

## EQUILIBRIUM ACOUSTIC VELOCITY IN VAPOR-LIQUID MIXTURE IN LAYER OF SPHERICAL PARTICLES

by

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*Theoretical and experimental study of acoustic velocity in vapor-liquid mixture that contained close-packed layer of spherical particles was performed with respect to non-stationary heat transfer between mixture and particles in compression half-wave. Theoretical model allows to explain the decay of equilibrium acoustic velocity compared to its adiabatic value in case of increasing of void fraction in the mixture. Calculated results are in agreement with experimental data, obtained in vertical channel with vapor-liquid filtration with close-packed layer of spherical particles of borosilicate glass, steel, and lead.*

Key words: *vapor-liquid mixture, layer of spherical particles, the equilibrium acoustic velocity, non-stationary heat transfer*

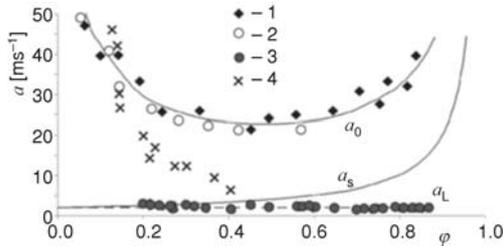
### Introduction

Two-phase vapor-liquid media are used as working substance in different types of apparatus in energy, chemical, cryogenic, and other areas of engineering. This explains interest to investigate problems of heat and mass transfer and hydrodynamics, especially wave dynamics in vapor- and gas-liquid systems [1-3]. Processes of interphase transfer increase dissipative properties of medium and inertial properties of gas inclusions leading to acoustic velocity dispersion. According to theory in the area of limited low frequencies of oscillations, approaching to equilibrium thermodynamic equilibrium acoustic velocity should occur. Its value for water with bubbles in case of atmospheric pressure and low concentration of bubbles has order about ~1 m/s. Also, experimental results [4-7] show that heat transfer has a negligibly small influence on acoustic velocity of vapor-liquid medium. For more complex case of vapor-gas mixture with solid particles and drops interphase impulse, mass and heat transfer was described in [8, 9]. Area of low frequency oscillations was allocated, where phase transitions lead to significant decrease of acoustic velocity.

New experimental results were presented in [10, 11], concerning low amplitude pressure perturbation propagation in low velocity vapor-liquid flow in cylindrical channel through

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**Figure 1. The acoustic perturbations speed: in the air-water medium ( $P_0 = 0.1$  MPa) with balls sized 8 mm (1), without balls (2), in the vapor-liquid medium with the same balls ( $P_0 = 0.2$  MPa) at the characteristic entrance perturbation frequency of 1.5-2.5 Hz (3), and 9-16 Hz (4); [11]**

layer of closed-packed spherical particles, fig. 1. Measured values of low frequency perturbations velocity at pressures  $P_0 = 0.2$  MPa and  $P_0 = 0.6$  MPa when  $x \rightarrow 0$  was comparable with equilibrium Landau acoustic velocity. Its theoretical values are  $a_L = 2.03$  m/s and  $a_L = 5.3$  m/s. Also, it was shown that with increasing of void fraction experimental data sufficiently deviate from theoretical curve for equilibrium acoustic velocity  $a_s$ , continuing to preserve its quantity around value  $a_L$ . In previous experiments [10] was shown that velocity at low frequency pressure perturbations can be lower than threshold value, defined by  $a_L$ . In [12] these experimental results are explained by influence of non-stationary heat transfer with stationary particles in the perturbation wave. Solution of heat transfer problem in solid body was used in case of changing boundary conditions at the particle surface. In this work such approach was developed and peculiarities of heat transfer, particle size, and material influence were examined according to influence to velocity of low frequency pressure perturbations in system "vapor-liquid-particles". Results of calculations were compared with experimental study.

### Thermodynamic equations

The average density of the vapor-liquid mixture  $\rho_{TP}$  is determined as:

$$\frac{1}{\rho_{TP}} = \frac{1}{\rho'} + x \left( \frac{1}{\rho''} - \frac{1}{\rho'} \right) \quad (1)$$

In the particular case, where the phase transitions are absent and  $x = \text{const}$ , for example, in an air-water mixture, calculating derivative  $(\partial P / \partial \rho)$  gives thermodynamically frozen speed value  $a_0$  (see fig. 1). Another case arises if we take  $\rho' = \text{const}$  and  $\rho'' = \text{const}$ , and assume that  $\rho_{TR}$  depends only on  $x$ .

Differentiation (1) with respect to  $P$  assuming that the mixture density depends only on changes of  $x$  leads to the expression:

$$\frac{\partial \rho_{TP}}{\partial P} = -\rho_{TP}^2 \frac{1}{\rho'' - \rho'} \frac{\partial x}{\partial P} \quad (2)$$

The problem is reduced to the calculation of the derivative  $(\partial x / \partial P)$ . For this reason following the technique described in [2] we write the differential entropy of a mixture as follows:

$$ds = \left( \frac{\partial s}{\partial P} \right)_x dT + \left( \frac{\partial s}{\partial x} \right)_T dx \quad (3)$$

which reduces to:

$$ds = \frac{c^*}{T} dT + \frac{L}{T} dx \quad (4)$$

where  $c^* = c''x + c'(1-x)$  is the amount of heat required to raise the temperature of 1 kg of mixture by 1 K while maintaining constant dryness  $x$ .

For adiabatic process the differential entropy on the left side of expression (4) is zero. Then the following transformations [2] will lead to well-known expression for speed of sound in thermodynamic equilibrium:

$$\left(\frac{\partial P}{\partial \rho}\right)_s = \frac{L^2}{\rho_{TP}^2 c^* T \left(\frac{1}{\rho''} - \frac{1}{\rho'}\right)^2} \quad (5)$$

Assuming that at low pressures  $1/\rho'' \gg 1/\rho'$ , we get the simplified expression:

$$a_s = \frac{L\rho''}{\rho_{TP} \sqrt{c^* T}} \quad (6)$$

If propagation of the pressure wave in two-phase environment is accompanied by heat exchange with the environment, then the left part of the expression (4) is not equal to zero, and this fact causes a displacement of the speed of sound from the values determined by the expression (6). To such a case refers propagation of low-frequency pressure disturbances in vapor-liquid mixture, which occupies a volume of porous space in close-packed layer of solid particles.

Movement to thermodynamic equilibrium in the compression zone is accompanied by the increase of the temperature of the mixture in accordance with the Clausius-Clapeyron equation, thereby creating a temperature difference between the vapor-liquid mixture and are solid particles within. At the same time, a part of the heat generated by condensation of the supercooled vapor, which is used to achieve the equilibrium temperature, is transmitted to the heat-conducting particles. Thus, the entropy of the vapor-liquid mixture decreases on the positive edge of the pressure, and the eq. (4) should be considered at  $ds < 0$ . Using the notation  $ds = dQ/T$ , we write:

$$-\frac{dQ}{T} = \frac{c^*}{T} dT + \frac{L}{T} dx \quad (7)$$

where  $Q$  describes the heat transmitted by 1 kg of liquid-vapor mixture to the solid filling in the process of unsteady heat transfer in the pressure front.

Let us represent  $dQ$  as:

$$dQ = c_{pr} dT \quad (8)$$

The parameter  $c_{pr}$  represents the effective heat capacity of the non-stationary process, describing the correlation between changes in the temperature of the mixture  $dT$  and the lost heat amount  $dQ$ . Thus,  $c_{pr}$  is the amount of heat, transmitted to the solid filling at non-stationary temperature increase by 1 K in 1 kg of the liquid-vapor mixture.

By substituting eq. (8) in eq. (7) we get:

$$0 = \frac{c^* + c_{pr}}{T} dT + \frac{L}{T} dx \quad (9)$$

Subsequent transformations using the eq. (9) lead to a modification of the expression of the velocity of sound in the equilibrium state in a vapor-liquid mixture under non-adiabatic conditions:

$$\tilde{a} = \frac{L\rho''}{\rho_{TP} \sqrt{\tilde{c}^* T}} \quad (10)$$

which differs from eq. (6) since it takes into account the additional heat capacity of the non-stationary process  $c_{pr}$ .

In the expression (10)

$$\tilde{c}^* = c^* + c_{pr} = c''x + c'(1-x) + c_{pr} \quad (11)$$

### Thermodynamic process

Let us consider a simplified heat transfer process between the two-phase mixture and filling of the close-packed spherical particles within. Heat of the vapor condensation is allocated on the interphase boundary between solid particles and liquid. Since the liquid layer on the surface of the particles create the additional thermal resistance, it is assumed that passing first pressure front the main amount of heat is transferred to the particles through the contact area with vapor. This area is determined by parameters of porous structure and vapor content of the mixture.

Let us  $F_x = F_0x$  represents the surface of the particles contacting vapor with volume  $v''x$ , where, in case of particle spherical shape:

$$F_0 = \frac{3v''(1-m)}{r_p m} \quad (12)$$

Heat flux density  $q_0(\tau)$  through the heat exchanging surface, for the total heat flux is:

$$Q(\tau) = F_0xq_0(\tau) \quad (13)$$

Specific heat flux  $q_0(\tau)$  on the surface of a spherical particle is defined according to the Fourier's law:

$$q_0(\tau)_{r=r_p} = -\lambda_p \left[ \frac{\partial T_p(\tau, r)}{\partial r} \right]_{r=r_p} \quad (14)$$

and can be calculated by solving equation of the transient heat conduction for a ball:

$$\rho_p c_p \frac{\partial T_p}{\partial \tau} = \nabla \cdot (\lambda_p \nabla T_p) \quad (15)$$

with the boundary conditions of the first type.

It is assumed that the changing temperature on the surface of the particle follows the equilibrium temperature of the vapor-liquid mixture, which depends on the known pressure profile  $P(\tau)$  in the compression front following Clausius-Clapeyron equation:

$$dT = \frac{T}{L} (v'' - v') dP \quad (16)$$

where the initial temperature is equal to the temperature of the mixture.

Effective heat capacity  $c_{pr}$  introduced in eq. (8), is a function of the process and changes over time. To simplify, when considering small disturbances let us move from the instantaneous values  $\bar{c}_{pr}$  to the averaged value  $\bar{c}_{pr}$  in the time period, equal to the duration of the compression front  $\tau_c$ . Then  $\bar{c}_{pr}$  is determined by the ratio of the amount of heat, that is transferred from 1 kg of the mixture to particles in time period  $\tau_c$  due to the increment of the equilibrium temperature of the mixture  $\Delta T_{TP}$  in the same period of time:

$$\bar{c}_{pr} = \frac{F_0x}{\Delta T_{TP}} \int_0^{\tau_c} q_0(\psi) d\psi \quad (17)$$

Using estimated relation (17) for  $\tilde{c}^*$ , expression (11) takes the following form:

$$\tilde{c}^* = c''x + c'(1-x) + \bar{c}_{pr} \quad (18)$$

From the expressions (10), (17), and (18) follows, that the heat transfer in the zone of the compression wave front between the two-phase mixture and solid particles actually leads to the appearance of the additional more effective heat capacity of the process  $c_{pr}$ , that causes the decrease of the equilibrium speed of sound in vapor-liquid mixture in comparison to the adiabatic case. The shift of the equilibrium speed in the presence of the external heat transfer process depends on its intensity and on the parameters of the solid fillings. Thus, the analysis shows the two-fold influence on  $\tilde{a}$  by the value of mass vapor content  $x$ . On one hand, the increase of the vapor content leads to the decrease of the density of the mixture  $\rho_{TP}$  in the denominator of the expression (10). On the other hand, increasing the surface of the heat exchange  $F_0x$  gives higher values of integral heat capacity  $\tilde{c}^*$ , which is also located in denominator (10). Therefore, the dependence  $\tilde{a}(x)$  in some cases may be non-monotonic. One of the important characteristics of the sound velocity is its dependence on the frequency of the signal. Characteristic frequency  $f$  determines the duration of the compression zone in the pressure wave. Increasing the frequency  $f$  decreases the interval of integration in the expression (17) and, therefore, the contribution of  $\tilde{c}_{pr}$  in total heat capacity  $\tilde{c}^*$  in the expression (18). At high frequencies, the contribution of the external heat exchange in compressibility of a vapor-liquid mixture decreases and the speed of sound is close to its adiabatic value. This fact is evident at fig. 1 if one follows trajectory of the experimental data (dots 4). At the same time in the range of low frequencies the contribution  $\tilde{c}_{pr}$  to the value of total heat capacity  $\tilde{c}^*$  can be significant, leading to substantial decrease of speed  $\tilde{a}$  compared to the equilibrium speed of sound in adiabatic conditions. Thus, we can talk about presence of the heat relaxation for the equilibrium velocity of sound in the vapor-liquid mixture in adiabatic conditions.

### Stationary mixture

When solving the 3-D heat transfer problem for solid particles, surrounded by vapor-liquid mixture, uncertainty of the choice of boundary conditions occurs for eq. (15), due to the uncertainty of phase distribution on the surface of the particle.

Based on the adopted assumption about primary diffusion of condensation heat to the particle through the interphase boundary “vapor-particle”, small changes of the wetted particle surface in time interval  $\tau_c$  are negligibly small, thus can be ignored. Thus, the following conditions are considered: temperature of wetted (“heat-insulated”) surface of the particle is assumed to be constant; the change in temperature of unwetted surface is defined by eq. (16). Figure 2 shows alternative boundary conditions with different distribution of “absorbent” and “isolated” spherical particle surface portions: a homogeneous – (a) and heterogeneous – (b, c, d). Cases considered correspond to the model and according to the possible boundary conditions of the

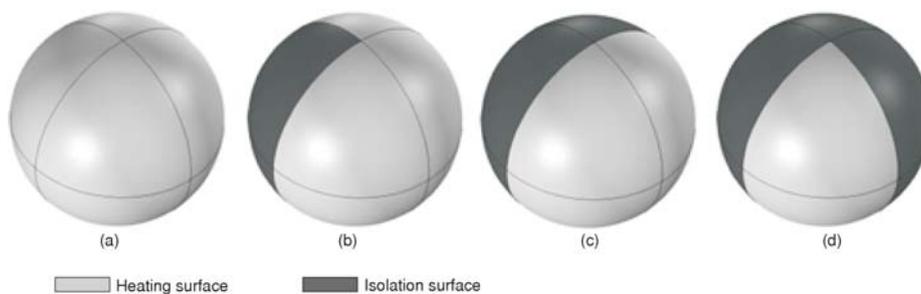


Figure 2. Distribution schemes of heat transfer surface of spherical particle; (a) 100% uniform distribution, (b) 75%, (c) 50%, (d) 25%, heterogeneous distribution

spatial distribution of phases. Increasing the area of “isolated” part of the ball corresponds to a decrease of vapor content  $\varphi$ .

Numerical calculations were performed with regard to backfill spherical particles of borosilicate glass and steel with a diameter of 2 mm and 4 mm, and also lead particles with diameter of 8 mm. The pressures of the vapor-liquid mixture in the pore space of the filling were 0.2 and 0.6 MPa.

Pressure front of disturbances, created in the vapor-liquid environment can be described by the first half-period of the harmonic function:

$$P(\tau) = P_0 + A_p \left[ 1 + \sin \left( 2\pi f\tau - \frac{\pi}{2} \right) \right] \quad (19)$$

where  $A_p = 0.05P_0$  is the amplitude of the sine wave signal and  $f$  – the oscillation frequency.

Heat transfer problem is solved using the computing package Comsol Multiphysics. Results of calculations of the specific heat amount, which is transferred to the solid particles in time period  $\tau_c$  at  $f = 1$  Hz, are shown in the tab. 1. Results clearly show the significant impact of the distribution of boundary conditions on the surface of the particle, as well as the influence of material and size of spherical particles on the intensity of the heat absorption. Small increase of the numeric values for  $P_0 = 0.6$  MPa can be explained by greater value of change of the equilibrium temperature in the mixture at the same relative disturbances of pressure.

**Table 1. Calculated quantity of heat supplied through the surface unit of particles with diameter  $d$  during the time  $\tau_c$**

Fraction of non-isolated part of the surface involved in heat transfer process, fig. 2	$\int_0^{\tau_c} q_0 d\psi$ [Jm <sup>-2</sup> ]				
	Material of sphere	$P_0 = 0.2$ MPa, $f = 1$ Hz		$P_0 = 0.6$ MPa, $f = 1$ Hz	
		$d = 2$ mm	$d = 4$ mm	$d = 2$ mm	$d = 4$ mm
100%, (a)	steel	4167.7	7402.3	5191.3	9252.1
	glass	1457.5	1872.9	1803.3	2317.3
	lead	1518.9	3076.1	1879.2	3805.9
75%, (b)	steel	5190.3	8569.2	6471.1	10735.0
	glass	1628.0	1951.0	2014.2	2413.9
	lead	2001.1	3864.7	2475.8	4781.6
50%, (c)	steel	6807.1	9691.2	8507.6	12179.2
	glass	1763.9	1998.8	2182.4	2473.0
	lead	2885.5	5189.6	3570.1	6420.8
25%, (d)	steel	10699.3	12032.4	13439.0	15181.4
	glass	2069.2	2121.7	2560.1	2625.1
	lead	5312.3	8654.5	6572.6	10707.7

Calculation of the integral in the right side of expression (17) allows to find  $c_{pr}$ , which directly affects the value of equilibrium speed of sound in the vapor-liquid mixture. Besides the pressure dependent specific surface  $F_0$  [m<sup>2</sup>kg<sup>-1</sup>], and vapor content  $x$  act, also, as correcting factors.

Table 2 shows values of the effective heat capacity of the heat exchange process in case of filling made from steel balls with diameter of 2 mm at two pressures. The dependence of the mass vapor content  $x$  on the volume  $\varphi$ , in the absence of the relative phase sliding, is determined by the expressions:

$$\frac{1}{\nu_{TP}} = \rho_{TP} = \rho'(1 - \varphi) + \rho''\varphi, \quad x = \frac{\nu_{TP} - \nu'}{\nu'' - \nu'} \quad (20)$$

Fast rise of  $c_{pr}$  is observed during the increase of vapor content in the mixture. At the same time, the value of  $c_{pr}$  becomes not only comparable with the effective heat capacity of the vapor-liquid mixture, but also, starting with the certain vapor content value, exceeds it manifold. Thus, the heat exchange with the filling particles in the pressure front can have significant influence on the value of the equilibrium speed of sound in the vapor-liquid environment.

**Table 2. Effective heat capacity of non-stationary heat transfer  $\bar{c}_{pr}$  (steel spheres wit 2 mm in diameter), [Jkg<sup>-1</sup>K<sup>-1</sup>]**

$\varphi$	$P_0 = 0.2 \text{ MPa}, f=1 \text{ Hz}$		$P_0 = 0.6 \text{ MPa}, f= 1 \text{ Hz}$	
	$x$	$\bar{c}_{pr}$	$x$	$\bar{c}_{pr}$
0.1	0.000133	825.3	0.000387	862.1
0.2	0.000299	1856.7	0.000871	1938.8
0.3	0.000513	3182.3	0.001493	3321.7
0.4	0.000798	4948.8	0.002320	5162.8
0.5	0.001196	7420.2	0.003476	7735.2
0.6	0.001793	11123.7	0.005204	11582.6
0.7	0.002786	17286.3	0.008072	17965.5
0.8	0.004766	29574.8	0.013759	30621.4

The results of calculations of the equilibrium speed of sound in the vapor-liquid mixture taking into account the non-adiabatic nature of the process in the steel and glass particle fillings are presented in fig. 3. The calculations are made at  $A_p = 0.05, P_0 = 0.6 \text{ MPa}, f = 1, 2, 4 \text{ Hz}$ . Heat energy dispersion in the heat-conducting particles in the phase of mixture compression in the pressure wave led to the significant decrease of the equilibrium speed. As was mentioned before, the magnitude of the speed decrease is influenced by the filling material and particles. The nature of the non-adiabatic speed of sound dependence on mixture void fraction  $\varphi$  in close-packed steel spherical particles looks more flat, than in case when the glass particles are used, discovering the slight excess over  $a_L$  in the small and medium volumetric vapor content areas.

The calculations performed also indicate that equilibrium speed in a system concerned depends on the pressure oscillation frequency, *i. e.* the heat dispersion of equilibrium speed of sound in non-adiabatic conditions is discovered. With the increase of the pressure oscillation frequency the duration of the non-stationary heat exchange during the compression of mixture is reduced, that leads deviation of the equilibrium speed of sound calculated in the adiabatic conditions.

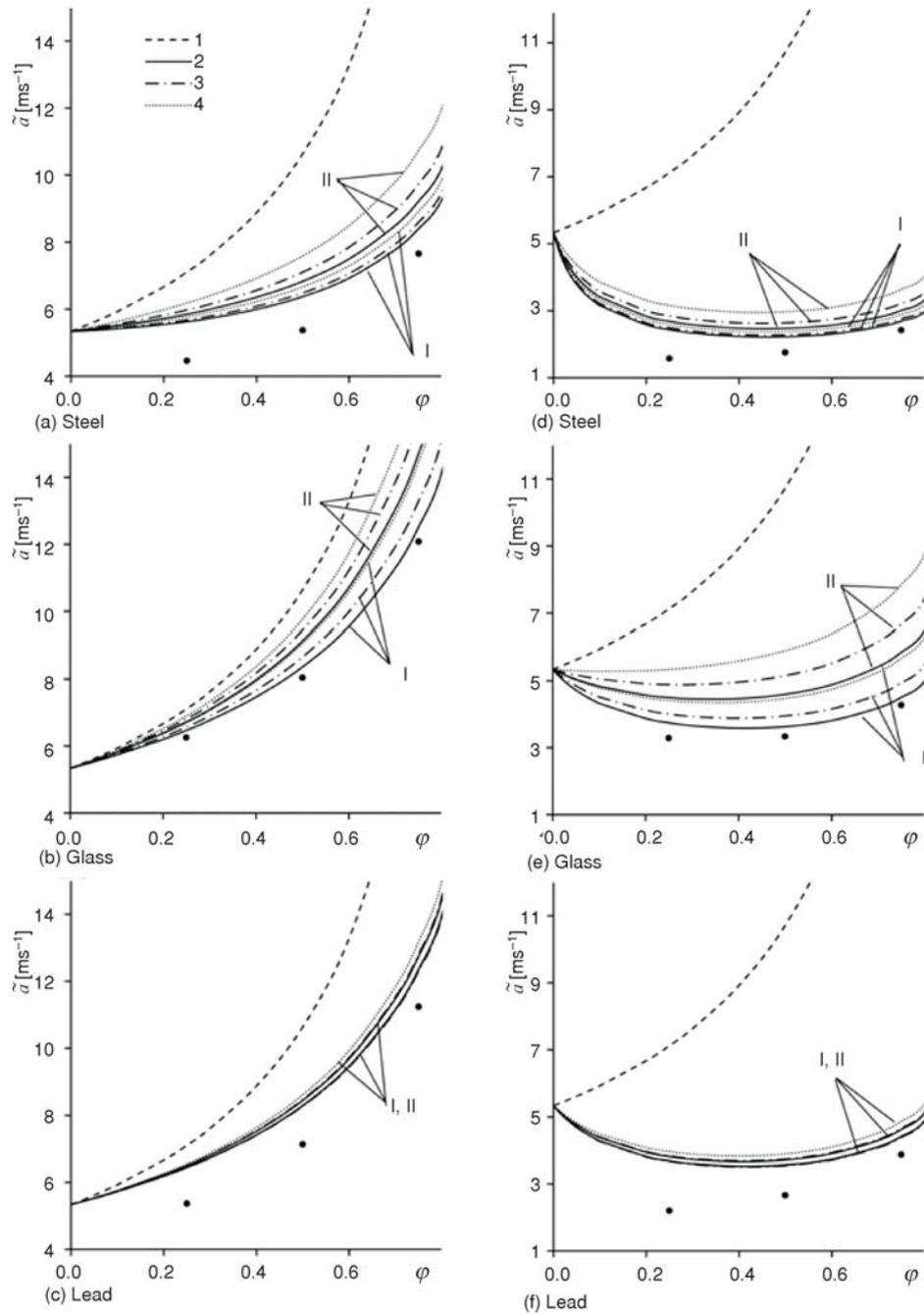


Figure 3. Equilibrium acoustic velocity of stationary (a, b, c) and moving with sleeping phase (d, e, f) of vapor-liquid mixture in granular layer. Pressure of the mixture  $P_0 = 0.6$  MPa. Without granular layer (1), with granular layer of particles with diameter 2 mm (I), 4 mm (II),  $f = 1(2), 2(3), 4(4)$  Hz. Dots show calculation results with respect to heat transfer scheme on fig. 1 (b, c, d) for  $\phi = 0.75, 0.5, 0.25$

Note that the curves 2-4 presented in fig. 3(a, b, c) are obtained considering the extremely idealized heat exchange, when it is believed, that the ball is completely surrounded by vapor or liquid fig. 2(a). Points at  $\varphi = 0.25, 0.5,$  and  $0.75$  in fig. 3 represent the calculated values of the equilibrium speed if particles diameter is 2 mm and  $f = 1$  Hz, obtained if we assume that the phase distribution on the surface of the particle is proportional to their volumetric parts. Thereafter, here we use the heat exchange layouts shown in fig. 2 for  $\varphi = 0.25, 0.5,$  and  $0.75,$  correspondingly. Calculations, that involves phase distributions on the surface of the particle have a significant impact on the value of the equilibrium speed, especially for the filling with heat-conducting particle material. From fig. 3a is obvious, that in the steel particle filling over the considerable interval of the volumetric vapor content, the value of the equilibrium speed, taking into consideration the irregularity of heat exchange on the surface of the particle, turns out to be considerably smaller than  $a_L$ . The glass particles has lesser ability to accumulate heat, see tab. 2, that causes not very significant but still pronounced, shift of the adiabatic speed fig. 3(b). Calculations made for the filling with lead particles, revealed slight differences in the value of the equilibrium speed, when particle diameter is changed in range from 2 mm to 4 mm and the characteristic frequency from 1 Hz to 4 Hz, fig. 3(c). Results obtained for other heat conducting material particles (copper, aluminum) showed the significant influence of non-adiabatic nature of the process on the value of the equilibrium speed of sound in the vapor-liquid mixture. Influence of the particle size and the disturbance frequency in the considered conditions turned out to be weak enough.

### Moving mixture

The flow of the two-phase liquid mixture in straight and complex shape channels is characterized by the difference in speeds of liquid and vapor phases. Two-phase mixture in this case can be considered to be quasi-homogenous environment. The correlation between speed of vapor and the speed of liquid is expressed using the coefficient of phase sliding  $K = w''/w'$ .

The value of the sliding coefficient depends on the orientation of the moving stream, parameters of the vapor-liquid mixture and the shape of the channel. Following recommendations given in [13] we considered the cases: vertical, inclined pipes, circular channels, and vertical rod bundles. From the dependences offered for the fillings which have significantly greater wetted surface area compared to the pipes, the most reasonable would be to apply the formula for the adiabatic flow of the vapor-liquid mixture in the rod bundles:

$$K = 1 + 2.27 \left( 1 - \frac{P}{P_{cr}} \right)^2 \left( \frac{\rho'}{\rho w} \right)^{0.7} \quad (21)$$

In the presence of the phase sliding, the relation between real volume  $\varphi$  and mass consumed vapor content  $x$  is set by expression:

$$\varphi = \left[ 1 + K \frac{\rho''(1-x)}{\rho'x} \right]^{-1} \quad (22)$$

Calculated equilibrium velocity of sound in the flow of vapor-liquid mixture are presented in fig. 3(d, e, f). The input are the same as in the stationary mixture, except the speed of the flow. When the mass flow rate is  $\rho w = 80$  [ $\text{kgm}^{-2}\text{s}^{-1}$ ] and the pressure  $P_0 = 0.6$  MPa the phase sliding coefficient calculated using formula (21), is  $K = 13$ . The selected values of pressure and mass flow rate are close to the range of operation parameters in which eq. (21) is valid. We can

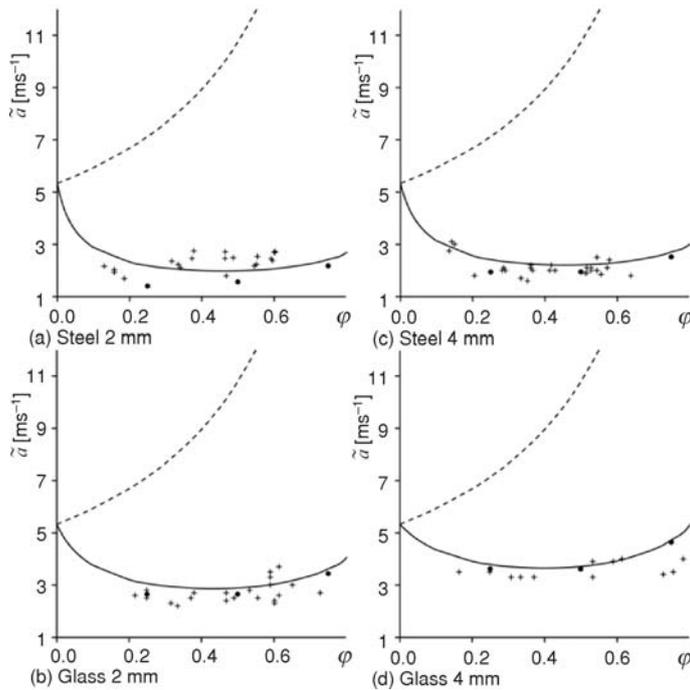
assume that the obtained value of the phase sliding coefficient most likely represents the estimation of below process, since the deceleration of a fluid on the inner surface of the porous structure, formed by spherical particles, is more significant than on the vertical bundles of the smooth rods.

Results presented in fig. 3(d, e, f) offer even more pronounced decrease of the equilibrium speed of sound in the whole flow vapor content change range in comparison with the stationary two-phase mixture fig. 3(a, b, c). Also the nature of behavior of  $\tilde{a}(\varphi)$  has changed. The flat minimum can be clearly seen in the volume vapor content middle value range. The values of speed in the minimum range are significantly less than on the lower border of the equilibrium speed of sound in adiabatic two-phase mixture. This qualitatively new effect was confirmed by the experiment.

### Experimental results

Experiments were carried out in a vertical cylindrical channel with the internal diameter of 33 mm and wall thickness of 7.5 mm. A detailed description of the layout of the experiment is given in the paper [11]. Forming disordered packing steel, glass, and lead balls with diameter 2 mm up to 8 mm were used as a filling.

Experiments were performed for pressures 0.2 MPa and 0.6 MPa; the average mass velocity in the channel was  $\rho w = 80 - 120$  [kgm<sup>-2</sup>c<sup>-1</sup>]. Carrier phase (this time liquid phase) intrinsic velocity was taken into account in the measurements. Based on the physical assumption of a



**Figure 4.** Velocity of small pressure perturbations in vapor-liquid flow with stationary steel and glass spherical particles.  $P_0 = 0.6$  MPa, + – experimental data; other symbols depicted in fig. 3

stronger braking of viscous fluid on developed inner surface of the porous structure, it is permissible to assume that the value of  $K$  in the fillings is substantially higher than it is indicated by expression (21). Calculations show that coefficient of the phase sliding in a close-packed filling calculated using formula (21) (valid for smooth pipe bundles) and calculated values of the equilibrium velocity are somewhat overstated in comparison with the experimental data.

So the overall increase of  $K$  by 1.7 times in most cases gives closer approximation of the theory to the experiment. Figure 4 shows the experimental data for the speed of small low-frequency disturbances of pressure and the results of calculations of the equilibrium velocity of sound

in the vapor-liquid mixture using steel and glass particle filling with a diameter of 2 mm and 4 mm at a pressure of 0.6 MPa. The theoretical curve was plotted taking into the account the heat transfer according to the layout shown in fig. 2(a), when a particle is entirely surrounded by vapor or liquid phase. Black dots show the calculated values for partial wetting of the ball using the layouts of heat transfer presented in fig. 2(b, c, d) for  $\varphi = 0.75, 0.5,$  and  $0.25,$  respectively. Figure 5 shows the results for the lead particle filling, with a diameter of 8 mm at a pressure of 0.6 MPa and 0.2 MPa. Given the uncertainty of the structure of the vapor-liquid flow, the received compliance of the theoretical model with the experiment can be considered as satisfactory.

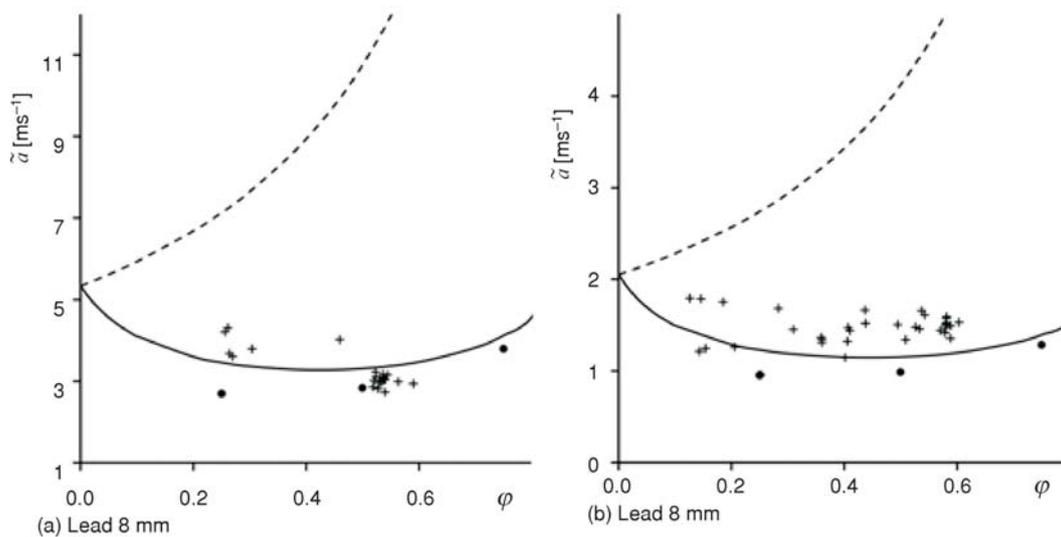


Figure 5. Velocity of small pressure perturbations in vapor-liquid flow with stationary lead spherical particles.  $P_0 = 0.6$  MPa (a),  $P_0 = 0.2$  MPa (b). + – experimental data, other symbols depicted in fig. 3

## Conclusions

Theoretical analysis of thermodynamic equilibrium acoustic velocity was performed in vapor-liquid system with respect to non-stationary heat transfer in closed-packed spherical particles for different conditions on particle surface heat transfer. For registration of mixture heat leaking, spending for heating of heat-conducting particles in the front of the pressure compression wave, formula for acoustic velocity included effective heat capacity. Theoretical results are in agreement with experimental data that allowed to explain anomalous low speeds of low frequency pressure perturbation in case of vapor-liquid filtering in closed-packed layer of solid heat-conducted particles. Difference of equilibrium acoustic velocity in stationary mixture (or in one-velocity flow) and acoustic velocity with slipping between liquid and vapor phases was shown. In the second case dependence of speed from true vapor void fraction has minimum that is lower than Landau velocity. Agreement of theoretical calculations and experimental data makes possible to conclude that basic theoretical principles are correct which were proposed for calculations of equilibrium acoustic velocity in stationary and moving vapor-liquid mixture in case of non-adiabatic conditions.

## Acknowledgment

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## Nomenclature

$A$	– perturbation amplitude, [ $\text{Nm}^{-2}$ ]
$a$	– acoustic speed, [ $\text{ms}^{-1}$ ]
$a_L$	– Landau's acoustic speed, [ $\text{ms}^{-1}$ ]
$c$	– specific heat capacity, [ $\text{Jkg}^{-1}\text{K}^{-1}$ ]
$\bar{c}_{pr}$	– effective heat capacity, [ $\text{Jkg}^{-1}\text{K}^{-1}$ ]
$F$	– area of surface, [ $\text{m}^2$ ]
$L$	– specific heat of vaporization, [ $\text{Jkg}^{-1}$ ]
$P$	– pressure, [ $\text{Nm}^{-2}$ ]
$Q$	– specific heat, [ $\text{Jkg}^{-1}$ ]
$r$	– particle radius, [m]
$s$	– specific mixture entropy, [ $\text{Jkg}^{-1}\text{K}^{-1}$ ]
$T$	– temperature, [K]
$x$	– dryness, [–]

## Greek symbols

$\lambda$	– thermal conductivity coefficient, [ $\text{Wm}^{-1}\text{K}^{-1}$ ]
$\rho$	– density, [ $\text{kgm}^{-3}$ ]
$\tau$	– time, [s]
$\varphi$	– void fraction, [–]

## Subscripts

p	– particle
pr	– process
S	– adiabatic
TP	– two-phase
'	– saturated liquid, vapor

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