743	833	688	734	585	695
0.010	0.060	1.033	1180	1257	1062	1216	985	1063	837	1017
0.005	0.080	1.017	1070	1119	963	1074	885	940	752	888
0.010	0.080	1.050	1500	1603	1350	1548	1240	1346	1054	1283
0.005	0.100	1.025	1300	1363	1170	1310	1080	1140	918	1074
0.010	0.100	1.083	1810	1922	1629	1856	1490	1606	1267	1525
0.010	0.120	1.113	2110	2232	1899	2154	1740	1857	1479	1760
0.010	0.160	1.150	2700	2850	2430	2764	2220	2353	1887	2244
0.010	0.200	1.188	3290	3441	2961	3355	2690	2819	2287	2714

[a] For copper Cu-ETP horizontally installed bus bars, T_{aT} = 35 °C, T_{cpT} = 65 °C, v_w = 0 m/s, ε = 0.9, α = 0, $\rho_{e20} = 1.78 \cdot 10^{-8}$ Ωm, α_L = 0.0038 K^{-1}, and f = 50 Hz; [b] For copper Cu-ETP horizontally installed bus bars, T_{aT} = 35 °C, T_{cpT} = 65 °C, v_w = 0 m/s, ε = 0.4, α = 0, $\rho_{e20} = 1.78 \cdot 10^{-8}$ Ωm, α_L = 0.0038 K^{-1}, and f = 50 Hz; [c] For horizontal bus bars installed with a vertical major axis; [d] For horizontal bus bars installed with a horizontal major axis.

In comparison to the values obtained by Coneybeer *et al.* [3], it has been demonstrated that ampacities generated by the BUSBAR.m programme match with their corresponding tabulated values to a larger extent. During the course of validation of the analytical algorithm it was demonstrated that differences between generated and tabulated ampacities are between 0 and ±27.36 %. Similar deviations were also obtained in the section *Application of developed algorithm*, whilst the largest ones were for bus bars installed with a horizontal major axis. All of the previous imposes the conclusion that tables with thermally incorrect ampacities have been used in the practice for years and that they need to be revised.

The proposed procedure of computation of allowable ampacities of bus bars excludes rough approximations, it is scientifically based, rather simple and provides more accurate results in comparison to the traditional one, which is based on the use of tables with correction factors for ampacities. Moreover, the authors intend to extend the procedure to other geometries and orientations of bus bars.

Table 4. Rated currents and allowable ampacities of single rectangular aluminum bus bars in the case of indoor installation

W_B	H_B	50 Hz k_s at 65 °C	Painted bus bars [a]				Bare bus bars [b]			
			I_{nom} [c]	I_{cp} [c]	I_{nom} [d]	I_{cp} [d]	I_{nom} [c]	I_{cp} [c]	I_{nom} [d]	I_{cp} [d]
[m]	[m]	[–]	[A]	[A]	[A]	[A]	[A]	[A]	[A]	[A]
0.005	0.012	1.000	160	176	144	173	139	154	118	149
0.010	0.012	1.000	257	276	231	275	224	237	190	236
0.005	0.020	1.000	254	266	229	259	214	228	182	219
0.010	0.020	1.005	393	405	354	399	331	345	281	338
0.005	0.030	1.000	356	374	320	362	295	317	251	302
0.010	0.030	1.010	536	558	482	545	445	471	378	456
0.005	0.040	1.005	456	478	410	461	376	402	320	381
0.010	0.040	1.017	677	705	609	686	557	591	473	569
0.005	0.050	1.006	556	581	500	559	455	485	387	459
0.010	0.050	1.025	815	848	734	824	667	707	567	677
0.005	0.060	1.010	655	682	590	655	533	566	453	534
0.010	0.060	1.033	951	988	856	956	774	819	658	782
0.005	0.080	1.017	851	880	766	844	688	723	585	681
0.010	0.080	1.050	1220	1260	1098	1217	983	1036	836	985
0.005	0.100	1.025	1050	1072	945	1030	846	876	719	823
0.010	0.100	1.083	1480	1510	1332	1458	1190	1235	1012	1169
0.015	0.100	1.150	1800	1833	1620	1778	1450	1499	1233	1429
0.010	0.120	1.113	1730	1754	1557	1693	1390	1427	1182	1349
0.015	0.120	1.188	2090	2118	1881	2052	1680	1723	1428	1638
0.010	0.160	1.150	2220	2240	1998	2172	1780	1803	1513	1718
0.015	0.160	1.238	2670	2684	2403	2610	2130	2160	1811	2068
0.010	0.200	1.188	2710	2710	2439	2637	2160	2161	1836	2076
0.015	0.200	1.290	3230	3230	2907	3144	2580	2580	2193	2479

[a] For aluminum ENAW-1350A horizontally installed bus bars, T_{aT} = 35 °C, T_{cpT} = 65 °C, v_w = 0 m/s, ε = 0.9, α = 0, $\rho_{e20} = 2.86 \cdot 10^{-8}$ Ωm, α_L = 0.004 K^{-1}, and f = 50 Hz; [b] For aluminum ENAW-1350A horizontally installed bus bars, T_{aT} = 35 °C, T_{cpT} = 65 °C, v_w = 0 m/s, ε = 0.35, α = 0, $\rho_{e20} = 2.86 \cdot 10^{-8}$ Ωm, α_L = 0.004 K^{-1}, and f = 50 Hz; [c] For horizontal bus bars installed with a vertical major axis; [d] For horizontal bus bars installed with a horizontal major axis.

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