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CALCULATION OF TEMPERATURE FIELD IN GAS FLOW WITH INTERNAL HEAT SOURCE

by

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Short paper
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Gas flow sequentially moving through three zones (input $z < 0$, internal heat release $0 \le z \le l$, and output $z > 1$) of a cylindrical channel was considered. Analytical solutions taking into account the influence of heat source limitation in the axial direction and intensity of air flow in this direction on thermal balance were obtained.

Key words: *convective heat transfer, gas flow, internal source, temperature, boundary problem*

Introduction

Convective heat transfer in a gas flow with internal source of heat was studied theoretically by various scientists [1-6]. Typically their solutions of energy equation were obtained only for the zone with internal heat release, which was considered as non-limiting in gas flow direction. However such approach can result in a significant mistake for gas temperature definition, especially in the case of comparatively short zone of internal heat release.

Initial equations

Let's consider a gas which moves in a cylindrical channel and consequently passes through three zones: input $z < 0$, internal heat release $0 \le z \le 1$, and output $z > 1$.

In initial approximation in the case of negligible heat input the physical properties of gas can be considered as constant and equal to average values for the given temperature interval. Due to the gas is mon atomic, the processes of vibrational relaxation are not taken into account. Axially symmetric distribution of internal heat sources in a channel is described by the function $E(r, z)$:

$$E(r,z) = \begin{cases} 0 & z < 0, \ z > l \\ F(r) & 0 \le z \le l \end{cases}$$

Calculations are based on the mean axial velocity U which is obtained by averaging over the velocity vector axial component time averaged radial co-ordinate.

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Based on mentioned assumptions, the process of convective heat transfer in concerned system is described by the differential equation:

$$\mathrm{Pe}\frac{\partial\theta}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)+\frac{\partial^2\theta}{\partial z^2};\quad z<0,\ z>\frac{l}{R}$$

$$\mathrm{Pe}\frac{\partial\theta}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)+\frac{\partial^2\theta}{\partial z^2}+\frac{q_V R^2}{\lambda}F(r),\quad 0<z\le\frac{l}{R}$$

Gas temperature on the wall T_{W} is considered to be constant and its radial profile is symmetric.

Besides, there are continuity conditions for T and $\partial T/\partial z$ at the boundary of internal heat release zone.

Solution method

Let's consider the solution of boundary problem concerning θ in detail.

The arbitrary function of source $F(r)$ can be expanded into a Fourier-Bessel series according to the functions $J_0(\mu_{\mathrm{i}}r)$, $0\le r\le 1$.

Therefore,

$$F(r)=\sum_{i=1}^{\infty}a_i J_0(\mu_{\mathrm{i}}r)$$

where

$$a_{\mathrm{i}}=\frac{1}{J_0^2(\mu_{\mathrm{i}})}\int_0^1 F(r)J_0(\mu_{\mathrm{i}}r)r\mathrm{d}r$$

Distribution function of excess temperature can be written as:

$$\theta(r,z)=\sum_{i=1}f_{\mathrm{i}}(z)J_0(\mu_{\mathrm{i}}r)$$

Kantorovitch-Galerkin method [7] was used to solve the boundary problem. This method reduces considered boundary condition problem to the system of typical differential equations by using orthogonality of the residual to basis functions $J_0(\mu_{\mathrm{i}}r)$.

Scalar product is:

$$(F,G)=\int_0^1 F(r)G(r)r\mathrm{d}r$$

Note that

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial J_0(\mu_{\mathrm{i}}r)}{\partial r}\right)=\mu_i^2 J_0''(\mu_i r)+\frac{1}{\mu_{\mathrm{i}}r}\mu_i^2 J_0'(\mu_{\mathrm{i}}r)$$

Since $J_0(\mu_{\mathrm{r}}r)$ is the solution of zero-order Bessel equation, we can get the function:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial J_0(\mu_{\mathrm{i}}r)}{\partial r}\right)=-\mu_i^2 J_0(\mu_{\mathrm{i}}r)$$

Therefore, taking into account the proposed approximation, the boundary value problem can be written in the form:

$$\mathrm{Pe}\sum_{j=1} J_0(\mu_\mathrm{i} r) f_i'(z) = -\sum_{i=1}\mu_i^2 J_0(\mu_\mathrm{i} r) f_\mathrm{i}(z) + \sum_{j=1} J_0(\mu_\mathrm{i} r) f_i''(z),\ z<0,\ z>\frac{1}{R}$$

$$\mathrm{Pe}\sum_{j=1} J_0(\mu_\mathrm{i} r) f_i'(z) = -\sum_{i=1}\mu_i^2 J_0(\mu_\mathrm{i} r) f_\mathrm{i}(z) + \sum_{j=1} J_0(\mu_\mathrm{i} r) f_i''(z) + \frac{q_V}{\lambda} R^2 \sum_{i=1} a_\mathrm{i} J_0(\mu_\mathrm{i} r),$$
$$0 \le z \le \frac{1}{R}$$

or after a number of scalar multiplications we can get the expression:

$$\mathrm{Pe} f'(z) = -\mu_i^2 f_\mathrm{i}(z) + f_i''(z) \quad z<0, \quad z>\frac{l}{R}$$
$$\mathrm{Pe} f'(z) = -\mu_i^2 f_\mathrm{i}(z) + f_i''(z) + \frac{q_V R^2}{\lambda} a_i \quad 0 \le z \le \frac{1}{R}$$

Results and discussion

The solution of this system of differential equations, taking into account join conditions f_i and f_i' at $z = 0$, $z = \mathrm{l}$, gives the following formulas for these values:

$$f_\mathrm{i}(z) = \frac{q_V R^2}{\lambda \mu_i^2}\begin{cases} \dfrac{A_i^- \exp^{A_i^+ z} \exp^{-\frac{l}{R}A_i^+} - 1}{A_i^+ - A_i^-} & z<0 \\ 1 + \dfrac{A_i^- \exp^{\left(z-\frac{l}{R}\right)A_i^+} - A_i^+ \exp^{zA_i^-}}{A_i^+ - A_i^-} & 0 \le z \frac{l}{R} \\ \dfrac{A_i^+ \exp^{A_i^- z} \exp^{-\frac{l}{R}A_i^-} - 1}{A_i^+ - A_i^-} & z > \frac{l}{R} \end{cases}$$

Here

$$A_i^+ = \frac{\mathrm{Pe} + \sqrt{\mathrm{Pe}^2 + 4\mu_i^2}}{2},$$
$$A_i^- = \frac{\mathrm{Pe} - \sqrt{\mathrm{Pe}^2 + 4\mu_i^2}}{2}$$

The last equation solves the task completely.

The obtained relationships show that when there is no gas flow (Pe = 0), the axial temperature profile is symmetric towards $z = 2R$. If it is sufficiently great, the $1/R$ ($l/R \approx 4$-5) reaches its maximum, with this effect being the faster the greater (l/R) value.

In the zone of internal heat release the θ decreases twice in comparison with its boundary value due to the heat efflux through the mechanism of heat transfer in the axial direction. In short channels ($\mathrm{l}/R > 3$) this process influences the whole zone of internal heat release.

In the presence of gas flow (Pe $\neq$ 0) the temperature profile is not symmetric. θ decreases at any point of the zone of internal heat release. The length of transition part increases.

The zone $z > \mathrm{l}/R$ influences θ value. Increase of Pe number from zero to some value Pe_κ, depending on l/R, results in θ increase at the end of zone $0 \le z \le l/R$. This effect results from maximum influence of the zone due to the absence of gas flow. It decreases with reduction of Pe number.

It should be noted that at large values of Pe number, depending on l/R, θ can be calculated in zone $0 \le z \le l/R$ with 3% accuracy without taking into account the zone $z > l/R$.

Within rather long zone of internal heat release ($l/R > 30$), with increase of Pe number from zero to some value, the temperature in the channel of this zone increases reaching the boundary value. Here the gas flow prevents the influence of output zone, while gas quench front has not reached the right side of the zone of internal heat release yet. With further increase of Pe number this front will reach the side, and the temperature will start decreasing.

Conclusions

The method presented is not the only one for temperature profile estimation. Analytical method was also proposed in work [8] for the determination of heavy particle temperature in plasmatron channel, using vast material with distribution of electron temperature.

Derived equations are used for the calculation of local and integral balance of gas energy, both at laminar and turbulent flow in a cylindrical channel.

Nomenclature

a – thermal diffusivity, [m^2s^{-1}]
$F(r)$ – function of source
J_0 – the Bessel function of order zero
l – length of the channel
Pe – Peclet number (= UR/a), [–]
q_V – power density of internal heat release on the channel axis, [Wm^{-3}]
R – radius of the cylindrical channel, [m]
r – radial co-ordinate, [m]
T – gas temperature, [K]
T_W – wall temperature, [K]
U – gas velocity, [ms^{-1}]
z – axial co-ordinate, [m]

Greeks symbols

θ – temperature difference (= $T - T_W$), [K]
λ – coefficient thermal conductivity, [$Wm^{-1}K^{-1}$]
μ_i – j-th root of equation $J_0(x) = 0$

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