FLOW AND HEAT TRANSFER OF THREE IMMISCIBLE FLUIDS IN THE PRESENCE OF UNIFORM MAGNETIC FIELD

by

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The magnetohydrodynamic flow of three immiscible fluids in a horizontal channel with isothermal walls in the presence of an applied magnetic field has been investigated. All three fluids are electrically conducting, while the channel plates are electrically insulated. The general equations that describe the discussed problem under the adopted assumptions are reduced to ordinary differential equations and closed-form solutions are obtained in three fluid regions of the channel. Separate solutions with appropriate boundary conditions for each fluid have been obtained and these solutions have been matched at the interface using suitable boundary conditions. The analytical results for various values of the Hartmann number, the ratio of fluid heights and thermal conductivities have been presented graphically to show their effect on the flow and heat transfer characteristics.

Keywords: *immiscible fluids, heat transfer, Hartmann number, magnetic field*

Introduction

The flow and heat transfer of electrically conducting fluids in channels and circular pipes under the effect of a transverse magnetic field occurs in magnetohydrodynamic (MHD) generators, pumps, accelerators, and flowmeters, and it has applications in nuclear reactors, filtration, geothermal systems and others. The interest in effects of outer magnetic field on heat-physical processes appeared in the early seventies of the last century. One of the first works in the field of mass and heat transfer in the presence of magnetic field was presented by Blum [1]. The flow and heat transfer of a viscous incompressible electrically conducting fluid between two infinite parallel insulating plates have been studied by many researchers [2-6].

The problem of convective MHD channel flow between two parallel plates subjected simultaneously to an axial temperature gradient and pressure gradient was studied numerically by Yang and Yu [7]. One decade ago, Bodosa and Borkakati [8] analyzed the problem of an unsteady 2-D flow of viscous incompressible and electrically conducting fluid between two parallel plates in the presence of a uniform transverse magnetic field, for the case of isothermal plates and one isothermal and other adiabatic. Ghosh [9] has obtained an analytical solution to the problem of steady and unsteady hydromagnetic flow of viscous incompressible electrically con-

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ducting fluid under the influence of constant and periodic pressure gradient in presence of inclined magnetic field. There are many other investigations, like Borkakati and Chakrabarty's [10] investigation of unsteady free convection MHD flow between two heated vertical parallel plates in induced magnetic field, or Aydin and Avci's [11] analytical investigation of laminar heat convection in a Couette-Poiseuille flow between two parallel plates with a simultaneous pressure gradient and an axial movement of the upper plate. Recently, Singa [12] has given an analytical solution to the problem of MHD free convective flow of an electrically conducting fluid between two heated parallel plates in the presence of an induced magnetic field.

All the mentioned studies refer to a single-fluid model. Most of the problems relating to the petroleum industry, geophysics, plasma physics, magneto-fluid dynamics and so forth involve multifluid flow situations. Because of that, Shail [13] studied Hartmann flow of conducting fluid and a non-conducting fluid layer contained in channel and his results predicted that an increase of the order of 30% can be achived in the flow rate for suitable ratios of heights and viscosities of the two fluids. Lohrasbi and Sahai [14] studied two-phase MHD flow and heat transfer in parallel plate channel with the fluid in one phase being conducting. There have been some experimental and analytical studies on hydrodynamic aspects of the two-fluid flow reported in the recent literature. Malashetty [15, 16] have studied the two fluid MHD flow and heat transfer in an inclined channel, and flow in an inclined channel containing porous and fluid layer. Recently, Umavathi [17, 18] have analyzed the MHD Poiseuille-Couette flow and heat transfer of two immiscible fluids between inclined parallel plates.

Due the importance of the two fluid flow models, in our previous paper [19] flow and heat transfer of two immiscible fluids in the presence of a uniform inclined magnetic field was investigated. While most of the previous studies mainly consider two-fluid flow, there is a model which discuss the combined effects of pressure gradient and electroosmosis for the three-fluid flow [20]. Multi-layer flows occur industrially in three main settings. Firstly, there are co-extrusion processes, where a product is made of more than one layer simultaneously. Secondly, there are film-coating processes, where a layer is applied to a fluid substrate. Thirdly, there are lubricated transport processes, where a lubricating fluid lies in a layer between the wall of a duct and the transported fluid [21]. The development of microfluidics platforms in recent years has led to an increase in the number of applications involving the flow of multiple immiscible layers of viscous electrolyte fluids [22]. Recent studies show that MHD flows can also be a viable option for transporting weakly conducting fluids in microscale systems, such as flow inside the microchannel networks of a lab-on-a-chip device [23, 24]. In microfludic devices, multiple fluids may be transported through a channel for various reasons. For example, an increase mobility of a fluid can be achieved by stratification of highly mobile fluid or mixing of two or more fluids in transit may be designed for heat and mass transfer applications. In that regard, magnetic field-driven micropumps are in increasing demand due to their long-term reliability, absence of moving parts, low power requirement, flow reversibility and mixing efficiency [25, 26].

The present work proposes a theoretical model of three electrically conductive fluid flow under the influence of uniform magnetic field.

Mathematical model

As already mentioned, the MHD flow of three immiscible fluids in a horizontal channel with isothermal walls in the presence of an applied magnetic field has been investigated. The fluids in the three regions have been assumed immiscible and incompressible and the flow has been steady, one dimensional, and fully developed. All three fluids have different viscosities μ_1, μ_2, and μ_3 and densities ρ_1, ρ_2, and ρ_3. The analytical solutions for velocities and temperature distribution have been obtained and computed for different values of the characteristic parameters. The physical model, shown in fig. 1, consists of two infinite parallel plates extending in the i and k-direction. Both infinite parallel plates are fixed. In the region I $h \le y \le 2\,h$ we have fluid of viscosity μ_1, electrical conductivity σ_1, thermal conductivity k_1, and specific heat capacity c_{p1}, in the region II $0 \le y \le h$, which has been filled by a layer of different fluid of viscosity μ_2, fluid properties are electrical conductivity σ_2, thermal conductivity k_2, and specific heat capacity c_{p2}, and in the last region III $-h \le y \le 0$, which has been filled by a layer of fluid of viscosity μ_3, properties are electrical conductivity σ_3, thermal conductivity k_3 and specific heat capacity c_{p3}.

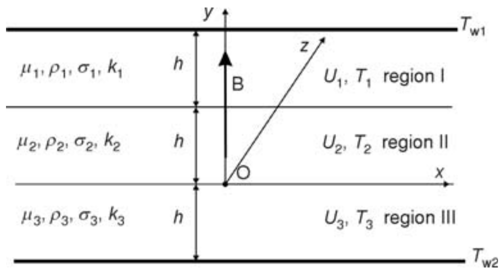

Figure 1. Physical model and co-ordinate system

A uniform magnetic field of the strength B is perpendicular to the fluid flow in $\vec{j}$ direction and we are considering 1-D flow in i -direction.

$$\vec{\mathrm{U}} = U\vec{\mathrm{i}} \tag{1}$$

$$\vec{\mathrm{B}} = B\vec{\mathrm{j}} \tag{2}$$

where $\vec{\mathrm{B}}$ is the magnetic field vector. The upper and lower plates have been kept at the two constant temperatures T_{w1} and T_{w2}, respectively, and the plates are electrically insulated. We are considering a stationary problem ($\partial f/\partial t = 0$). The described MHD three fluid flow problem is mathematically presented with a continuity equation:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0, \text{ because of } \frac{\partial V}{\partial y} = 0 \text{ we have } \Rightarrow U = U,(y), \tag{3}$$

– momentum equation (for all three fluid layers)

$$\mu_i \frac{d^2 U_i}{\mathrm{d}y^2} - \sigma_i B^2 U_i - \frac{\partial p}{\partial x} = 0, \quad i = 1, 2, 3 \tag{4}$$

– energy equation

$$\rho c_p \left(\frac{\partial T}{\partial t} + \vec{\mathrm{U}} \nabla T \right) = k \nabla T + \mu \phi + \frac{\vec{\mathrm{J}}^2}{\sigma} \tag{5}$$

– where current density vector $\vec{\mathrm{J}}$ is

$$\vec{\mathrm{J}} = \sigma (\vec{\mathrm{E}} + \vec{\mathrm{U}} \times \vec{\mathrm{B}}) \tag{6}$$

and $\vec{\mathrm{E}}$ is the vector of applied electric field, which is neglected in present study.

Energy equation now has the following form:

$$k \frac{\mathrm{d}^2 T}{\mathrm{d}y^2} + \left(\frac{\mathrm{d}U}{\mathrm{d}y} \right)^2 + \sigma B^2 U^2 = 0 \tag{7}$$

and, as we can see, the temperature changes only in j direction, because the temperature on the plates is constant in i direction.

Now, we can write the energy equation for all three fluid layers:

$$k_i \frac{\mathrm{d}^2 T_i}{\mathrm{d}y^2} \mu_i \left(\frac{\mathrm{d}U_i}{\mathrm{d}y} \right)^2 + \sigma_i B^2 U_i^2 = 0, \quad i = 1, 2, 3 \tag{8}$$

The fluid and thermal boundary conditions for this problem are represented by equations:

$$U_1(2h) = 0, \quad U_1(h) = U_2(h), \quad U_2(0) = U_3(0), \quad U_3(-h) = 0$$
$$\mu_1 \frac{\mathrm{d}U_1}{\mathrm{d}y} = \mu_2 \frac{\mathrm{d}U_2}{\mathrm{d}y} \quad \text{for} \quad y = h, \quad \mu_2 \frac{\mathrm{d}U_2}{\mathrm{d}y} = \mu_3 \frac{\mathrm{d}U_3}{\mathrm{d}y} \quad \text{for} \quad y = 0 \tag{9}$$

and

$$T_1(2h) = T_{\mathrm{W1}}, \quad T_1(h) = T_2(h), \quad T_2(0) = T_3(0), \quad T_3(-h) = T_{\mathrm{W2}}$$
$$k_1 \frac{\mathrm{d}T_1}{\mathrm{d}y} = k_2 \frac{\mathrm{d}T_2}{\mathrm{d}y} \quad \text{for} \quad y = h, \quad k_2 \frac{\mathrm{d}T_2}{\mathrm{d}y} = k_3 \frac{\mathrm{d}T_3}{\mathrm{d}y} \quad \text{for} \quad y = 0 \tag{10}$$

Now the following transformations have been used to transform previous equations to dimensionless form:

$$y^* = \frac{y}{h}, \quad U_i^*(y) = \frac{U_i}{\overline{U}_1}, \quad P_i = \frac{h^2}{\mu_i \overline{U}_1} \left(-\frac{\partial p}{\partial x} \right), \quad \theta_i = \frac{T_i - T_{\mathrm{W2}}}{T_{\mathrm{W1}} - T_{\mathrm{W2}}} \tag{11}$$

where $\overline{U}_1$ is the average velocity in channel and after transformations we obtain the following momentum equation:

$$\frac{\mathrm{d}^2 U_i^*}{\mathrm{d}y^{*2}} - \mathrm{Ha}_i^2 U_i^* + P_i = 0 \tag{12}$$

where Ha is the Hartmann number, defined as:

$$\mathrm{Ha}_i = Bh \sqrt{\frac{\sigma_i}{\mu_i}} \tag{13}$$

The general solution of eq. (12) is:

$$U_i^* = A_i \cosh(\mathrm{Ha}_i y^*) + B_i \sinh(\mathrm{Ha}_i y^*) + \frac{P_i}{\mathrm{Ha}_1^2} \tag{14}$$

Now we have to determine constants A_i and B_i using the boundary conditions in dimensionless form:

$$U_1^*(2) = 0, \quad U_1^*(1) = U_2^*(1), \quad U_2^*(0) = U_3^*(0), \quad U_3^*(-1) = 0$$
$$\alpha_1 \frac{\mathrm{d}U_1^*}{\mathrm{d}y^*} = \frac{\mathrm{d}U_2^*}{\mathrm{d}y^*} \quad \text{for} \quad y^* = 1, \quad \alpha_1 = \frac{\mu_1}{\mu_2}, \quad \alpha_2 \frac{\mathrm{d}U_2^*}{\mathrm{d}y^*} = \frac{\mathrm{d}U_3^*}{\mathrm{d}y^*} \quad \text{for} \quad y^* = 0, \quad \alpha_2 = \frac{\mu_2}{\mu_3} \tag{15}$$

The solution of transformed momentum equation, with boundary conditions has the following form:

$$U_i^* = A_i \cosh(\mathrm{Ha}_i y^*) + B_i \sinh(\mathrm{Ha}_i y^*) + C_i, \quad C_i = \frac{P_i}{\mathrm{Ha}_i^2} \tag{16}$$

where

$$A_1 = -\frac{P_1}{\mathrm{Ha}_1^2} \frac{1}{\cosh(2\mathrm{Ha}_1)} - (M_3 + M_1 N_2)\mathrm{tgh}(2\mathrm{Ha}_1) - (M_2 - M_1 N_1) B_2 \mathrm{tgh}(2\mathrm{Ha}_1) \tag{17}$$

$$B_1 = M_3 + M_1 N_2 + (M_2 - M_1 N_1) B_2 \tag{18}$$

$$A_2 = N_2 - N_1 B_2 \tag{19}$$

$$B_2 = \frac{N_2 \cosh(\mathrm{Ha}_3) - M_5}{\alpha_2 \dfrac{\mathrm{Ha}_2}{\mathrm{Ha}_3} \sinh(\mathrm{Ha}_3) + N_1 \cosh(\mathrm{Ha}_3)} \tag{20}$$

$$A_3 = \frac{P_2}{\mathrm{Ha}_2^2} - \frac{P_3}{\mathrm{Ha}_3^2} + N_2 - N_1 B_2 \tag{21}$$

$$B_3 = \alpha_2 \frac{\mathrm{Ha}_2}{\mathrm{Ha}_3} B_2 \tag{22}$$

and

$$M = [\sinh(\mathrm{Ha}_1) - \mathrm{tgh}(2\mathrm{Ha}_1)\cosh(\mathrm{Ha}_1)]^{-1} \tag{23}$$

$$M_1 = M \cosh(\mathrm{Ha}_2) \tag{24}$$

$$M_2 = M \sinh(\mathrm{Ha}_2) \tag{25}$$

$$M_3 = M\left[\frac{P_2}{\mathrm{Ha}_2^2} - \frac{P_1}{\mathrm{Ha}_1^2}\left[1 - \frac{\cosh(\mathrm{Ha}_1)}{\cosh(2\mathrm{Ha}_1)}\right]\right] \tag{26}$$

$$M_4 = \alpha_1 \mathrm{Ha}_1 [\cosh(\mathrm{Ha}_1) - \mathrm{tgh}(2\mathrm{Ha}_1)\sinh(\mathrm{Ha}_1)] \tag{27}$$

$$M_5 = \frac{P_3}{\mathrm{Ha}_3^2}[\cosh(\mathrm{Ha}_3) - 1] - \frac{P_2}{\mathrm{Ha}_2^2}\cosh(\mathrm{Ha}_3) \tag{28}$$

$$N = [M_1 M_4 - \mathrm{Ha}_2 \sinh(\mathrm{Ha}_2)]^{-1} \tag{29}$$

$$N_1 = [M_2 M_4 - \mathrm{Ha}_2 \cosh(\mathrm{Ha}_2)]N \tag{30}$$

$$N_2 = \left[\frac{\alpha_1 P_1}{\mathrm{Ha}_1} \frac{\sinh(\mathrm{Ha}_1)}{\cosh(2\mathrm{Ha}_1)} - M_3 M_4\right]N \tag{31}$$

Using the same transformations, we obtain the second energy equation in the following form:

$$\frac{\mathrm{d}^2\theta_i}{\mathrm{d}y^{*2}} + \mathrm{Pr}_i \mathrm{Ec}_i \left(\frac{\mathrm{d}U_i^*}{\mathrm{d}y^*}\right)^2 + \mathrm{Pr}_i\, \mathrm{Ec}_i \mathrm{Ha}_i^2 U_i^{*2} = 0 \tag{32}$$

where

– Prandtl number
$$\mathrm{Pr}_i = \frac{\mu_i c_{pi}}{k_i} \tag{33}$$

– Eckert number
$$\mathrm{Ec}_i = \frac{\overline{U}_1^2}{c_{pi}(T_{\mathrm{W}1} - T_{\mathrm{W}2})} \tag{34}$$

The general solution of eq. (32) is:

$$\theta_i = -\mathrm{Pr}_i\, \mathrm{Ec}_i \left\{\frac{1}{4}(A_i^2 + B_i^2)\cosh(2\mathrm{Ha}_i y^*) + \frac{1}{2} A_i B_i \sinh(2\mathrm{Ha}_i y^*) + \right.$$
$$\left. +2C_i[A_i \cosh(\mathrm{Ha}_i y^*) + B_i \sinh(\mathrm{Ha}_i y^*)] + \frac{1}{2} C_i^2 \mathrm{Ha}_i^2 y^{*2}\right\} + D_i y^* + E_i \tag{35}$$

The dimensionless boundary conditions that were used for the determination of the constants D_i and E_i are:

$$\theta_1(2)=1,\quad \theta_1(1)=\theta_2(1),\quad \theta_2(0)=\theta_3(0),\quad \theta_3(-1)=0$$
$$\frac{\mathrm{d}\theta_1}{\mathrm{d}y^*}=\beta_1\frac{\mathrm{d}\theta_2}{\mathrm{d}y^*}\quad \text{for}\quad y^*=1,\quad \beta_1=\frac{k_2}{k_1},\quad \frac{\mathrm{d}\theta_2}{\mathrm{d}y^*}=\beta_2\frac{\mathrm{d}\theta_3}{\mathrm{d}y^*}\quad \text{for}\quad y^*=0,\quad \beta_2=\frac{k_3}{k_2} \tag{36}$$

Using previous conditions, we have determined the constants D_i and E_i $(i = 1, 2, 3)$ in the form:

$$D_1=C_5+\beta_1C_6+\beta_1\beta_2D_3 \tag{37}$$

$$D_2=C_6+\beta_2D_3 \tag{38}$$

$$D_3=-\frac{C_2-C_1+C_3+C_4+C_5+C_6(1+\beta_1)}{\beta_2+\beta_1\beta_2+1} \tag{39}$$

$$E_1=C_1-2D_1 \tag{40}$$

$$E_2=C_3+C_4+D_3 \tag{41}$$

$$E_3=C_4+D_3 \tag{42}$$

and

$$C_1=1+\Pr_1\,\mathrm{Ec}_1\left\{\frac{1}{4}(A_1^2+B_1^2)\cosh(4\mathrm{Ha}_1)+\frac{1}{2}A_1B_1\sinh(4\mathrm{Ha}_1)+\right.$$
$$\left.+2C_1[A_1\cosh(2\mathrm{Ha}_1)+B_1\sinh(2\mathrm{Ha}_1)]+2C_1^2\mathrm{Ha}_1^2\right\} \tag{43}$$

$$C^*\ \Pr_1\,\mathrm{Ec}_1\left\{\frac{1}{4}(A_1^2+B_1^2)\cosh(2\mathrm{Ha}_1)+\frac{1}{2}A_1B_1\sinh(2\mathrm{Ha}_1)+\right.$$
$$\left.+2C_1[A_1\cosh(\mathrm{Ha}_1)+B_1\sinh(\mathrm{Ha}_1)]+\frac{1}{2}C_1^2\mathrm{Ha}_1^2\right\} \tag{44}$$

$$C^{**}=\Pr_2\,\mathrm{Ec}_2\left\{\frac{1}{4}(A_2^2+B_2^2)\cosh(2\mathrm{Ha}_2)+\frac{1}{2}A_2B_2\sinh(2\mathrm{Ha}_2)+\right.$$
$$\left.+2C_2[A_2\cosh(\mathrm{Ha}_2)+B_2\sinh(\mathrm{Ha}_2)]+\frac{1}{2}C_2^2\mathrm{Ha}_2^2\right\} \tag{45}$$

$$C_2=C^*-C^{**} \tag{46}$$

$$C^3=\Pr_2\,\mathrm{Ec}_2\left[\frac{1}{4}(A_2^2+B_2^2)+2C_2A_2\right]-\Pr_3\,\mathrm{Ec}_3\left[\frac{1}{4}(A_3^2+B_3^2)+2C_3A_3\right] \tag{47}$$

$$C_4=\Pr_3\,\mathrm{Ec}_3\left\{\frac{1}{4}(A_3^2+B_3^2)\cosh(2\mathrm{Ha}_3)-\frac{1}{2}A_3B_3\sinh(2\mathrm{Ha}_3)+\right.$$
$$\left.+2C_3[A_3\cosh(\mathrm{Ha}_3)-B_3\sinh(\mathrm{Ha}_3)]+\frac{1}{2}C_3^2\mathrm{Ha}_3^2\right\} \tag{48}$$

$$Q^*=\Pr_1\,\mathrm{Ec}_1\mathrm{Ha}_1\left\{\frac{1}{2}(A_1^2+B_1^2)\sinh(2\mathrm{Ha}_1)+A_1B_1\cosh(2\mathrm{Ha}_1)+\right.$$
$$\left.+2C_1[A_1\sinh(\mathrm{Ha}_1)+B_1\cosh(\mathrm{Ha}_1)]+C_1^2\mathrm{Ha}_1\right\} \tag{49}$$

$$Q^{**}=\beta_1\Pr_2\,\mathrm{Ec}_2\mathrm{Ha}_2\left\{\frac{1}{2}(A_2^2+B_2^2)\sinh(2\mathrm{Ha}_2)+A_2B_2\cosh(2\mathrm{Ha}_2)+\right.$$
$$\left.+2C_2[A_2\mathrm{sihn}(\mathrm{Ha}_2)+B_2\cosh(\mathrm{Ha}_2)]+C_2^2\mathrm{Ha}_2\right\} \tag{50}$$

$$C_5=Q^*-Q^{**} \tag{51}$$

$$C_6 = \mathrm{Pr}_2\,\mathrm{Ec}_2\,\mathrm{Ha}_2\,(A_2 B_2 + 2C_2 B_2) - \beta_2\,\mathrm{Pr}_3\,\mathrm{Ec}_3\,\mathrm{Ha}_3(A_3 B_3 + 2C_3 B_3) \tag{52}$$

Results and discussion

In the previous section, we have defined the mathematical model for flow and heat transfer of three immiscible fluids in the presence of uniform magnetic field.

Using this model, the graphs of velocity and temperature are generated for different Hartmann numbers and for different ratios of fluid viscosity μ and thermal conductivity k.

Figures 2 and 3 illustrate the effect of the Hartmann number on velocity and temperature profiles. The influence of the Hartmann number on the velocity profile was more pronounced in the channel region III than in the region II, and much more than in the region I. The region III contains the fluid with the greatest electrical conductivity. It was found that for large values of Hartmann number, flow can be almost completely stopped in the region III, while in the region I, velocity decrease is significant.

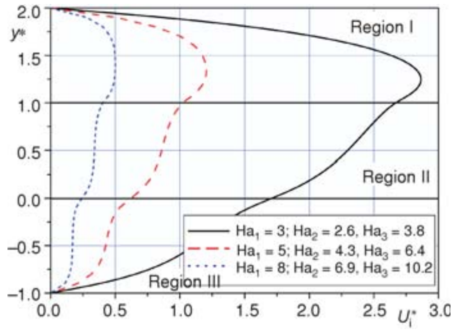

Figure 2. Velocity profiles for different values of Hartmann numbers

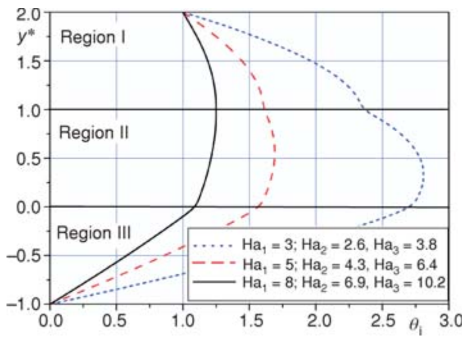

Figure 3. Temperature profiles for different values of Hartmann numbers

The effect of increasing the Hartmann number on temperature profiles in all three channel regions was in equalizing the fluid temperatures.

The effect of the ratio of fluids viscosities in regions I, II, and III on the velocity profiles is shown in fig. 4 to 7. In the case of parameter α_1 alteration, the change in velocity is significant in all three fluid regions, while in the case of parameter α_2 velocity significantly changes only in the region III, while in other two regions remains almost constant. These results are given for the same value of Lorentz force intensity (Hartmann number is constant) for all three fluids, and it can be concluded that the dominant effect of changes in fluid viscosity occurs in region III, while this influence in other two regions is much less pronounced.

Figures 5 and 7 also show the effect of the ratio of viscosity of fluids in regions I, II, and III to temperature alteration. In first considered case a significant change in temperature is noticed in all three fluid regions. Temperature rise in the middle of the channel is a consequence of viscous dissipation and large velocity gradients and constant Joule heating effect. The total temperature increases due to mutual effects of fluids at the interface.

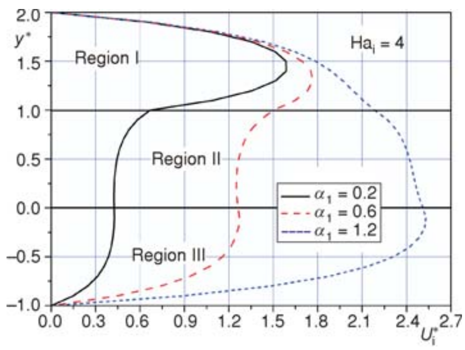

Figure 4. Velocity profiles for different ratios of fluids viscosities $\alpha_1 = \mu_1/\mu_2$

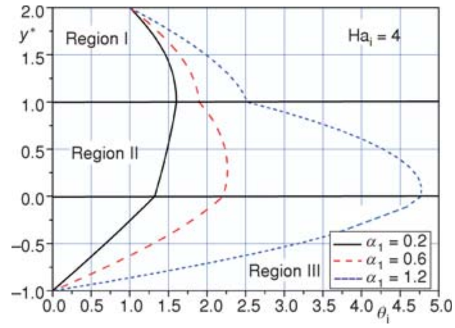

Figure 5. Temperature profiles for different ratios of fluids viscosities $\alpha_1 = \mu_1/\mu_2$

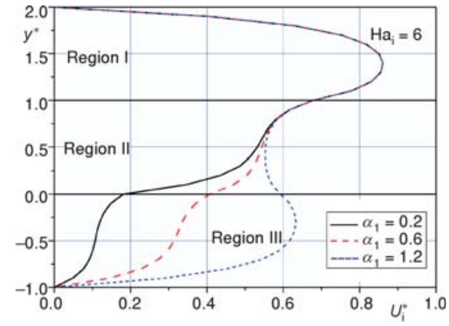

Figure 6. Velocity profiles for different ratios of fluids viscosities $\alpha_2 = \mu_2/\mu_3$

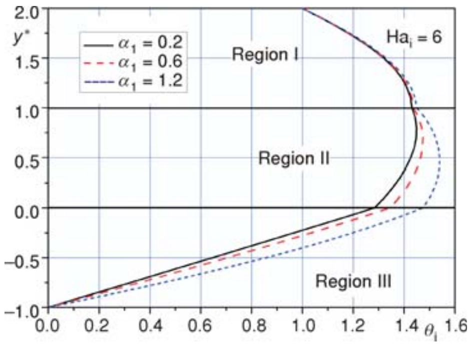

Figure 7. Temperature profiles for different ratios of fluids viscosities $\alpha_2 = \mu_2/\mu_3$

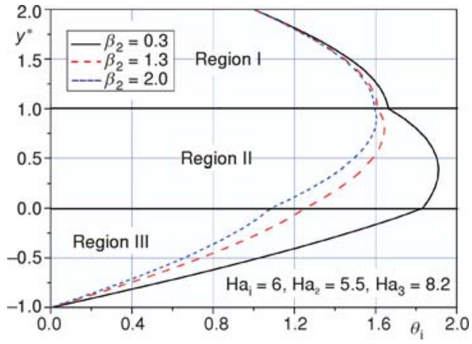

Figure 8. Temperature profiles for different ratios of thermal conductivities $\beta_2 = k_3/k_2$

In second considered case (change of α_2) there is no significant change in temperature. Smaller temperature changes occur in the regions II and III, while in the region I, the temperature remain almost constant.

Figures 8 and 9 shows the temperature distribution over the channel height for different values of the fluids thermal conductivities ratios β_1 and β_2. Decrease of thermal conductivities ratios cause an increase of dimensionless temperature in all three fluid regions. In the case when the parameter β_1 significantly increases the conduction plays dominant role in heat transfer. In the case of increasing the parameter β_2 viscous dissipation and Joule heating effects are still present (fig. 8).

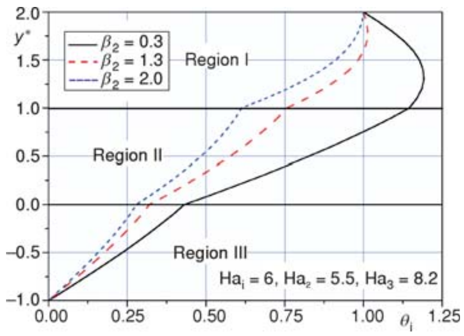

Figure 9. Temperature profiles for different ratios of thermal conductivities $\beta_2 = k_2/k_1$

Influence of Eckert and Prandtl number can be displayed together taking their product (fig. 10). The product of these two numbers is usually called Brinkman number, and it represents the ratio of the kinetic energy dissipated in the fluid and the conduction of heat in fluid or from it.

When the product of EcPr is significantly lower than 1, the energy dissipation can be neglected in comparison with the heat conduction. As this product increases the energy dissipation becomes an important parameter in the process of heat transfer and plays an important role in the temperature distribution in the fluid stream and the total heat transfer.

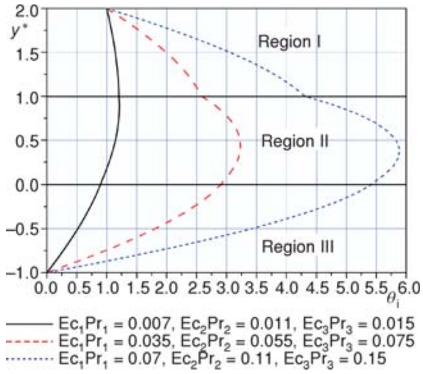

Figure 10. Velocity profiles for different values of Eckert and Prandtl number

Conclusions

The problem of MHD flow and heat transfer of three immiscible fluids between parallel plates in the presence of an applied magnetic field was investigated analytically. All three fluids were assumed Newtonian and electrically conducting. Closed form solutions for dimensionless velocity and temperature of each fluid were obtained taking into consideration suitable interface matching conditions and boundary conditions. The results were numerically evaluated and presented graphically for three fluids. Only the part of the results is presented for various values of Hartmann number and ratios of viscosities and thermal conductivities. The obtained results show that the control of flow and heat transfer for observed case can be realized by changing the magnetic field intensity and defined ratios of fluid properties.

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Nomenclature

$\vec{B}$	–	magnetic field vector, [T]
B_0	–	strength of applied magnetic field, [T]
c_p	–	specific heat capacity, [Jkg^{-1}K^{-1}]
$\vec{E}$	–	electric field vector, [Vm^{-1}]
$\mathrm{Ha_i}$	–	Hartmann number in region *i*
h_i	–	region *i* height, [m]
J	–	current density vector, [Am^{-2}]
k_i	–	thermal conductivity of fluid *i*, [WK^{-1}m^{-1}]
p	–	pressure, [Pa]
T	–	temperature, [K]
U_i	–	fluid velocity in region *i* , [ms^{-1}]
X	–	longitudinal co-ordinate, [m]
Y	–	transversal co-ordinate, [m]

Greek symbols

α	–	viscosities ratio of fluids
β	–	ratio of thermal conductivities
θ_i	–	dimensionless temperature in region *i*
μ_i	–	dynamic viscosity in region *i* , [kgm^{-1}s^{-1}]
ρ_i	–	density of fluid in region *i* , [kgm^{-3}]
σ_i	–	electrical conductivity region , [Sm^{-1}]
$\varnothing$	–	dissipative function

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