A MODIFICATION OF LEVEL SET RE-INITIALIZED METHOD FOR THE SHOCK WAVES THROUGH THE AIR BUBBLE

by

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Original scientific paper
DOI: 10.2298/TSCI1405525Z

For the re-initialization problem of the level set method, a new re-initialization formula of smoothing parameter based on the traditional implicit method is proposed. The improved method is applied to describe the process of the shock through the air bubble by means of numerical simulation. Numerical results show that the method is superior to the traditional level set method.

Key words: *air bubble problem, level set method, re-initialization, smoothing parameter*

Introduction

In practical problems, we often deal with the issue of interface, such as the movement of free surface, discontinuity or various internal interfaces. The numerical simulation of moving interfaces has a profound practical significance and application value, and there are many excellent algorithms. Level set method is the earliest one proposed by Osher and Sethian in 1988 [1, 2]. The basic idea in level set method is that the interface is captured implicitly by level set function. In other words, it is the zero isocontour of level set function, which is the interface position. We can determine fluid type based on the symbols of level set function. Interface tracking aims at fine processing near the interface as far as possible with the hopes the calculation results will be reach the required accuracy in the interface and near interface. The level set function can be obtained by solving level set equation, namely the symbol distance function at any grid point to the interfaces in calculation region. So, the geometric features such as location, normal and curvature of moving interfaces are known. Then the control equation of physical quantities could be solved. Some key problems [3] of numerical simulation about level set interface capturing method are how to keep the interface position stationary after level set function evolvement, and how to make the level set function revert to sign distance function. In fact, these problems are the re-initialization of level set function. It aims at removing the numerical oscillation when the gradient of level set function is too big or too small. Re-initialization algorithms [4] in general can be divided between implicit method of partial differential equation and the explicit method based on fast marching method. Some scholars have proposed many improved re-initialization algorithms [5, 6] under the two frameworks.

In this paper, for keeping the interface position stationary, reviewing with traditional implicit re-initialization method and some references [7-9], a new formula of smoothing pa-

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rameter is provided. Its validity is validated through numerical examples, and the modified interface capturing method is applied in shock waves through the air bubble problems. The numerical results show that the improved re-initialization level set method is more accurate and efficient than the traditional one.

Modified level set re-initialized method

The process of level set re-initialization is solving one Hamilton-Jacobi system. As two-dimensional problem has the form:

$$\begin{cases} \dfrac{\partial \phi}{\partial \tau} = \mathrm{sign}(\phi_0)\left[1 - \sqrt{\left(\dfrac{\partial \phi}{\partial x}\right)^2 + \left(\dfrac{\partial \phi}{\partial y}\right)^2}\right] \\ \phi(x, y, 0) = \phi_0(x, y) \end{cases} \tag{1}$$

where ϕ_0 is the initial value of level set function, sign(ϕ_0) – the signed distance function, and τ – the evolution variable.

When $\tau \to \infty$, ϕ keeps distance function properties relative to the initial value $\phi(\vec{x}, 0)$, that is $|\nabla \phi| = 1$. In order to easily solve eq. (1), the singed function will be smoothed as:

$$\mathrm{sign}_\varepsilon(\phi_0) = \frac{\phi_0}{\sqrt{\phi_0^2 + \varepsilon^2}} \tag{2}$$

where ε is the smoothing parameter.

The selection of smoothing parameter is very important to re-initialization results. As an example, consider $\varepsilon = \Delta x$ in [4]. While this approach [10] can reduce many numerical oscillations problem near the interface, but it is not very good for maintaining interface location in the process of re-initialization. Based on the previous analysis, a way of smooth parameter selection is given below in order to reduce error and improve accuracy.

Because the level set function is conducted solution on fixed Euler grid, therefore, it cannot always guarantee interface through the grid nodes. When the interface is located in an interval $[x_i, x_{i+1}] \times [y_i, y_{i+1}]$, ϕ satisfies $\phi_{ij}^0 > 0 > \phi_{i+1,j}^0, \phi_{ij}^0 > 0 > \phi_{i,j+1}^0$. Suppose Euler forward difference scheme is used to time discrete, and the one-order upwind difference scheme is used to space discrete. The relations of equations as flow after a time step:

$$\phi_{ij}^1 = \phi_{ij}^0 - \mathrm{sign}_\varepsilon(\phi_{ij})\Delta t\left[\sqrt{\left(\frac{\phi_{ij}^0 - \phi_{i+1,j}^0}{\Delta x}\right)^2 + \left(\frac{\phi_{ij}^0 - \phi_{i,j+1}^0}{\Delta y}\right)^2} - 1\right] \tag{3}$$

$$\phi_{i+1,j}^1 = \phi_{i+1,j}^0 - \mathrm{sign}_\varepsilon(\phi_{i+1,j})\Delta t\left[\sqrt{\left(\frac{\phi_{ij}^0 - \phi_{i+1,j}^0}{\Delta x}\right)^2 + \left(\frac{\phi_{ij}^0 - \phi_{i,j+1}^0}{\Delta y}\right)^2} - 1\right] \tag{4}$$

$$\phi_{i,j+1}^1 = \phi_{i,j+1}^0 - \mathrm{sign}_\varepsilon(\phi_{i,j+1})\Delta t\left[\sqrt{\left(\frac{\phi_{ij}^0 - \phi_{i+1,j}^0}{\Delta x}\right)^2 + \left(\frac{\phi_{ij}^0 - \phi_{i,j+1}^0}{\Delta y}\right)^2} - 1\right] \tag{5}$$

where

$$\mathrm{sign}_{\varepsilon}(\phi_{ij})=\frac{\phi_{ij}^{0}}{\sqrt{\phi_{ij}^{0}\phi_{ij}^{0}+\varepsilon_{ij}^{2}}},\quad \mathrm{sign}_{\varepsilon}(\phi_{i+1,j})=\frac{\phi_{i+1,j}^{0}}{\sqrt{\phi_{i+1,j}^{0}\phi_{i+1,j}^{0}+\varepsilon_{i+1,j}^{2}}},$$

$$\mathrm{sign}_{\varepsilon}(\phi_{i,j+1})=\frac{\phi_{i+1,j}^{0}}{\sqrt{\phi_{i,j+1}^{0}\phi_{i,j+1}^{0}+\varepsilon_{i+1,j}^{2}}}$$

By eq. (1)

$$0<sign_{\varepsilon}(\phi_{ij})\le 1,\quad -1\le sign_{\varepsilon}(\phi_{i+1,j})<0,\quad -1\le sign_{\varepsilon}(\phi_{i,j+1})<0 \tag{6}$$

Set $\phi_{ij}^{0}-\phi_{i+1,j}^{0}=m\Delta x$ and $\phi_{ij}^{0}-\phi_{i,j+1}^{0}=n\Delta y$, so $\left|\phi_{ij}^{0}\right|+\left|\phi_{i+1,j}^{0}\right|=m\Delta x$, and $\left|\phi_{ij}^{0}\right|+\left|\phi_{i,j+1}^{0}\right|=$ $=n\Delta x$, m, n > 0. Suppose level set function is linear distribution in the interval $[x_i,x_{i+1}]\times[y_i,y_{i+1}]$, that is, level set function is linear distribution along x- and y-direction. We can find out the location of the initial interface:

$$\begin{aligned} x_{\phi=0}^{0}&=x_i+\frac{\left|\phi_{ij}^{0}\right|}{\left|\phi_{ij}^{0}\right|+\left|\phi_{i+1,j}^{0}\right|}\Delta x=x_i+\frac{\phi_{ij}^{0}}{\phi_{ij}^{0}-\phi_{i+1,j}^{0}}\Delta x,\\ y_{\phi=0}^{0}&=y_j+\frac{\left|\phi_{ij}^{0}\right|}{\left|\phi_{ij}^{0}\right|+\left|\phi_{i,j+1}^{0}\right|}\Delta y=y_j+\frac{\phi_{ij}^{0}}{\phi_{ij}^{0}-\phi_{i,j+1}^{0}}\Delta y \end{aligned} \tag{7}$$

Using appropriate small time step to ensure $\phi_{ij}^{1}>0>\phi_{i+1,j}^{1}$ and $\phi_{ij}^{1}>0>\phi_{i,j+1}^{1}$, after one iteration, if:

$$\phi_{ij}^{1}-\phi_{i+1,j}^{1}=m'\Delta x,\quad \phi_{ij}^{1}-\phi_{i,j+1}^{1}=n'\Delta x, m',n'>0 \tag{8}$$

then the interface location is:

$$\begin{aligned} x_{\phi=0}^{1}&=x_i+\frac{\left|\phi_{ij}^{1}\right|}{\left|\phi_{ij}^{1}\right|+\left|\phi_{i+1,j}^{1}\right|}\Delta x=x_i+\frac{\phi_{ij}^{1}}{\phi_{ij}^{1}-\phi_{i+1,j}^{1}}\Delta x,\\ y_{\phi=0}^{1}&=y_j+\frac{\left|\phi_{ij}^{1}\right|}{\left|\phi_{ij}^{1}\right|+\left|\phi_{i,j+1}^{1}\right|}\Delta y=y_j+\frac{\phi_{ij}^{1}}{\phi_{ij}^{1}-\phi_{i,j+1}^{1}}\Delta y \end{aligned} \tag{9}$$

To keep the interface position should have:

$$x_{\phi=0}^{0}=x_{\phi=0}^{1} \tag{10}$$

then:

$$\frac{\phi_{ij}^{0}}{\phi_{i+1,j}^{0}}=\frac{\phi_{ij}^{1}-\phi_{ij}^{0}}{\phi_{i+1,j}^{1}-\phi_{i+1,j}^{0}},\quad \frac{\phi_{ij}^{0}}{\phi_{i,j+1}^{0}}=\frac{\phi_{ij}^{1}-\phi_{ij}^{0}}{\phi_{i,j+1}^{1}-\phi_{i,j+1}^{0}} \tag{11}$$

By type eq. (3), eq. (4), and eq. (5), so that:

$$\frac{\phi_{ij}^0}{\phi_{i+1,j}^0}=\frac{\text{sign}_\varepsilon(\phi_{ij})}{\text{sign}_\varepsilon(\phi_{i+1,j})},\quad \frac{\phi_{ij}^0}{\phi_{i,j+1}^0}=\frac{\text{sign}_\varepsilon(\phi_{ij})}{\text{sign}_\varepsilon(\phi_{i,j+1})} \tag{12}$$

that is:

$$\phi_{i+1,j}^0\phi_{i+1,j}^0+\varepsilon_{i+1,j}^2=\phi_{ij}^0\phi_{ij}^0+\varepsilon_{ij}^2,\quad \phi_{i,j+1}^0\phi_{i,j+1}^0+\varepsilon_{i,j+1}^2=\phi_{ij}^0\phi_{ij}^0+\varepsilon_{ij}^2 \tag{13}$$

It can be deduced value formula of smooth parameter from eq. (13):

$$\varepsilon_{ij}^2=\phi_{i+1,j}^0\phi_{i+1,j}^0=\left(\left|\phi_{i+1,j}^0-\phi_{i,j}^0\right|-\left|\phi_{ij}^0\right|\right)^2=$$

$$=\left[\left(\frac{\left|\phi_{i+1,j}^0-\phi_{i,j}^0\right|}{\Delta x}-\frac{\left|\phi_{ij}^0\right|}{\Delta x}\right)\Delta x\right]^2=\left(\left|\nabla\phi^0\right|_{ij}-\frac{\left|\phi_{ij}^0\right|}{\Delta x}\right)^2(\Delta x)^2 \tag{14}$$

In the same way there is:

$$\varepsilon_{ij}^2=\phi_{i,j+1}^0\phi_{i,j+1}^0=\left(\left|\phi_{i,j+1}^0-\phi_{i,j}^0\right|-\left|\phi_{ij}^0\right|\right)^2=$$

$$=\left[\left(\frac{\left|\phi_{i,j+1}^0-\phi_{i,j}^0\right|}{\Delta y}-\frac{\left|\phi_{ij}^0\right|}{\Delta y}\right)\Delta y\right]^2=\left(\left|\nabla\phi^0\right|_{ij}-\frac{\left|\phi_{ij}^0\right|}{\Delta y}\right)^2(\Delta y)^2 \tag{15}$$

The addition of eq. (14) and eq. (15):

$$\varepsilon_{ij}^2=\frac{1}{2}\left[\left|\nabla\phi^0\right|_{ij}-\left(\frac{1}{\Delta x}+\frac{1}{\Delta y}\right)\left|\phi_{ij}^0\right|\right]^2[(\Delta x)^2+(\Delta y)^2] \tag{16}$$

If taking $\Delta x = \Delta y$, eq. (16) becomes:

$$\varepsilon_{ij}^2=\left(\left|\nabla\phi^0\right|_{ij}-\frac{2}{\Delta x}\left|\phi_{ij}^0\right|\right)^2(\Delta x)^2 \tag{17}$$

It can be solved $\left|\nabla\phi^0\right|_{ij}$ by central difference scheme. Because $\varepsilon_{ij}^2\le\left|\phi_{ij}\right|^2$, so:

$$\frac{\sqrt{2}}{2}\le\left|\text{sign}_\varepsilon(\phi_{ij})\right|=\left|\frac{\phi_{ij}}{\sqrt{\phi_{ij}^2+\varepsilon_{ij}{}^2}}\right|\le 1 \tag{18}$$

Numerical example

Example 1. Single bubble problem

Consider a region [0,100] × [0, 40], assume that the density of bubbles is relatively small, and the Mach number of shock wave through single bubble is 1.22 [11-12], as shown in fig. 1. Single bubble center is located in (50, 10), the radius is 8. The location of shock wave is in $x = 80$, its spread direction to the left.

The non-dimensionalized initial conditions are:

$$x \ge 80:\ \rho = 1.3764,\ u = -0.394,\ v = 0.0,\ p = 1.5698;$$

$$x < 80:\ \rho = 1.0,\ u = 0.0,\ v = 0.0,\ p = 1.50;$$

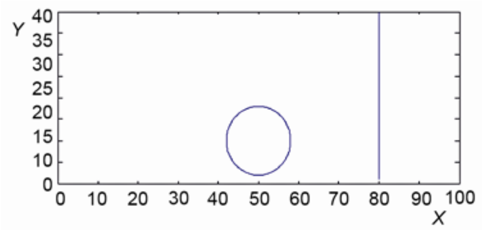

Figure 1. The calculation region of shock wave through single bubble problem (initial moment)

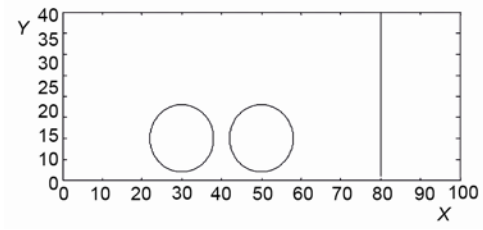

Figure 2. The calculation region of shock wave through two bubbles problem (initial moment)

$$(x - 50.0)^2 + (y - 10.0)^2 \leq 64.0: \rho = 1.138, u = 0.0, v = 0.0, p = 1.0, \gamma = 1.4.$$

Level set equation of the initial value is:

$$\phi_1(x, y, 0) = \sqrt{(x - 50.0)^2 + (y - 10.0)^2} - 8.0$$

Grid number: 400 × 100, space step: $\Delta x = \Delta y = 0,3$, CFL condition: CFL = 0.4. Utilizing the modified re-initialization level set method to capture interface, here are density contours, contour lines and isocontour of shock wave through single bubble at $t = 37.4$, $n = 1500$ (figs. 3, 4, and 5).

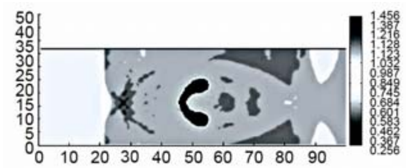

Figure 3. Density contours

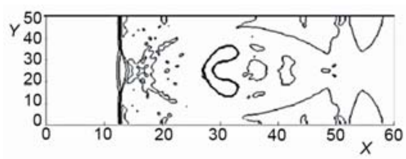

Figure 4. 15 contour lines

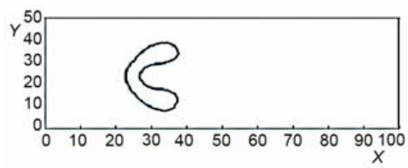

Figure 5. Isocontour map of level set function

As can be seen from figs. 2, 3, and 4, because of the bubble with large bubbles, low density and sound velocity, the shock Mach number is reduced, but the speed is faster than before when the shock wave into the bubble. The plane near to bubble occurs bifurcate by shock wave, and the gens rarefaction wave on the right side of bubble are grown. At the same time, we also found that the aberrations of bubble have taken place when shock waves in the air bubble on the left side of the interface transmission, and the right of the interface distortion is more apparent.

Example 2. Two bubbles problems

As shown in fig. 2, two bubbles centers are separately located in (50, 10) and (30, 10), and the non-dimensionalized initial conditions are same as the previous example.

Level set equations of the initial value are:

$$\phi_1(x, y, 0) = \sqrt{(x - 50.0)^2 + (y - 10.0)^2} - 8.0, \quad \phi_2(x, y, 0) = \sqrt{(x - 30.0)^2 + (y - 10.0)^2} - 8.0$$

Utilizing the modified re-initialization level set method to capture interface, here are density contours, contour lines and isocontour of shock wave through two bubbles at $t = 44.5$, $n = 2000$ (figs. 6, 7, and 8).

Figures 6, 7, and 8 show that the shock wave through two bubbles was more complicated than through single bubble, but the deformation situation of bubble is similar.

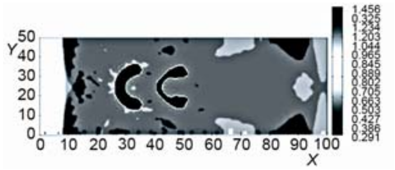

Figure 6. Density contours

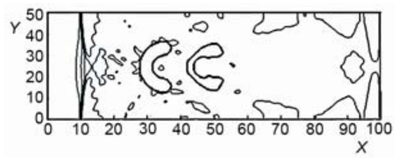

Figure 7. 15 contour lines

From the two numerical examples it is obvious that the effect of interface capture after shock wave through bubble is very well by modified re-initialized level set method. The study further demonstrates that the effectiveness and applicability of modified re-initialized level set method are verified, and the resolution is greatly enhanced.

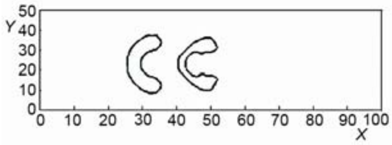

Figure 8. Isocontour map of level set function

Acknowledgments

This work was financially supported by the National Natural Science Foundation of China (10872085), the education department of Sichuan Province major project (11ZA026), the education department of Sichuan Province project (12ZB266), the education department of Sichuan Province university innovation team (13TD0001).

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Paper submitted: July 28, 2013
Paper revised: March 30, 2014
Paper accepted: July 12, 2014