

ANALYTICAL SOLUTION OF CONJUGATE TURBULENT FORCED CONVECTION BOUNDARY LAYER FLOW OVER PLATES

by

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A conjugate (coupled) forced convection heat transfer from a heated conducting plate under turbulent boundary layer flow is considered. A heated plate of finite thickness is cooled under turbulent forced convection boundary layer flow. Because the conduction and convection boundary layer flow is coupled (conjugated) in the problem; a semi-analytical solution based on differential transform method is presented for solving the non-linear integro-differential equation occurring in the problem. The main conclusion is that in the conjugate heat transfer case the temperature distribution of the plate is flatter than the one in the non-conjugate case. This feature is more pronounced under turbulent flow when compared with the laminar flow.

Key words: *analytical solution, differential transform method, conjugate, coupled heat transfer*

Introduction

When the temperature distribution of the solid body and that of the fluid flow are coupled, the resulting problem is known *conjugate heat transfer*. This problem, for the first time, was studied by Perelman [1] and Luikov *et al.* [2]. Perelman [1] used the method of the asymptotic solution to solve the integral equation occurring in the conjugate heat-transfer problem. A generalized Fourier sine transform was presented by Luikov *et al.* [2] for the semi-infinite plate. Trevino and Linan [3] modeled the external heating of a flat plate cooled under a convective laminar flow with and accounted for the axial heat conduction in the plate by solving the integro-differential equation using perturbation. Determination of plate temperature in case of combined conduction, convection and radiation heat exchange is carried out by Sohal and Howel [4]. Later, Karvinen [5] presented an approximate method is presented by for calculating heat transfer from a flat plate in forced flow and compared the results with experimental data and previous results obtained in [4] for the case of combined convective heat exchange with the environment, conduction in the plate and internal heat sources. Forced convection conjugate heat transfer in a laminar plane wall jet was considered by Kanna and Das [6]. A problem of conduction-convection in fins [7, 8] and in cavities [9, 10] and the combined effect of conduction and radiation in a T-Y shaped fin [11] are carried out in recent years.

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Applying the superposition principle and Green's function, Nagasue [12] carried out a solution for conjugate heat transfer in a circular tube. Chiu *et al.* [13] considered the conjugate heat transfer in a horizontal heated channel experimentally and numerically. Transient conjugated heat transfer to laminar flow in a tube or channel was studied by Karvinen [14]. Numerical and asymptotic solution was conducted by Escandon *et al.* [15] to solve a steady-state problem of a conjugate heat transfer in an electro-Osmotic fully developed laminar flow. Optimization under the platform of conjugate heat transfer has also been studied [16-18]. Recently, Lindstedt and Karvinen [19] carried out a detailed analysis on conjugate heat transfer in a plate that one surface was kept at constant temperature and the other cooled by forced or natural convection. Interested readers can refer to the perfect review of Dorfman and Renner [20] and other papers [21-27] for acquiring more details about the history and characteristics of conjugate heat transfer problems.

In the recent years, semi-analytical treatments are used to solve the complex equations occurring in heat transfer and fluid mechanic studied. The most common treatments are the homotopy analysis method [28, 29], homotopy perturbation method [30, 31], variational iteration method [32] and the differential transform method (DTM) [33-37]. The DTM was first proposed by Zhou [33]. The DTM requires no symbolic computations of the necessary derivatives of the data functions, unlike traditional Taylor series method. The purpose of the present work is present a new algorithm for solving the integro-differential equation occurring in a turbulent conjugate heat transfer problems. To do this, a turbulent forced convection conjugate heat transfer from a good conducting plate is modeled. Because turbulent flow usually occurs in practical cooling systems cooled by forced convection flow, the present work may be of sufficient interests as well as the interesting research recently carried out by Hajmohammadi *et al.* [35], who assumed laminar forced convection flow for the studied problem.

Mathematical description of the problem

Shown in fig. 1 is a schematic of a conjugate force convection flow over a conducting solid plate. An incompressible fluid is flowing over the upper surface of the plate. The flow is assumed to be turbulent, steady and 2-D in $x-y$ plane with the free stream of velocity, U_∞ , and free stream temperature, T_∞ . The lower surface of the plate is heated with uniform and constant heat flux, q'' . The left and the right sides of the rectangular plate are insulated whereas a constant heat flux is traveling through the bottom side toward the top of the plate. The upper side of the solid is in direct contact with the flowing fluid and heat transfer to the fluid occurs via this surface. For the sake of simplicity, the effects of buoyancy and viscous dissipation are neglected. The physical properties of the fluid and the solid are assumed to be uniform, isotropic, and constant. Given the axial heat conduction occurring in a plate of finite thickness, b , the heat flux distribution at the interface surface (upper surface of the plate), and many be q'' , differs with that at the bottom, q''_u correlated to the temperature distribution of the interface surface, T_s , by the classical expression [38]:

$$q''_u(x) = 0.0287 \frac{k_f}{x} \text{Pr}^{0.6} \text{Re}_x^{0.8} \left\{ \int_0^x \left[1 - \left(\frac{\xi}{x} \right)^{9/10} \right]^{-1/9} \frac{dT_s}{d\xi} d\xi + \Delta T_0 \right\} \quad (1)$$

The first term in eq. (1) is related to the integration of the temperature variation along the plate and the second term corresponds to the temperature jump in the leading edge of the plate, ΔT_0 , x – the axial location measured from the leading edge of the plate, k_f – the

thermal conductivity of fluid, Pr – the Prandtl number, and the local Reynolds number, Re_x is defined:

$$Re_x = \frac{U_\infty x}{\nu} \quad (2)$$

To arrive at eq. (1), it is assumed that the velocity profiles are polynomial, there is no gravity and no external force, and all thermo-physical properties are constant. However, it must be noted that in real practical systems, the properties may vary with temperature. Besides, eq. (1) is valid for high Reynolds number and not for the transient situation. It is further assumed that the temperature of the plate is uniform in the transversal co-ordinate, y , that is:

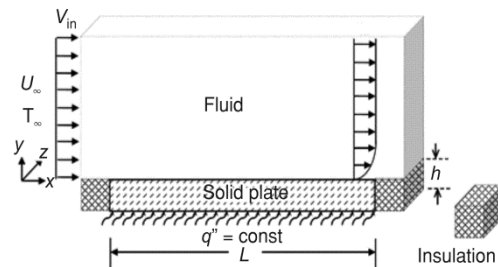


Figure 1. Conjugate forced convection flow over a solid conducting plate

$$\frac{1}{Bi_b} = \frac{q_{\text{transversal conduction}}}{q_{\text{convection}}} \approx \frac{k_s l \left(\frac{\Delta T}{b} \right)}{hl(\Delta T)} = \frac{k_s}{hb} \gg 1 \quad (3)$$

where Biot number, Bi_b , is defined based on the height of the plate, b .

Substituting the convective heat transfer coefficient, h , from the classical relations [38], eq. (3) is rewritten:

$$0.0287 \left(\frac{k_s}{k_f} \right) Re_l^{-0.8} Pr^{-0.6} \gg 1 \quad (4)$$

where l is the length of the plate, λ – the aspect ratio of the plate, and the ratio of the height to the length of the plate, b/l . Applying the energy balance in a control volume of width, dx , and height, b , (with uniform temperature along the transversal co-ordinate), the energy equation is obtained:

$$q'' + k_{s,\infty} b \frac{\partial^2 T}{\partial x^2} = q''_u(x) \quad (5)$$

where q''_u is calculated from eq. (1). To formulate the problem under study, it is natural to define the following dimensionless variables:

$$(\xi, \hat{x}) = \frac{(\xi, x)}{l}; \quad \theta = \frac{Pr^{0.6} Re_l^{0.8} (T_s - T_\infty)}{0.0287 q'' l / k_f} \quad (6)$$

and also

$$\sigma = 0.0287 \left(\frac{k_{s,\infty}}{k_f} \right) Pr^{-0.6} Re_l^{-0.8} \quad (7)$$

$$\sigma_\lambda = \sigma \lambda \quad (8)$$

In this study, the parameter, σ_λ , is called the *axial conduction parameter* which represents the ratio of the ability of the plate to carry heat in the stream-wise direction to the ability of the fluid to carry heat from the plate. We also call a plate as *good conducting plate*, if in addition to meet the condition described in eq. (4), the magnitude of the parameter σ_λ is sufficiently large and is defined:

$$\sigma_{\lambda} = 0.0287 \left(\frac{k_{s,\infty}}{k_f} \right) \text{Pr}^{-0.6} \text{Re}_l^{-0.8} \lambda \frac{k_s b \left(\frac{\Delta T}{l} \right)}{hl(\Delta T)} \approx \frac{q_{\text{axial conduction}}}{q_{\text{convection}}} = \frac{k_s}{hl} \lambda = \frac{1}{\text{Bi}_l} \lambda \gg 0 \quad (9)$$

where the Biot number, Bi_l , is defined based on the length of the plate, l . We will later prove that the aforementioned power series solution derived for the plate temperature, is quickly convergent when the plate under consideration is a *good conducting* plate, $\sigma_{\lambda} \gg 0$. Utilizing eqs. (1) - (9), the governing equation for dimensionless temperature distribution of the good conducting plate can be obtained:

$$\sigma_{\lambda} \frac{\partial^2 \theta}{\partial \hat{x}^2} = -1 + \frac{\theta^0}{\hat{x}^{0.2}} + \frac{1}{\hat{x}^{0.2}} \int_0^{\hat{x}} \left[1 - \left(\frac{\xi}{\hat{x}} \right)^{9/10} \right]^{-1/9} \frac{\partial \theta}{\partial \xi} d\xi \quad (10)$$

with the boundary conditions:

$$\left. \frac{\partial \theta}{\partial \hat{x}} \right|_{\hat{x}=0} = 0 \quad (11)$$

$$\left. \frac{\partial \theta}{\partial \hat{x}} \right|_{\hat{x}=1} = 0 \quad (12)$$

Due to obvious reasons, the solution of non-linear, integro-differential equation (10) is not easily tractable by pure analytical means and, therefore, has to be obtained numerically or via a recursive semi-analytical method. In what follows, it is shown that the DTM is capable of providing a quick and highly accurate power series solution of eq. (10) satisfying the boundary conditions (11) and (12).

Solution of the problem with DTM

The basic definitions and the fundamental theorems of DTM and its applicability for various types of equations are given in [33-37]. For the convenience of the reader, in what follows, a brief review of the DTM is presented. The differential transform of a function $f(x)$ is defined:

$$\tilde{f}(k) = \left(\frac{1}{k!} \right) \frac{d^k f(x)}{dx^k} \Big|_{x=x_0} = 0 \quad (13)$$

where $f(x)$ stands for the original function and $\tilde{f}(k)$ is the differential transform function. The inverse differential transform of a sequence, $\tilde{f}(k)$ is approximated:

$$f(x) \approx \sum_{k=0}^N \tilde{f}(k) (x - x_0)^k \quad (14)$$

where N is the highest order of the power series solution, eq. (14). The fundamental operations performed by DTM are listed in tab. 1. Applying these fundamental operations to

Table 1. Fundamental operations of DTM

Original function	Transformed function
$f(t) = \alpha u(t) \pm \beta v(t)$	$\tilde{f}(k) = \alpha \tilde{u}(k) \pm \beta \tilde{v}(k)$
$f(t) = u(t)v(t)$	$\tilde{f}(k) = \sum_{k_1}^k \tilde{u}(k_1) \tilde{v}(k - k_1)$
$f(x) = \frac{d^k u(x)}{dx^k}$	$\tilde{f}(k) = (k+1)(k+2)\dots(k+m) \tilde{u}(k+m)$
$f(x) = \int_{x_0}^x u(x) dx$	$\tilde{f}(k) = \frac{\tilde{u}(k-1)}{k}, \quad k \geq 1$
$f(x) = x^m$	$\tilde{f}(k) = \delta(k-m), \quad \delta(k-m) = \begin{cases} 1 & k=m \\ 0 & k \neq m \end{cases}$
$f(x) = \exp(x)$	$\tilde{f}(k) = \frac{\lambda^k}{k!}$

the given problem, a recurrence relation is obtained. Then solving this recurrence relation and using the differential inverse transform, we can obtain the solution of the problem.

As an initial step, we substitute eq. (14) into the integral of eq. (10). Owing that here $x_0 = 0$ (leading edge of the plate) and $\theta^0 = \theta(0)$, we obtain:

$$\sigma_\lambda \frac{\partial^2 \theta}{\partial \hat{x}^2} = -1 + \frac{\theta(0)}{\hat{x}^{0.2}} + \frac{1}{\hat{x}^{0.2}} \sum_{j=1}^N j \theta(j) \int_0^{\hat{x}} \left[1 - \left(\frac{\xi}{\hat{x}} \right)^{9/10} \right]^{-1/9} \hat{x}^{j-1} d\xi \quad (15)$$

where θ stands for the differential transform of function, θ . Using the variable transformation, $u = 1 - (\xi/\hat{x})^{9/10}$, eq. (15) is reduced to the following expression:

$$\sigma_\lambda \frac{\partial^2 \theta}{\partial \hat{x}^2} = -1 + \frac{\theta(0)}{\hat{x}^{0.2}} + \frac{10}{9} \sum_{j=1}^N j \tilde{\theta}(j) B\left(\frac{8}{9}, \frac{10j}{9}\right) \hat{x}^{j-0.2} \quad (16)$$

where $B(m, n)$ is the numerical beta function defined:

$$B(m, n) = \int_0^1 u^{m-1} (1-u)^{n-1} du \quad (17)$$

Applying the fundamental operations performed by DTM (listed in tab. 1) to eq. (16), the following relation is obtained:

$$\begin{aligned} \sigma_\lambda (k+1)(k+2) \tilde{\theta}(k+2) = \\ -\delta(k) + \tilde{\theta}(0) \delta\left(k + \frac{1}{5}\right) + \frac{10}{9} \sum_{j=1}^N j \tilde{\theta}(j) B\left(\frac{8}{9}, \frac{10j}{9}\right) \delta\left(k - j + \frac{1}{5}\right) \end{aligned} \quad (18)$$

Re-arranging and simplifying eq. (18), gives the following recurrence relations:

$$\tilde{\theta}\left(\frac{9}{5}\right) = \frac{25}{36\sigma_\lambda} \tilde{\theta}(0) \quad (19)$$

$$\tilde{\theta}(2) = -\frac{1}{2\sigma_\lambda} \quad (20)$$

$$\tilde{\theta}(k) = \frac{10}{9\sigma_\lambda k(k-1)} B\left[\frac{8}{9}, \frac{10\left(k - \frac{9}{5}\right)}{9}\right] \left(k - \frac{9}{5}\right) \tilde{\theta}\left(k - \frac{9}{5}\right) \quad k \geq \frac{18}{5} \quad (21)$$

The differential transform of the boundary conditions, eqs. (11) and (12), can easily be obtained:

$$\tilde{\theta}(1) = 0 \quad (22)$$

$$\sum_{j=1}^N j \tilde{\theta}(j) = 1 \quad (23)$$

The numerical value of constant $\tilde{\theta}(0)$ is determined by substituting the given number of series solution components up to N into relation (23) and solving the resulting equation for $\tilde{\theta}(0)$ when the value of σ_λ is known. Substituting the numerical value of $\tilde{\theta}(0)$ in power series solution coefficients, and utilizing the inverse differential transform, eq. (14), we may construct the following power series solution up to $N = 8$ for the plate non-dimensional temperature, when $\sigma_\lambda = 1$ and $\lambda = 0.1$, as following:

$$\begin{aligned} \sigma(\hat{x}) = & 0.7719409502 + 0.5360701043\hat{x}^{9/5} - 0.5\hat{x}^2 + 0.06822159130\hat{x}^{18/5} \\ & - 0.05651356282\hat{x}^{19/5} + 0.00365322472\hat{x}^{27/5} - 0.002806422020\hat{x}^{28/5} \\ & + 0.000108500\hat{x}^{36/5} - 0.000078859278\hat{x}^{37/5} + 0.000002058733474\hat{x}^9 \end{aligned} \quad (24)$$

The local Nusselt number of the flow can be determined from:

$$\text{Nu}(\hat{x}) = \frac{hx}{k_f} = \frac{1}{0.0287} \text{Pr}^{0.6} \text{Re}_l^{0.8} \hat{x}^{0.8} \frac{\tilde{\theta}(0) + \frac{10}{9} \sum_{k=9/5} k \tilde{\theta}(k) B\left(\frac{8}{9}, \frac{10k}{9}\right) \hat{x}^k}{\theta(\hat{x})} \quad (25)$$

Substituting eq. (24) into eq. (25), gives the local Nusselt number:

$$\text{Nu}(\hat{x}) = \frac{1}{0.0287} \text{Pr}^{0.6} \text{Re}_l^{0.8} \hat{x}^{0.8} \frac{\tilde{\theta}(0) + \frac{10}{9} \sum_{k=9/5} k \tilde{\theta}(k) B\left(\frac{8}{9}, \frac{10k}{9}\right) \hat{x}^k}{\theta(\hat{x})} \quad (26)$$

To characterize the convergence trend of the present series solution it is clear that each term in the series solution should be less than the next subsequent term, meaning that:

$$\tilde{\theta}(k) < \tilde{\theta}\left(k - \frac{3}{2}\right) \quad (27)$$

which in the view of eq. (23) it gives the following relation:

$$\frac{4}{3\sigma_\lambda k(k-1)} B\left(\frac{2}{3}, \frac{4k}{3}\right) \left(k - \frac{3}{2}\right) < 1 \quad (28)$$

Equation (28) demonstrates that; in order to have a reasonable and fast convergence of the presented series solution, the magnitude of σ_λ must be sufficiently large, depending on the number of the terms contributing in the series solution.

Results and discussion

The effect of σ_λ on the 1-D temperature distribution of the plate, $\theta(\hat{x})$, is shown in fig. 2 when $\lambda = 0.1$. Figure 2 reveals that, increasing σ_λ results in flattening the plate temperature distribution. As a consequence of high axial heat conduction in the plate, when $\sigma_\lambda = 50$ the plate temperature is nearly uniform along the x co-ordinate.

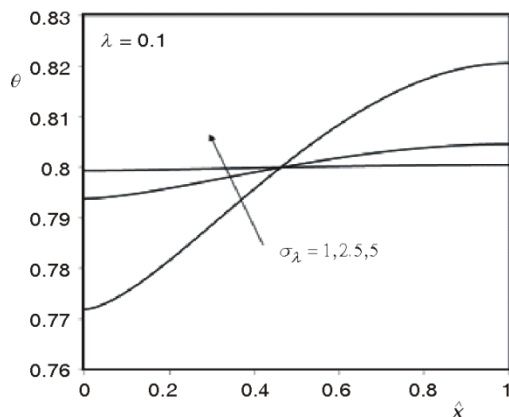


Figure 2. Effect of σ_λ on the 1-D temperature distribution of the plate, $\theta(\hat{x})$ when $\lambda = 0.1$

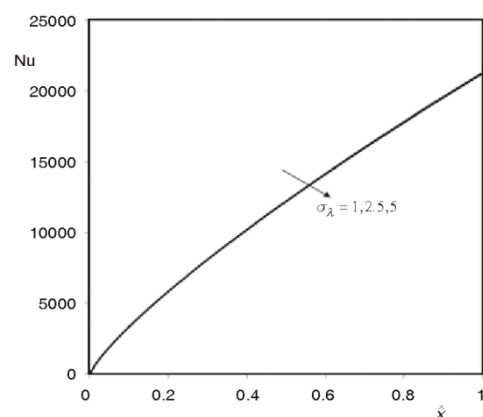


Figure 3. The variation of Nusselt number of the flow in the axial co-ordinate for several values of σ_λ

The variations of Nusselt number of the flow in the axial coordinate is plotted in fig. 3 for several values of σ_λ . Based on this figure, the variations of Nusselt number of the flow is not sensitive with the variations of σ_λ , however, the Nusselt number in the conjugate laminar flow is affected by the variation of σ_λ [15]. This feature is justified as: quite different in the laminar flow, the thermal boundary conditions of the plate have a negligible effect on the heat transfer coefficient and the Nusselt number, in the turbulent flow.

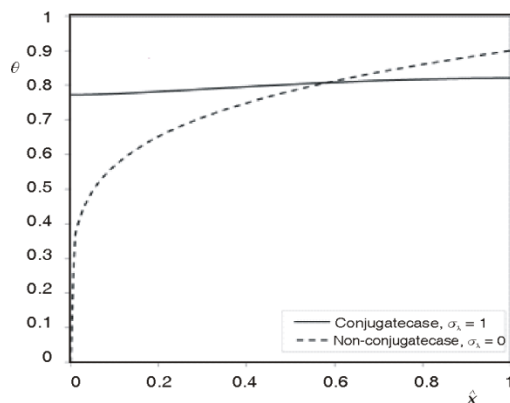


Figure 4. The comparison between the plate temperature distribution of the conjugate and non-conjugate convection heat transfer, $\sigma_\lambda = 1$ and $\sigma_\lambda = 0$, respectively

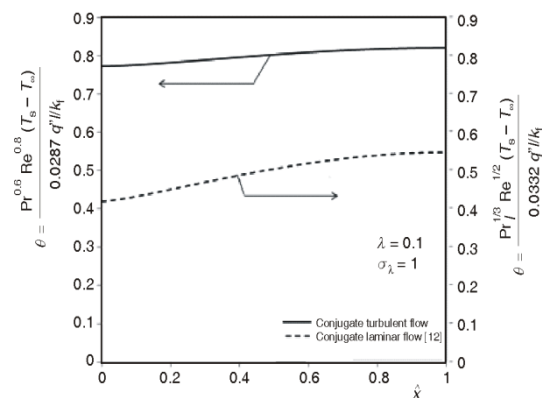


Figure 5. The comparison between the plate temperature distribution of the plate cooled under laminar forced convection flow with the ones in the case of turbulent flow

Figure 4 show the comparison of the temperature distribution of plate for the case of conjugate convection heat transfer ($\sigma_\lambda = 1$, flow over a plate of finite thickness) with those for the non-conjugate convection heat transfer ($\sigma_\lambda = 0$, flow over a very thin plate). Figure 4 reveals that the temperature distribution in the conjugate case is flatter than temperature distribution at the non-conjugate case. The flattered temperature distribution in the conjugate case are compared in two cases; laminar forced convection and turbulent forced convection. It is observed that the effect of σ_λ , or in other words, the plate thickness, on flattering the temperature distribution is more pronounced under in the conjugate turbulent flow when compared with the temperature distribution in the conjugate laminar flow. This feature is extremely significant, because in the cooling systems, the objective is to spread heat as much as possible, such that the temperature distribution tends to be uniform at the entire devices.

Conclusions

Owing that turbulence often occurs in real practical systems, consideration is given to the conjugate heat transfer from a heated plate of finite thickness when the flow is supposed to be turbulent. The mathematical formulation of the problem derived in this study deals mainly with the 1-D variation of the plate temperature and Nusselt number of the flow. The integro-differential equation occurring in problem is solved by DTM.

As a significant physical conclusion of this study we conclude that the u of the conjugate heat transfer over a flat plate is higher than the non-conjugate case, because the temperature distribution is more uniform in the conjugate case. It is further concluded that the use of flat plate in reducing the hot spot temperature of the plate is of greater advantage in turbulent case when compared with the laminar flow situation.

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Nomenclature

B	– beta function
b	– plate thickness
H	– convective heat transfer coefficient
K	– thermal conductivity
L	– length,
N	– highest order of the series solution
Pr	– Prandtl number
q''	– heat flux
q_u''	– heat flux on the upper surface of the plate
Re_x	– Reynolds number
T	– local temperature
T_s	– temperature distribution of the plate
T_∞	– free stream temperature
U_∞	– free stream velocity
x	– Cartesian co-ordinate

Greek symbols

δ	– dfined in table 1.
θ	– dimensionless temperature, defined in Eq. (6)
λ	– plate aspect ratio (b/l)
ν	– kinematic viscosity
σ	– dimensionless parameter, defined in eq. (7)
σ_λ	– dimensionless parameter, defined in eq. (8)

Scripts

f	– fluid
s	– solid plate
$(\wedge)$	– dimensionless variables defined in eq. (6)
$(\sim)$	– differential transformed variables

References

- [1] Perelman, Y. L., On Conjugate Problems of Heat Transfer, *International Journal of Heat and Mass Transfer*, 3 (1961), 4, pp. 293-303
- [2] Luikov, A. V., et al., Analytical Methods of Solution of Conjugate Problems in Convective Heat Transfer, *International Journal of Heat and Mass Transfer*, 14 (1971), 8, pp. 1047-1056
- [3] Trevino, C., Linan, A., External Heating of a Flat Plate in a Convective Flow, *International Journal of Heat and Mass Transfer*, 27 (1984), 7, pp. 1067-1073
- [4] Sohal, M. S., Howel, J. R., Determination of Plate Temperature in Case of Combined Conduction, Convection and Radiation Heat Exchange, *International Journal of Heat and Mass Transfer*, 16 (1973), 11, pp. 2055-2066
- [5] Karvinen, R., Some New Results for Conjugated Heat Transfer in a Flat Plate, *International Journal of Heat and Mass Transfer*, 21 (1978), 9, pp. 1261-1264
- [6] Kanna, P. R. Das, M. K., Conjugate Forced Convection Heat Transfer from a Flat Plate by Laminar Plane Wall Jet Flow, *International Journal of Heat and Mass Transfer*, 48 (2005), 14, pp. 2896-2910
- [7] Hajmohammadi, M. R., et al., Optimal Architecture of Heat Generating Pieces in a Fin, *J. Mech. Sci. and Tech.*, 27, (2013), 4, pp. 1143-1149
- [8] Alebrahim, A. Bejan, A., Constructal Trees of Circular Fins for Conductive and Convective Heat Transfer, *International Journal of Heat and Mass Transfer*, 42, (1999), 19, pp. 3585-359
- [9] Pouzesh, A., et al., Investigations on the Internal Shape of Constructal Cavities Intruding a Heat Generating Body, *Thermal Science*, 19 (2015), 2, pp. 609-618
- [10] Hajmohammadi, M. R., et al., Valuable Reconsideration in the Constructal Design of Cavities, *Energy Conv. and Management*, 66 (2013), Feb., pp. 33-40
- [11] Hajmohammadi, M. R., et al., Radiation Effect on Constructal Design Analysis of a T-Y-Shaped Assembly of Fins, *J. Therm.Sci. & Tech.*, 7 (2012), 4, pp. 677-692
- [12] Nagasue, H., Steady State Heat Transfer with Axial Conduction in Laminar Flow in a Circular Tube with a Specified Temperature or Heat Flux Wall, *International Journal of Heat and Mass Transfer*, 24 (1981), 11, pp. 1823-1832
- [13] Chiu, W. K., et al., Experimental and Numerical Study of Conjugate Heat Transfer in a Horizontal Channel Heated from Below, *J. Heat Transfer, ASME*, 123 (2001), 4, pp. 688-694
- [14] Karvinen, R., Transient Conjugated Heat Transfer to Laminar Flow in a Tube or Channel, *International Journal of Heat and Mass Transfer*, 31 (1988), 6, pp. 1326-1328
- [15] Escandon, J. P., et al., Theoretical Conjugate Heat Transfer Analysis in a Parallel Flat Plate Micro-Channel, *International Journal of Thermal Sciences*, 50 (2011), 6, pp. 1022-1030

- [16] Hajmohammadi, M. R., *et al.*, Effect of a Thick Plate on the Excess Temperature of Iso-Heat Flux Heat Sources Cooled by Laminar Forced Convection Flow; Conjugate Analysis, *Numerical Heat Transfer, Part A*, 66 (2014), 2, pp. 205-216
- [17] Hajmohammadi, M. R., *et al.*, Detailed Analysis for the Cooling Performance Enhancement of a Heat Source under a Thick Plate, *Eng. Conv. and Management*, 76 (2013), Dec., pp. 691-700
- [18] Hajmohammadi, M. R., *et al.*, Improvement of Forced Convection Cooling Due to the Attachment of Heat Sources to a Conducting Thick Plate, *J Heat Transf. Trans. ASME*, 135 (2013), 12, pp. 124504-124507
- [19] Lindstedt, M., Karvinen, R., Conjugate Heat Transfer in a Plate – One Surface at Constant Temperature and the other Cooled by Forced or Natural Convection, *International Journal of Heat and Mass Transfer*, 66 (2013), Nov., pp. 489-495
- [20] Dorfman, A., Renner, Z., Conjugate Problems in Convective Heat Transfer: Review, *Mathematical Problems in Engineering*, 2009 (2009), ID 927350
- [21] Karvinen, R., Use of Analytical Expressions of Convection in Conjugated Heat Transfer Problems, *J. Heat Transfer*, 134 (2012), 3, pp. 031007-031015
- [22] Sofiane, T., Toufik, B., Numerical Study of the Conjugate Heat Transfer in a Horizontal Pipe Heated by Joulean Effect, *Thermal Science*, 16 (2012), 1, pp. 53-67
- [23] Rachid, S., *et al.*, Computational Analysis of Transient Turbulent Flow and Conjugate Heat Transfer Characteristics in a Solar Collector Panel with Internal, Rectangular Fins and Baffles, *Thermal Science*, 14 (2010), 1, pp. 221-234
- [24] Kumar, T., *et al.*, Conjugated Heat Transfer Analysis of Gas Turbine Vanes Using Maccormack's Technique, *Thermal Science*, 12 (2008), 3, pp. 65-73
- [25] Mamun, A. A., *et al.*, MHD-Conjugate Heat Transfer Analysis for a Vertical Flat Plate in Presence of Viscous Dissipation and Heat Generation, *Int. Comm. Heat Mass Transfer*, 35 (2008), 10, pp. 1275-1280
- [26] Mobedi, M., Conjugate Natural Convection in a Square Cavity with Finite Thickness Horizontal Walls, *Int. Comm. Heat Mass Transfer*, 35 (2008), 4, pp. 503-513
- [27] Karuppa, R., Srikanth, G., Shell Side Numerical Analysis of a Shell and Tube Heat Exchanger Considering the Effects of Baffle Inclination Angle on Fluid Flow Using CFD, *Thermal Science*, 16 (2012), 4, pp. 1165-1174
- [28] Dinarvand, S., *et al.*, Homotopy Analysis Method for Mixed Convective Boundary Layer Flow of a Nanofluid over a Vertical Circular Cylinder, *Thermal Science*, 19 (2015), 2, pp. 549-561
- [29] Rashidi, M. M., Erfani, E., New Analytical Method for Solving Burgers' and Nonlinear Heat Transfer Equations and Comparison with HAM, *Computer Physics Communications*, 180 (2009), 9, pp. 1539-1544
- [30] Hetmaniok, E., *et al.*, Solution of the Inverse Heat Conduction Problem with Neumann Boundary Condition by Using the Homotopy Perturbation Method, *Thermal Science*, 17 (2013), 3, pp. 643-650
- [31] He, J.-H., A Note on the Homotopy Perturbation Method, *Thermal Science*, 14 (2010), 2, pp. 565-568
- [32] Khaleghi, H., *et al.*, Application of Variational Iteration and Homotopy-Perturbation Methods to Nonlinear Heat Transfer Equations with Variable Coefficients, *Numerical Heat Transfer, Part A*, 52 (2007), 1, pp. 25-42
- [33] Zhou, J. K., *Differential Transformation and Its Applications for Electrical Circuits* (in Chinese), Huazhong University Press, Wuhan, China, 1986
- [34] Moradia, A., *et al.*, Investigation of Heat Transfer and Viscous Dissipation Effects on the Jeffery-Hamel Flow of Nanofluids, *Thermal Science*, 19 (2015), 2, pp. 563-578
- [35] Hajmohammadi, M. R., *et al.*, Semi-Analytical Treatments of Conjugate Heat Transfer, *Journal of Mechanical Engineering Science*, 227 (2012), 3, pp. 492-503
- [36] Yaghoobi, H., Torabi, M., The Application of Differential Transformation Method to Nonlinear Equations Arising in Heat Transfer, *Int. Comm. Heat Mass Transfer*, 38 (2011), 6, pp. 815-820
- [37] Hajmohammadi, M. R., Nourazar, S. S., Conjugate Forced Convection Heat Transfer from a Heated Flat Plate of Finite Thickness and Temperature-Dependent Thermal Conductivity, *Heat Transfer Engineering*, 35 (2014), 9, pp. 863-874
- [38] Bejan, A., *Convection Heat Transfer*, 3rd ed., John Wiley and Son, N. Y., USA, 2004