RAYLEIGH TAYLOR INSTABILITY IN DUSTY MAGNETIZED FLUIDS WITH SURFACE TENSION FLOWING THROUGH POROUS MEDIUM

by

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In this paper we investigate the effect of surface tension on hydromagnetic Rayleigh-Taylor (R-T) instability of two incompressible superimposed fluids in a porous medium with suspended dust particles immersed in a uniform horizontal magnetic field. The relevant linearized perturbation equations have been solved using normal mode technique and the dispersion relation is derived analytically for the considered system. The dispersion relation is influenced by the simultaneous presence of medium porosity, suspended dust particles, permeability, magnetic field and surface tension. The onset criteria of R-T stability and instability are obtained and discussed. The growth rate of R-T instability is calculated numerically and it is affected by the simultaneous presence of surface tension and magnetic field. The effects of various parameters on the growth rate of the R-T instability are discussed.

Key words: *Rayleigh-Taylor, porous medium, suspended dust particles, hydromagnetic instability, fluid dynamics*

Introduction

The hydromagnetic Rayleigh-Taylor (R-T) instability occurs at the interface of two fluids when a heavy fluid is supported by a lighter one. The problems of R-T instability are widely discussed in astrophysics, experiments of thermonuclear fusion, inertial confinement fusion, geophysics, magnetohydrodynamic experiments, and space physics. It is also discussed in astrophysical fluids and fusion pellet implosions [1, 2]. Chandrasekhar [3] has given a detail study of R-T instability for the fully ionized incompressible medium under various assumptions. Kull and Anisimov [4] discussed the ablative R-T instability in an incompressible fluid model. Kalra [5] studied the effect of finite Larmor radius (FLR) corrections on the stability of superposed fluids. Bhatia [6] discussed the R-T instability of two superposed conducting fluids of uniform densities and suggested that viscous forces have stabilizing influence. Gupta and Bhatia [7] discussed the R-T instability of two viscous superposed partially-ionized plasmas of uniform densities and found that viscosity has a stabilizing influence.

The effect of suspended particles is important in the dynamics of Martian atmosphere and astrophysical problems. It may also be important in the study of fluid stability problems and

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it is used in industrial and chemical engineering. Michael [8] has studied the Kelvin-Helmholtz (K-H) instability of a dusty gas and found that dust reduces the growth rate of the disturbance. Sharma and Sharma [9] have discussed the R-T instability of two superposed conducting fluids in the presence of suspended particles. Chhajlani *et al*. [10] studied the R-T instability of a stratified magnetized medium in the presence of suspended particles. Sanghvi and Chhajlani [11] have investigated the hydromagnetic K-H instability in the presence of suspended particles and FLR effect. Sharma and Chhajlani [12] discussed the effect of rotation on the R-T instability of two superposed magnetized conducting fluids in the presence of suspended dust particles. Prajapati *et al*. [13] studied the K-H of two superposed magnetized fluids with suspended dust particles.

In recent years, stability problems of two superposed fluids in porous medium became an important field due to its several applications in geophysics and astrophysics. Porosity might be useful for the study of physical properties of comets, meteorites, and interplanetary dust in astrophysical context McDonnel [14]. Sharma and Sunil [15] studied the R-T instability of ionized plasma in a porous medium in the presence of variable magnetic field. Sharma and Kumar [16] studied the effect of R-T instability of viscous-viscoelastic fluids through porous media. Sunil and Chand [17] discussed the R-T instability of plasma in the presence of variable magnetic field and suspended particles in porous media. Wurm *et al.* [18] discussed the importance of gas flow through porous bodies for the formation of planetesimals. Kumar *et al*. [19] studied the R-T instability of rotating viscoelastic fluids in porous media in the presence of variable magnetic fields. Kumar [20] discussed the instability of streaming fluids in porous medium in hydromagnetic fluid. Sharma *et al.* [21] have discussed the effect of surface tension and rotation on R-T instability of two superposed fluids with suspended particles. Prajapati and Chhajlani [22] investigated the combined K-H and R-T instability of two superposed streaming fluids with suspended dust particles flowing through a porous medium. Singh and Dixit [23] discussed the stability of rotating viscous viscoelastic superposed fluids in presence of suspended particles in a porous medium.

The objective of the present paper is to study the effect of surface tension and suspended particles on the stability of two superposed incompressible conducting fluids, when the whole system is porous and immersed in 2-D magnetic fields. In the previous discussed problems, Prajapati and Chhajlani [22] discussed the effect of surface tension in a non-conducting medium without magnetic field. Sharma *et al.* [21] discussed the effect of surface tension and rotation on R-T instability of two superposed fluids with suspended particles in non-conducting and non-porous medium. In this paper, we extend the previous problems of R-T instability in the presence of 2-D magnetic fields and find the stability conditions. We also study, the behaviour of growth rate of R-T instability depending upon various physical parameters.

Model equations of the problem

Consider a Cartesian geometry with co-ordinates (x, y, z) having axes with the z-direction as vertical. Suppose that a plane interface of discontinuity exist at $z = 0$, separating the two semi-infinite homogeneous incompressible conducting superposed fluids in a porous medium of porosity ε and medium permeability k_1. Let the mixture of hydromagnetic fluid and non-conducting suspended dust particles be superposed in the porous medium under the action of gravity $\vec{g} = (0, 0, g)$ and dimensional uniform magnetic field $\vec{H} = (H_x, H_y, 0)$. Let $\rho, \mu, p,$ and $\vec{u} = (u, v, w)$ denote the density, viscosity, pressure, and velocity of the fluid, respectively.

It is assumed that suspended dust particles are non-conducting, of spherical shape and are uniform in size. Force exerted on the fluid by the dust is assumed proportional to the relative velocity, and is given by $KN(\vec{v}-\vec{u})$, where N is the particle number density and K is Stokes coefficient of resistance, given by $K = 6\pi a\mu$ for spherical particles, where a is the particle radius. The inter particle distance is assumed to be very large as compared to the diameter of the particle, so that particle interaction can be ignored. The linearized perturbation equations using the MHD fluid model neglecting initial fluid velocity are written [9, 10, 12]:

$$\frac{\rho}{\varepsilon}\frac{\partial\vec{u}}{\partial t} = -\nabla\delta p + \frac{KN}{\varepsilon}(\vec{v}-\vec{u}) + \vec{g}\delta\rho - \frac{\mu}{k_1}\vec{u} +$$

$$+\frac{\mu_0}{4\pi}[(\nabla\times\vec{h})\times\vec{H} + (\nabla\times\vec{H})\times\vec{h}] + \sum T_s\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta z_s\delta(z-z_s) \tag{1}$$

$$\varepsilon\frac{\partial\delta\rho}{\partial t} + (\vec{u}\nabla)\rho = 0 \tag{2}$$

$$\varepsilon\frac{\partial\vec{h}}{\partial t} = (\vec{H}\nabla)\vec{u} - (\vec{u}\nabla)\vec{H} \tag{3}$$

$$\nabla\vec{u} = 0 \tag{4}$$

$$\nabla\vec{h} = 0 \tag{5}$$

$$\left\{\tau\left[\frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\vec{u}\nabla)\right] + 1\right\}\vec{v} = \vec{u} \tag{6}$$

where $\vec{h} = (h_x, h_y, h_z)$, δp, and $\delta\rho$ are the perturbations in magnetic field, fluid pressure, and density of fluid, respectively, and $\tau = m/6\pi a\mu$ denotes the relaxation time for the suspended dust particles. In eq. (1) $\delta(z - z_s)$ denotes Dirac's function and T_s is the surface tension of interfacial surface.

We assume the perturbation in the physical variables to be dependent on spatial and temporal co-ordinates (x, y, t), and of the form [9]:

$$f(z) \sim \exp[i(k_x x + k_y y + nt)] \tag{7}$$

where k_x and k_y are the horizontal wave numbers ($k^2 = k_x^2 + k_y^2$) and n is the growth rate of the harmonic perturbations.

Dispersion relation

In eq. (1) variable $\vec{v}$ is eliminated by using eq. (6) and then employing eq. (7) in eqs. (1)-(6). The set of linearized perturbed equations can then be written in the component forms:

$$\rho\left[\frac{i}{\varepsilon}(n) + \frac{\nu}{k_1} + \left(\frac{\alpha_0}{\varepsilon}\right)\frac{(in)}{\tau(in)+1}\right]u = -ik_x\delta p + \frac{\mu_0 H_y}{4\pi}(ik_y h_x - ik_x h_y) \tag{8}$$

$$\rho\left[\frac{i}{\varepsilon}(n) + \frac{\nu}{k_1} + \left(\frac{\alpha_0}{\varepsilon}\right)\frac{(in)}{\tau(in)+1}\right]v = -ik_y\delta p + \frac{\mu_0 H_x}{4\pi}(ik_x h_y - ik_y h_x) \tag{9}$$

$$\rho\left[\frac{i}{\varepsilon}(n) + \frac{\nu}{k_1} + \left(\frac{\alpha_0}{\varepsilon}\right)\frac{(in)}{\tau(in)+1}\right]w = -D\delta p - g\delta\rho + \frac{\mu_0 H_x}{4\pi}(ik_x h_z - Dh_x) +$$

$$+\frac{\mu_0 H_x}{4\pi}(ik_y h_z - Dh_y) - \sum T_s(k_x^2 + k_y^2)\delta z_s\delta(z-z_s) \tag{10}$$

$$(\varepsilon in)\delta\rho = -wD\rho \tag{11}$$

$$i\varepsilon nh_{\mathrm{x}} = (ik_{\mathrm{x}}H_{\mathrm{x}} + ik_{\mathrm{y}}H_{\mathrm{y}})u \tag{12}$$

$$i\varepsilon nh_{\mathrm{y}} = (ik_{\mathrm{x}}H_{\mathrm{x}} + ik_{\mathrm{y}}H_{\mathrm{y}})v \tag{13}$$

$$i\varepsilon nh_{\mathrm{z}} = (ik_{\mathrm{x}}H_{\mathrm{x}} + ik_{\mathrm{y}}H_{\mathrm{y}})w \tag{14}$$

$$ik_{\mathrm{x}}u + ik_{\mathrm{y}}v + Dw = 0 \tag{15}$$

$$ik_{\mathrm{x}}h_{\mathrm{x}} + ik_{\mathrm{y}}h_{\mathrm{y}} + Dh_{\mathrm{z}} = 0 \tag{16}$$

where D stands for the operator d/dz, $\alpha_0 = mN/\rho$, and $\nu = \mu/\rho$ denotes the mass concentration of the particles and kinematic viscosity of the fluid, respectively.

The dispersion relation governing the vertical component of velocity w, for the assumed configuration can be obtained by solving eqs. (8)-(10) with the help of eqs. (11)-(16) and written:

$$\left[\frac{i}{\varepsilon}(n) + \frac{\nu}{k_1} + \left(\frac{\alpha_0}{\varepsilon}\right)\frac{(in)}{\tau(in)+1}\right][D(\rho Dw) - k^2\rho w] - \\ -\frac{\mu_0 i}{4\pi}\left[\frac{(H_{\mathrm{x}}k_{\mathrm{x}} + k_{\mathrm{y}}H_{\mathrm{y}})^2}{4\pi\varepsilon n}\right](D^2 - k^2)w - \frac{igk^2 w}{\varepsilon n}\left[D\rho - \frac{k^2}{g}\sum T_{\mathrm{s}}(z - z_{\mathrm{s}})\right] = 0 \tag{17}$$

Equation (17) represents the differential form of the general dispersion relation. It incorporates the effect of the surface tension, the 2-D uniform magnetic field, the suspended dust particles, the kinetic viscosity (ν) and the porosity of the medium (ε). The dispersion relation (17) is identical with eq. (6) obtained by Prajapati and Chhajlani [22] excluding the effect of magnetic field in static R-T configuration ($U_0 = 0$). Thus, the presence of magnetic field in both x and y directions modifies the results of this problem, thus leading to a better explanation of the static R-T configuration of two superposed magnetized fluids in the presence of suspended dust particles flowing through a porous medium. This dispersion relation is solved employing the boundary conditions for the considered interface, as discussed in continuation.

Boundary conditions at the interface

Consider two superposed fluids of uniform density ρ_1 (lower fluid) and ρ_2 (upper fluid), uniform kinematic viscosities, in a constant porosity medium, in the presence of uniform magnetic fields separated by a horizontal boundary at $z = 0$. We have assumed the number density of suspended particles in both the regions $z < 0$ and $z > 0$ to be the same. In case of constant fluid density, eq. (17) reduces to:

$$(D^2 - k^2)w = 0 \tag{18}$$

The general solutions of eq. (18) for the lower and upper fluids can be written:

$$w_1 = A_1(\varepsilon n)e^{kz} \quad (z < 0) \tag{19}$$

$$w_2 = A_2(\varepsilon n)e^{-kz} \quad (z > 0) \tag{20}$$

where A_1 and A_2 are arbitrary constants. Applying the boundary conditions, given by Chandrasekhar [3] at the common interface of two magnetized fluids which are:

(1) The velocity w should vanish when $z \to \infty$ (for the upper fluid) and $z \to -\infty$ (for lower fluid).
(2) $w(z)$ is continuous at $z = 0$, hence $A_1 = A_2 = A$.
(3) The jump condition at the interface $z = 0$ between the fluids is obtained by integrating eq. (17) over an infinitesimal element of z including 0, to get:

$$\Delta_0\left[\rho Dw\left\{\frac{ni}{\varepsilon}+\frac{\nu}{k_1}+\frac{\alpha_0(in)}{\varepsilon[\tau(in)+1]}\right\}\right]-\frac{\mu_0 i}{4\pi\varepsilon n}\{k_x H_x + k_y H_y\}^2\Delta_0(Dw)- -\frac{igk^2}{n\varepsilon}\left[\Delta_0(\rho)-\frac{k^2}{g}T_s\right]w_{z=0}=0 \tag{21}$$

Using eqs. (19) and (20) in eq. (21) we obtain the following dispersion relation:

$$\sigma^3+\sigma^2\left[f_s\left(1+\frac{2mN}{\rho_1+\rho_2}\right)+\frac{\varepsilon}{k_1}(\beta_1\nu_1+\beta_2\nu_2)\right]+\sigma\left[\frac{\varepsilon f_s}{k_1}(\beta_1\nu_1+\beta_2\nu_2)+2(k_s V_A+k_y V_B)^2- -gk(\beta_2-\beta_1)+\frac{k^3 T}{\rho_1+\rho_2}\right]+f_s\left[2(k_x V_A+k_y V_B)^2-gk(\beta_2-\beta_1)+\frac{k^3 T}{\rho_1+\rho_2}\right]=0 \tag{22}$$

where $f_s = 1/\tau$, is the relaxation frequency, $\sigma = in$, $\alpha_{1,2} = mN/\rho_{1,2}$, $\beta_1 = \rho_1/(\rho_1 + \rho_2)$, $\beta_2 = \rho_2/(\rho_1 + \rho_2)$, $\nu_1 = \mu_1/\rho_1$, and $\nu_2 = \mu_2/\rho_2$. In the eq. (22) the Alfven velocity is defined:

$$V_{A,B}^2=\frac{\mu_0 H_{x,y}^2}{4\pi(\rho_1+\rho_2)} \tag{23}$$

Equation (22) represents the effect of surface tension with combined influence of suspended particles, porosity, kinematic viscosity, and two directional magnetic fields on the R-T instability of two superposed conducting incompressible fluids. If we ignore the effect of magnetic field in the dispersion relation (22), it reduces to what was obtained by Prajapati and Chhajlani [22]. If we ignore the effects of surface tension and transverse magnetic field, $V_B = 0$, then this dispersion relation reduces to eq. (32) that was obtained by Sunil and Chand [17]. For the infinite permeable medium along with $\varepsilon = 1$, $\nu_1 = \nu_2 = V_{A,B} = 0$, the above dispersion relation reduces to the result obtained in Sharma *et al.* [21] excluding the effect of rotation. In the absence of porosity, kinetic viscosity and surface tension, the previous dispersion relation is similar to that obtained by Prajapati *et al.* [13]. It is clear that the combined effect of surface tension, suspended dust particles, porosity, and 2-D magnetic fields modify the present dispersion relation of R-T instability of two superimposed magnetized fluids.

Discussion of dispersion relation

Stable configuration ($\beta_1 > \beta_2$) or ($\rho_1 > \rho_2$)

Equation (22) represents the dispersion relation of two superimposed magnetized partially ionized and conducting fluids with suspended particles flowing through a porous medium. In this potentially stable arrangement, when the lower fluid is heavier than the upper fluid, it clearly represents a stable configuration. We apply the necessary condition of the Hurwitz criterion [24] in eq. (22), according to which we find that all the coefficients of the polynomial equations are positive and real. In order to satisfy the sufficient condition the Hurwitz minors are calculated which are found to be positive. Hence the system is potentially stable for the above considered configuration.

Unstable configuration ($\beta_2 > \beta_1$) or ($\rho_2 > \rho_1$)

The system is stable or unstable depending on the following conditions:

(a) If $2(k_x V_A + k_y V_B)^2 + k^3 T/(\rho_1 + \rho_2) > gk(\beta_2 - \beta_1)$, the system is stable because there is no positive real root.

(b) If $2(k_x V_A + k_y V_B)^2 + (k^3 T)/(\rho_1 + \rho_2) < gk(\beta_2 - \beta_1)$, the system is unstable because the constant term is negative, therefore allowing at least one positive real root.

The occurrence of a positive root implies that the system is unstable. When we put $k_x = k\sin\theta$ and $k_y = k\cos\theta$ in the condition (b), then it may be written:

$$2k^2(\sin\theta V_A + \cos\theta V_B)^2 + \frac{k^3 T}{\rho_1 \rho_2} < gk(\beta_2 - \beta_1) \tag{24}$$

where θ is the inclination of the wave vector k to the direction of the magnetic field $\vec{H}$.

If we substitute $T_{\text{eff}} = 2(P_2 + P_1)(V_A \sin\theta + V_B \cos\theta)^2/k$ then condition of instability (24) becomes:

$$\left[\left(\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\right) - \frac{k^2(T + T_{\text{eff}})}{g(\rho_2 + \rho_1)}\right] > 0 \tag{25}$$

where T_{eff} is the effective surface tension of magnetic field due to the tension along the lines of force. Condition (25) represents the combined effect of surface tension and 2-D magnetic fields and these results are very much similar to those given in Chandrasekhar [3].

The condition of instability (25) is satisfied for $\rho_2 > \rho_1$ when wave number k is taken to be in range $0 < k < k_c$, where:

$$k_c = \frac{2\pi}{\lambda_c} = \sqrt{\frac{(\rho_2 - \rho_1)g}{T + T_{\text{eff}}}} \tag{26}$$

The arrangement remains stable for all disturbances with $k > k_c$, hence the system is unstable for sufficiently long wavelengths. Since the value of critical wave number k_c decreases with an increase of magnetic field, the system is unstable for small magnetic fields. For large magnetic fields the system tends to move towards stability. Similarly, surface tension also has a stabilizing effect on the system. Thus both the magnetic field and surface tension succeed in stabilizing a potentially unstable arrangement.

Now, assuming that both fluids have different dynamic viscosities then from eq. (22) we get:

$$\sigma^3 + \sigma^2\left[f_s(1 + 2\alpha') + \frac{\varepsilon(\nu_1' + \nu_2')}{k_1}\right] + \sigma\left[\frac{\varepsilon f_s(\nu_1' + \nu_2')}{k_1} + 2(V_A \sin\theta + V_B \cos\theta)^2 k^2 - \right.$$
$$\left. - gk(\beta_2 - \beta_1) + \frac{k^3 T}{\rho_1 + \rho_2}\right] + f_s\left[2(V_A \sin\theta + V_B \cos\theta)^2 k^2 - gk(\beta_2 - \beta_1) + \frac{k^3 T}{\rho_1 + \rho_2}\right] = 0 \tag{27}$$

where $\nu_1' = \mu_1/(\rho_1 + \rho_2)$, $\nu_2' = \mu_2/(\rho_1 + \rho_2)$, and $\alpha' = mN/(\rho_1 + \rho_2)$.

Under the consideration of equal dynamic viscosities, the condition of R-T instability will not be changed and it is clear that the growth rate is a function of relaxation frequency and mass concentration of suspended dust particles. Thus the growth rate will be affected by these parameters but condition of instability remains unchanged.

When we neglect the porosity of the medium, eq. (27) reduces to:

$$\sigma^3+\sigma^2[f_s(1+2\alpha')]+\sigma\left\{2(V_\text{A}\sin\theta+V_\text{B}\cos\theta)^2k^2-gk(\beta_2-\beta_1)+\frac{k^3T}{\rho_1+\rho_2}\right\}+$$
$$+f_\text{s}\left[2(V_\text{A}\sin\theta+V_\text{B}\cos\theta)^2k^2-gk(\beta_2-\beta_1)+\frac{k^3T}{\rho_1+\rho_2}\right]=0 \tag{28}$$

In the absence of porosity or dynamic viscosity, condition of instability remains the same. Hence, there is no effect of porosity and dynamic viscosity on the conditions of instability in a magnetized system.

Now, we can write the dispersion relation (27) in dimensionless form and try to find out the combined effect of magnetic field, surface tension, porosity, density difference of fluids, and suspended dust particles on the growth rate of R-T instability. The dimensionless equation can be obtained by using the substitutions:

$$\sigma^*=\frac{\sigma}{\sqrt{gk}},\quad f_\text{s}^*=\frac{f_\text{s}}{\sqrt{gk}},\quad v_1^*=\frac{v_1'}{\sqrt{gk}},\quad v_2^*=\frac{v_2'}{\sqrt{gk}},$$
$$V_\text{A}^*=\sqrt{\frac{k}{g}}V_\text{A},\quad V_\text{B}^*=\sqrt{\frac{k}{g}}V_\text{B},\quad T^*=\frac{k^2T}{g(\rho_1+\rho_2)} \tag{29}$$

Using eq. (29), eq. (27) can be written as:

$$\sigma^{*3}+\sigma^{*2}\left[f_s^*(1+2\alpha')+\frac{\varepsilon(v_1^*+v_2^*)}{k_1}\right]+\sigma^*\left[\frac{\varepsilon(v_1^*+v_2^*)f_s^*}{k_1}+2(V_A^*\sin\theta+V_B^*\cos\theta)^2-\right.$$
$$\left.-(\beta_2-\beta_1)+T^*\right]+f_s^*[2(V_A^*\sin\theta+V_B^*\sin\theta)^2-(\beta_2-\beta_1)+T^*]=0 \tag{30}$$

Our interest is to investigate the effects of various physical parameters on the growth rate of unstable magnetized system. To accomplish this, we have solved eq. (30) numerically to determine the values of growth rate against the relaxation frequency of suspended particles for the various values of one parameter, taking fixed values of other parameters of dynamic viscosity, density of suspended particles, density difference of fluids, medium porosity, surface tension, and magnetic field.

In fig. 1, growth rate (σ^*) of unstable R-T instability is plotted against the relaxation frequency (f_s^*) of suspended particles for the different values of medium porosity ε = 0.15, 0.25, and 0.35 with and without uniform magnetic field ($V_\text{A}^*=V_\text{B}^*$ = 0.15 and $V_\text{A}^*=V_\text{B}^*$ = 0), represented by solid and dashed line, respectively, taking arbitrary fixed values of surface tension T^* = 0.3, and α' = 0.2, v_1^* = 0.3, v_2^* = 0.4, k_1 = = 0.1, $(\beta_2-\beta_1)$ = 0.5 and θ = 45°. We notice that the growth rate decreases as the medium porosity increases in the presence or absence of mag-

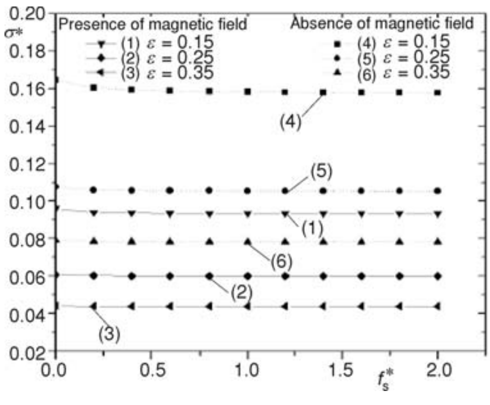

Figure 1. The growth rate (σ*) *vs.* relaxation frequency of suspended particles (f_s^*) in variation of medium porosity (ε) in presence and absence of magnetic field for the fixed values of: α' = 0.2, v_1^* = 0.3, v_2^* = 0.4, k_1 = 0.1, T^* = 0.3, $\beta_2-\beta_1$ = 0.5, $V_\text{A}^*=V_\text{B}^*$ = (0.15, 0), and θ = 45°

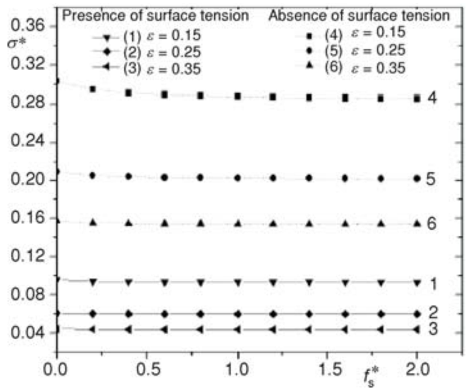

Figure 2. The growth rate (σ^*) *vs.* relaxation frequency of suspended particles (f_s^*) in variation of medium porosity (ε) in presence and absence of magnetic field for the fixed values of: $\alpha' = 0.2$, $\nu_1^* = 0.3$, $\nu_2^* = 0.4$, $k_1 = 0.1$, $T^* = (0.3, 0)$, $\beta_2 - \beta_1 = 0.5$, $V_A^* = V_B^* = 0.15$, and $\theta = 45°$

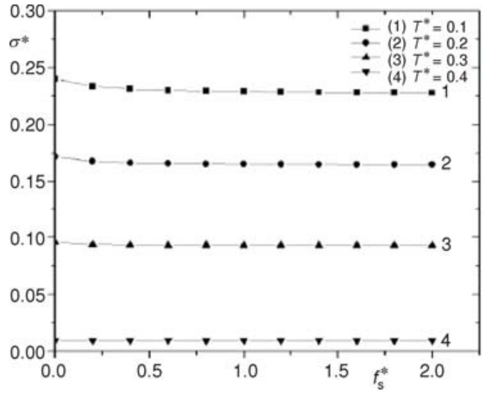

Figure 3. The growth rate (σ^*) *vs.* relaxation frequency of suspended particles (f_s^*) in variation of surface tension (T^*) for the fixed values of: $\alpha' = 0.2$, $\nu_1^* = 0.3$, $\nu_2^* = 0.4$, $k_1 = 0.1$, $\beta_2 - \beta_1 = 0.5$, $\varepsilon = 0.15$, $V_A^* = V_B^* = 0.15$, and $\theta = 45°$

netic field. Thus the medium porosity influences the stability of the system. Similarly fig. 2 is plotted for the different values of medium porosity $\varepsilon = 0.15$, 0.25, and 0.35 with and without surface tension at fixed value of uniform magnetic field ($V_A^* = V_B^* = 0.15$), represented by solid and dashed lines, respectively, taking arbitrary fixed values of other parameters as previously. We find that the growth rate decreases as the medium porosity increases, which shows the stabilizing behaviour of medium porosity in the absence or presence of surface tension.

In fig. 3 the growth rate is plotted against the relaxation frequency of suspended particles for different values of surface tension in the porous medium with the fixed values of other parameters. Figure 3 demonstrates that the growth rate is suppressed as the surface tension increases, which shows the stabilizing behaviour of surface tension. We therefore, conclude with the help of figs. 2 and 3 that the surface tension has stabilizing behaviour in the porous medium.

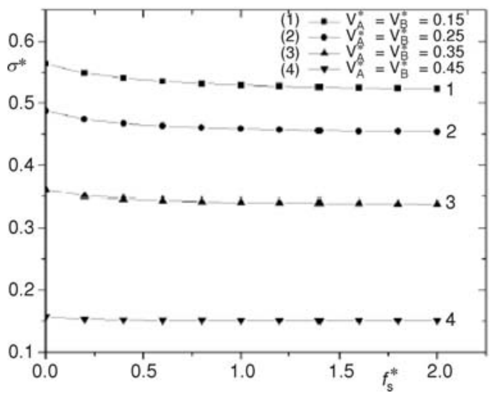

Figure 4. The growth rate (σ^*) *vs.* relaxation frequency of suspended particles (f_s^*) in variation of magnetic field ($V_A^* = V_B^*$) for the fixed values of: $\alpha' = 0.2$, $\nu_1^* = 0.3$, $\nu_2^* = 0.4$, $k_1 = 0.1$, $\beta_2 - \beta_1 = 1.2$, $\varepsilon = 0.15$, $T^* = 0.2$, and $\theta = 45°$

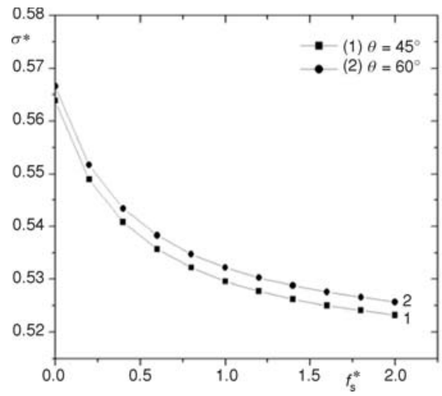

Figure 5. The growth rate (σ^*) *vs.* relaxation frequency of suspended particles (f_s^*) in variation of angle (θ) for the fixed values of: $\alpha' = 0.2$, $\nu_1^* = 0.3$, $\nu_2^* = 0.4$, $k_1 = 0.1$, $\beta_2 - \beta_1 = 1.2$, $\varepsilon = 0.15$, $T^* = 0.2$, and $V_A^* = V_B^* = 0.15$

In fig. 4, the growth rate decreases with increasing uniform magnetic field in the presence of suspended particles and surface tension when the flow domain is porous. It indicates the stabilizing behaviour of magnetic field. The same stabilizing behaviour of magnetic field is seen in fig. 1. In fig. 5 it is seen that growth rate increases as the angle of inclination increases (θ = 45°, 60°) in the presence of uniform magnetic field ($V_A^* = V_B^*$ = 0.15), which shows that stability of the system depends on the orientation of magnetic field.

In figs. 6-8 we emphasized the role of dynamic viscosity of the fluid in the porous medium with suspended particles. In fig. 6, the growth rate of R-T instability is plotted against

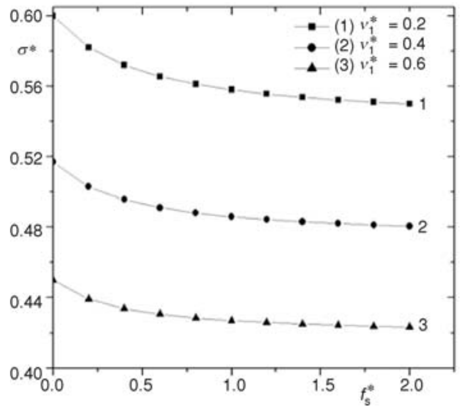

Figure 6. The growth rate (σ*) *vs.* relaxation frequency of suspended particles (f_s^*) in variation of dynamic viscosity of lower fluid (ν_1^*) for the fixed values of: α' = 0.2, ν_2^* = 0.3, k_1 = 0.1, $\beta_2 - \beta_1$ =1.2, ε = 0.15, T^* = 0.3, θ = 45°, and $V_A^* = V_B^*$ = 0.15

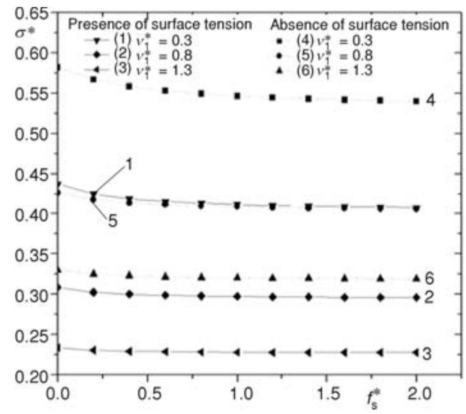

Figure 7. The growth rate (σ*) *vs.* relaxation frequency of suspended particles (f_s^*) in variation of dynamic viscosity of upper fluid (ν_2^*) in presence and absence of surface tension for the fixed values of: α' = 0.2, , ν_1^* = 0.4 , k_1 = 0.1, $\beta_2 - \beta_1$ =1.2, ε = 0.15, T^* = 0.3, θ = 45°, and $V_A^* = V_B^*$ = 0.25

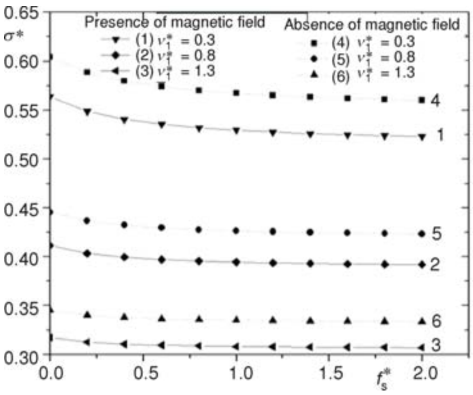

Figure 8. The growth rate (σ*) *vs.* relaxation frequency of suspended particles (f_s^*) in variation of dynamic viscosity of upper fluid (ν_2^*) in presence and absence of magnetic field for the fixed values of: α' = 0.2, ν_1^* = 0.4, k_1 = 0.1, $\beta_2 - \beta_1$ = 1.2, ε = 0.15, T^* = 0.2, θ = 45°, and $V_A^* = V_B^*$ = 0.25

the relaxation frequency of suspended particles for the different values of dynamic viscosity of the lower fluid ν_1^* = 0.2, 0.4, and 0.6 , with fixed dynamic viscosity of upper fluid, ν_2^* = 0.3, in the presence of surface tension, magnetic field and permeability. We find that the growth rate decreases with increasing dynamic viscosity of the lower fluid, considering the fixed values of other parameters. In fig. 7, the growth rate is plotted for different values of dynamic viscosity of the upper fluid, ν_2^* = 0.3, 0.8, and 1.3, with fixed dynamic viscosity of lower fluid, ν_1^* = 0.4, in the absence and presence of surface tension in the porous medium, represented by dashed line and solid line, respectively. We find that the growth rate decreases with increasing dynamic viscosity of upper fluid. Similarly, in fig. 8 we have plotted the growth rate of R-T instability for the same values of dynamic viscosity of the upper fluid, considered in fig. 7, in the absence and presence of uniform magnetic field ($V_A^* = V_B^* = 0.25$) represented by dashed line and solid line, respectively. In all three cases discussed we find that dynamic viscosities of upper and lower fluids have stabilizing influences.

In figs. 9 and 10, the growth rate σ^* of unstable R-T instability is plotted against the relaxation frequency f_s^* of suspended particles for the different number densities of suspended particles in the absence and presence of surface tension, respectively, and find that the growth rate decreases with increasing number densities of suspended particles, which shows the stabilizing behaviour of suspended particles in the porous medium.

Therefore, from the previousgraphical representations we find that all the parameters, surface tension, kinetic viscosity, porosity of the medium, number density of suspended particles, and magnetic field have stabilizing influence.

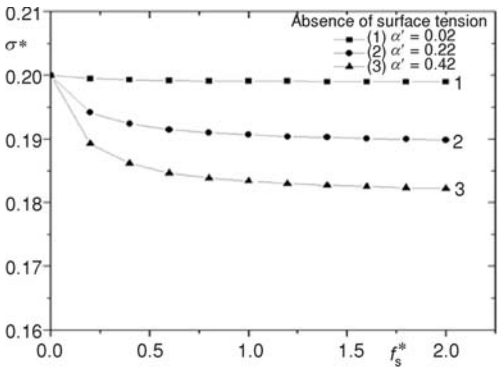

Figure 9. The growth rate (σ^*) *vs.* relaxation frequency of suspended particles (f_s^*) in variation of density of suspended particles (α') in absence of surface tension for the fixed values of: $v_1^* = 0.3$, $v_2^* = 0.4$, $k_1 = 0.1$, $\beta_2 - \beta_1 = 0.5$, $\varepsilon = 0.15$, $T^* = 0$, $\theta = 45°$, and $V_A^* = V_B^* = 0.25$

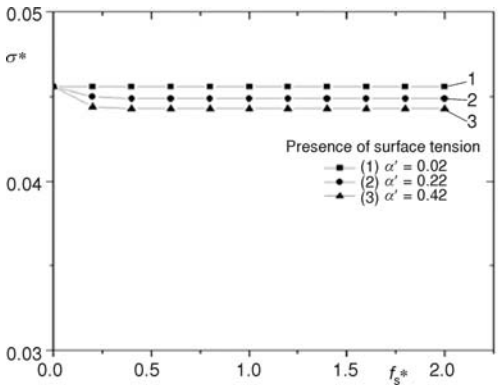

Figure 10. The growth rate (σ^*) *vs.* relaxation frequency of suspended particles (f_s^*) in variation of density of suspended particles (α') in presence of surface tension for the fixed values of $v_1^* = 0.3$, $v_2^* = 0.4$, $k_1 = 0.1$, $\beta_2 - \beta_1 = 0.5$, $\varepsilon = 0.15$, $T^* = 0.2$, $\theta = 45°$, and $V_A^* = V_B^* = 0.25$

Conclusions

In the present work, we have investigated the combined effects of surface tension and suspended dust particles on hydromagnetic R-T instability of two incompressible superimposed magnetized fluids flowing through porous media. The MHD equations are modified and linearized for the considered system and a general dispersion relation is obtained using the normal mode analysis. We found that the dispersion characteristics are strongly affected by the presence of medium porosity, suspended dust particles, permeability, magnetic field, and surface tension. The onset criteria of R-T stability and instability are obtained, and shown to depend upon effective surface tension and two directional magnetic fields. The potential stable and unstable configurations are also discussed and critical wave number for the stable and unstable arrangements are calculated.

The numerical calculations and the graphs show that the growth rate of unstable R-T instability varies with respect to the relaxation frequency of suspended dust particles. We observe that the growth rate decreases as the medium porosity increases in both the magnetized and un-magnetized cases. Hence, the medium porosity has stabilizing influence on the growth rate of the instability. Surface tension is also observed to have a similar effect on the growth rate of

R-T instability, while the kinematic viscosity has its usual damping effect on the growth rate of unstable R-T modes.

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Nomenclature

a – suspended dust particle radius, [m]
f_s – relaxation frequency of suspended dust particles, [s^{-1}]
$\vec{g}$ – acceleration due to gravity, [ms^{-2}]
$\vec{H}$ – magnetic field, (H_x, H_y), [Am^{-1}]
$\vec{h}$ – perturbed magnetic field, (h_x, h_y, h_z), [Am^{-1}]
k_1 – permeability of porous medium, [m^{-2}]
k_x, k_y – horizontal wave numbers, [m^{-1}]
m – mass of suspended dust particles, [kg]
N – number density, [m^{-3}]
p – fluid pressure, [Pa]
T – surface tension, [$kgms^{-2}$]
t – time, [s]
$\vec{u}$ – fluid velocity, (u, v, w), [ms^{-1}]
$\vec{v}$ – suspended dust particle velocity, [ms^{-1}]
V_A, V_B – Alfven velocity [ms^{-1}]

Greek symbols

β – density ratio
ε – porosity of porous medium
θ – inclination of the wave vector k to the direction of the magnetic field
μ – dynamic viscosity, [$kgm^{-1}s^{-1}$]
μ_0 – magnetic permeability, [Hm^{-1}]
ν_1 – kinematic viscosity of lower fluid, [m^2s^{-1}]
ν_2 – kinematic viscosity of upper fluid, [m^2s^{-1}]
ρ – density, [kgm^{-3}]
σ – growth rate, [s^{-1}]
τ – relaxation time of suspended dust particles, [s]

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