SIMULTANEOUS RECONSTRUCTION OF TEMPERATURE FIELD AND RADIATIVE PROPERTIES BY INVERSE RADIATION ANALYSIS USING STOCHASTIC PARTICLE SWARM OPTIMIZATION

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Simultaneous reconstruction of temperature field and radiative properties including scattering albedo and extinction coefficient is presented in a two-dimensional (2-D) rectangular, absorbing, emitting and isotropically scattering gray medium from the knowledge of the exit radiative intensities received by charge-coupled device (CCD) cameras at boundary surfaces. The inverse problem is formulated as a non-linear optimization problem and solved by stochastic particle swarm optimization. The effects of particle swarm size, generation number, measurement errors, and optical thickness on the accuracy of the estimation, and computing time were investigated and the results show that the temperature field and radiative properties can be reconstructed well for the exact and noisy data, but radiative properties are harder to obtain than temperature field. Moreover, the extinction coefficient is more difficult to reconstruct than scattering albedo.

Keywords: Simultaneous reconstruction, Temperature field, Radiative properties, Particle swarm optimization

1. Introduction

The inverse radiation analysis in the participating medium has many applications, such as the determination of the radiative properties of the medium, the estimation of the temperature distribution and so on. A comprehensive review of inverse radiation problems has been given by McCormick [1]. Lots of work has been reported on the estimation of temperature field or source term distribution under the assumption of known radiative properties of the medium, such as the work of Li and Özişik [2], Siewert [3,4], Li [5-7], Liu et al. [8-10], Zhou et al. [11] and Liu et al. [12-16]. Simultaneous estimation of temperature profile and properties of boundary surface were also conducted by some researchers, e.g., Li and Özişik [17] presented a method for simultaneous estimation of the unknown temperature distribution and the diffuse surface reflectivity in the participating medium, and Liu et al. [18,19] conducted the simultaneous identification of the temperature profile or sources and the boundary emissivities in one-dimensional (1-D) semitransparent medium.

Simultaneous estimation of temperature field and the radiative properties of the participating medium or soot volume fraction of flame have aroused many interests of researchers. Liu et al. [20] carried out the simultaneous identification of temperature profile and uniform absorption coefficient in 1-D semitransparent medium. Zhou et al. [21] conducted simultaneous estimation of temperature, absorption and scattering coefficient profile in 1-D medium from the boundary intensity and boundary
temperature profiles. Snelling et al. [22] developed a multiwavelength flame emission technique for high spatial resolution determination of soot temperature and soot volume fraction in axisymmetric laminar diffusion flames. Ayrancı et al. [23] determined soot temperature, volume fraction and refractive index from flame emission spectrometry for an optically thin axisymmetric flame. Zhou et al. [24] reconstructed temperature distribution, absorptivity of wall surface and absorption coefficient of medium in a 2-D furnace system filled with gray emitting/absorption medium. Lou et al. [25] used flame images to obtain the flame emissivity and the uniform radiative properties of the particulate media in pulverized-coal-fired boiler furnaces by modified Tikhonov regularization method and iteration algorithm. Liu et al. presented inverse radiation analysis for simultaneous estimation of temperature field and uniform radiative properties in participating medium using LSQR method and 2-D searching [26], and also used CCD cameras to simultaneously measure three-dimensional (3-D) soot temperature and volume fraction fields in axisymmetric or asymmetric small unconfined flames with optically thin medium [27, 28]. Lou et al. [29] reported simultaneous measurement of temperature field, uniform absorption coefficient of medium and emissivity of wall surface by inverse analysis. Wang et al. [30] reconstructed 2-D temperature and particle concentration distribution from the image of the pulverized coal flame simultaneously using optimization method by ignoring scattering of the medium, but in the large pulverized coal-fired furnace, the scattering should not be neglected in most space of the furnace [31]. In the real situation, the distribution of radiative properties is generally non-uniform, but simultaneous estimation of 2-D or 3-D temperature distribution and non-uniform radiative properties is still a formidable work. Therefore, the available studies of simultaneous estimation of 2-D temperature distribution and radiative properties in participating medium are only limited to the uniform radiative properties [24-26, 29]. Lou et al. [29, 32] have reported that using the uniform radiative properties to reconstruct the temperature distribution in the inhomogeneous system may take some errors, and have proved that the maximum reconstruction error of temperature is less than 6%.

In recent years, as an alternative to gradient-based methods, genetic algorithm (GA) and particle swarm optimization (PSO) algorithm have increasingly been applied to the inverse radiation problem [33-42]. In the present study, we used stochastic PSO for simultaneous reconstruction of temperature field and radiative properties including scattering albedo and extinction coefficients with different kinds of optical thicknesses in a large 2-D rectangular, absorbing, emitting and isotropically scattering medium from the knowledge of the exit radiative intensities received by CCD cameras at the boundary surfaces. The effects of particle swarm size, generation number, measurement errors, and optical thickness on the accuracy of the estimation, and computing time were investigated.

2. Analysis

2.1. Direct problem

For the participating medium the radiative heat transfer equation can be written as follows [43],

$$\frac{dI_{\lambda}}{d\tau_{\lambda}} + I_{\lambda} = S_{\lambda}(\tau_{\lambda}, \hat{s})$$  \hspace{1cm} (1)
with \( S_\lambda(\tau_\lambda, \hat{s}) = (1 - \omega) I_{b\lambda} + \frac{\omega}{4\pi} \int_4 \Phi_\lambda(\hat{s}_i) S_\lambda(\hat{s}_i, \hat{s}) d\Omega \),

where \( \hat{s} \) is a unit local direction vector, \( S \) is the local radiative source, \( \omega \) is the local scattering albedo, \( \tau_\lambda \) is the optical thickness, \( \Phi_\lambda \) is the scattering phase function, and \( \Omega \) is the solid angle.

Considering the following system as shown in Fig.1, integration of Eq. (1) over a single radiation ray in one volume element from \( \tau_{in} \) to \( \tau_{out} \) results in,

\[
I_\lambda(\tau_{out}) = I_\lambda(\tau_{in}) e^{-\tau} + \int_{\tau_{in}}^{\tau_{out}} S_\lambda(\tau'_i, \hat{s}) e^{-(\tau_{out} - \tau'_i)} d\tau'_i
\]  

(2)

where \( \tau = \tau_{out} - \tau_{in} \).

It was pointed out by Lockwood and Shah [44] that the in-scattering term is zero for gas flames, negligible for oil flames, while for coal flames it is often acceptable to ignore it with the effect of particle scattering largely being accounted for by the out-scattering term. This approximation was later utilized in numerical simulations for coal-fired utility boilers [45,46] and in their studies comparisons between numerical simulation and measurements were also carried out. The temperature and radiative properties are assumed to be uniform within one volume element. Based on the above considerations, Eq. (2) can be simplified to

\[
I_\lambda(\tau_{out}) = I_\lambda(\tau_{in}) e^{-\tau} + (1 - \omega) I_{b\lambda} (1 - e^{-\tau})
\]  

(3)

Now considering the whole ray \( j \) from one side of the system to the CCD camera, based on Eq. (3) the intensity received by CCD camera can be written as

\[
I_{\lambda j} = I_{\lambda j} e^{-(\tau_{out} - \tau_{in})} + (1 - \omega_j) I_{b\lambda j} (1 - e^{-\tau_j}) + \cdots + (1 - \omega_s) I_{b\lambda s} (1 - e^{-\tau_s})
\]  

(4)

where \( s \) is the \( s \)th volume element of ray traversing and \( S \) is the total volume element number of ray traversing.

If at the CCD camera location the discrete direction number is assumed to be \( M \), the following equations can be obtained based on Eq. (4),

\[
f_1(I_{b\lambda 1}, I_{b\lambda 2}, \ldots, I_{b\lambda N}) = I_{\lambda 1}
\]

\[
\vdots
\]

\[
f_j(I_{b\lambda 1}, I_{b\lambda 2}, \ldots, I_{b\lambda N}) = I_{\lambda j}
\]

\[
\vdots
\]

\[
f_M(I_{b\lambda 1}, I_{b\lambda 2}, \ldots, I_{b\lambda N}) = I_{\lambda M}
\]

(5)

where \( N \) is the total number of volume element.

Usually in the high temperature furnace the water-wall temperatures are much lower than inner medium temperatures, rewrite Eq. (5) to the matrix form,

\[
AI_b = I_{CCD}
\]  

(6)

where \( A \) is the coefficient matrix, \( I_b \) is the unknown blackbody intensity vector from which the temperatures can be obtained, and \( I_{CCD} \) is the monochromatic intensity vector received by CCD cameras, which is the input data in the reconstruction process.

In the direct problem, for the known temperature field and radiative properties including scattering albedo and extinction coefficient (absorption and scattering coefficients), the intensities received by CCD cameras can be obtained by Eq. (6).
2.2. Inverse problem

For the inverse problem, the temperature field and radiative properties are regarded as unknown and the intensities received by CCD cameras are known. The temperature field and radiative properties can be reconstructed simultaneously by using the intensities received by CCD cameras.

Simultaneous reconstruction of temperature field and radiative properties including scattering albedo and extinction coefficient is a non-linear optimization problem. Stochastic PSO was used to directly solve this non-linear optimization problem. The simple principle of stochastic PSO algorithm was described as follows [39, 42]. Each individual in particle swarm, referred to as a particle. Every particle moves its position in search domain and updates its velocity according to its own flying experience and neighbor’s flying experience, aiming at a better position for itself and a global fitness.

At generation $t$, the $i$th particle $X_i(t)$ can be described as $X_i(t)=[x_{i1}(t), \ldots, x_{ij}(t), \ldots, x_{in}(t)]^T$, the velocity is described as $V_i(t)=[v_{i1}(t), \ldots, v_{ij}(t), \ldots, v_{in}(t)]^T$, the individual best position is expressed as $P_i(t)=[p_{i1}(t), \ldots, p_{ij}(t), \ldots, p_{in}(t)]^T$, and the global best position of the swarm is expressed as $P_g(t)=[p_{g1}(t), \ldots, p_{gj}(t), \ldots, p_{gn}(t)]^T$, where $x_{ij}(t)$ is the position of the $i$th particle with the $j$th dimension, $v_{ij}(t)$ is the velocity of the $i$th particle with $j$th dimension, $p_{ij}(t)$ is the local best position of the $i$th particle with $j$th dimension, $p_{gj}(t)$ is the best of all positions with $j$th dimension, $n$ is the number of optimization variables.

For the stochastic PSO, the velocity and position of the particle update according to the following equations,

$$V_i(t+1)=c_1r_1[P_i(t)-X_i(t)]+c_2r_2[P_g(t)-X_i(t)] \quad (7)$$
$$X_i(t+1)=X_i(t)+V_i(t+1) \quad (8)$$

where, $c_1$, $c_2$ are acceleration coefficients, $r_1$, $r_2$ are uniform random variables in the range $[0, 1]$.

The objective function is defined as below,

$$f(T, \text{albedo}, \kappa_{es}) = \| A I_b - I_{CCD} \|^2 + \mu \cdot \Psi(T) \quad (9)$$

where $\Psi(T)$ is the smoothing function of temperature field, $\mu$ is the weight coefficient.

3. Results and discussion

As shown in Fig.1, the size of 2-D rectangular enclosure was 10m×10m. The viewing angle of
the CCD camera was assumed to be 120° and for the high temperature system, the walls could be assumed to be black and cold for simplicity. The wavelength used here was 700nm, which was in the visible wavelength range. For the non-linear optimization problem, more discrete volume elements are divided, more computing time is consumed. Considering the computing time and the practical use, the discrete volume elements of system were assumed to be 5×5 in order to reduce the computing time and complexity. In the practical use, when the coarse temperature field is obtained, the main features of the system are obtained, and then more delicate temperature distribution can be deduced by using 2-D interpolation, which already has matured algorithms. The discrete direction number was 25 for each CCD camera.

The measured intensities received by CCD cameras were simulated by adding random errors of normal distribution with zero average value and mean square deviation $\sigma$ to the exact intensities which can be obtained from the direct problem.

$$I_{\text{measured,j}} = \mu + \sigma \xi I_j + I_j$$

where $I_{\text{measured,j}}$ is the element of the simulated measured intensity vector and $I_j$ is the element of the exact intensity vector. Average value $\mu$ equals zero, $\xi$ is a random variable of standard normal distribution and the probability lying in the range of -2.576<\xi<2.576 is 99%. $j=1,2,\ldots,M$.

An assumed temperature field and radiative properties including scattering albedo and extinction coefficient were provided for testing the method developed here. The assumed temperature field was shown in Fig. 4. The scattering albedo was assumed to be 0.5. Two kinds of extinction coefficients were assumed to be a) 0.2 m$^{-1}$ and b) 0.6 m$^{-1}$. The corresponding optical thicknesses $\tau$ were a) 2.0 and b) 6.0. The assumed optical thicknesses were all optically thick.

The relative error of the temperature in each volume element is defined as,

$$E_{\text{rel,i}} = 100 \left| \frac{T_i^{\text{recon}} - T_i^{\text{exact}}}{T_i^{\text{exact}}} \right|$$

where $T_i^{\text{recon}}$ and $T_i^{\text{exact}}$ represent reconstructed and exact temperature respectively, $i=1,2,\ldots,N$.

The maximum, mean, and minimum relative errors of temperature distribution are defined as,

$$\max = \max \{ E_{\text{rel,i}} \}, i = 1,2,\ldots,N, \quad \text{mean} = \sum_{i=1}^{N} E_{\text{rel,i}} / N,$$

$$\min = \min \{ E_{\text{rel,i}} \}, \quad i = 1,2,\ldots,N$$

3.1. Case 1

In this case, the scattering albedo and extinction coefficient are assumed to be 0.5 and 0.2 m$^{-1}$. The optical thickness is 2.0. The searching spaces are [1576K, 1800K] for temperature, [0.1, 0.9] for scattering albedo, and [0.1 m$^{-1}$, 0.8 m$^{-1}$] for extinction coefficient.

3.1.1. Effect of particle swarm size on the fitness of objective function

The effect of particle swarm size on fitness of objective function was examined. Three kinds of swarm size were used, i.e., 30, 50 and 100, as shown in Fig.2. No measurement errors were considered here. The fitness of objective function was the lowest for swarm size 50 in the same generation number. In the generation 20000, the fitnesses of swarm size 50 and 100 are almost the same and it also be noticed from the tendency that after generation 20000 the fitness of swarm size 100 will
decrease lower than that of swarm size 50. The swarm size 50 seems the best candidate of three kinds of swarm sizes before generation 20000. Moreover, larger swarm size is used, more computing time is consumed.

3.1.2. Effects of swarm size and generation on the reconstruction accuracy of radiative properties

Effects of swarm size and generation on the reconstruction accuracy of radiative properties were checked, as shown in Fig. 3. The measurement errors were assumed to be zero here. For all three kinds of swarm sizes, the reconstruction accuracy of albedo and extinction coefficient increases with the generation. For the scattering albedo reconstruction, before generation 10000 the performances of swarm size 50 and 100 are almost the same, better than that of swarm size 30, and after generation 10000, there are no obvious differences for three kinds of swarm sizes. For the extinction coefficient reconstruction, before generation 15000 the results from swarm size 50 are best, and after generation 15000, there are no obvious differences for three kinds of swarm sizes. In the generation 10000, albedo and extinction coefficient can be reconstructed accurately. Computing time increases linearly with the generation for each swarm size. For the same generation, the computing time increases with the swarm size.

3.1.3. Effect of generation on the reconstruction accuracy of temperature field

Measurement errors were assumed to be zero here. It was found that all the three kinds of swarm sizes can obtain satisfying reconstructed temperature field. Particle swarm size 30 was used for examining the effect of generation on the reconstruction accuracy of temperature field for less computing time. The results are shown in Fig. 4. The reconstruction accuracy of temperature field increases with the generation. For the generation 10000, the reconstructed temperature field agrees well with the assumed one.

3.1.4. Effect of measurement errors on the reconstruction accuracy of radiative properties

Effect of measurement errors on the reconstruction accuracy of radiative properties was examined. Swarm size 50 and generation 15000 were used here. As shown in Fig. 5, it was seen that the albedo and extinction coefficient can be reconstructed well under different measurement errors. The measurement errors can affect the reconstruction accuracy but the relative errors can be maintained at a certain level and did not increase fast with the measurement errors. The reconstruction relative errors of extinction coefficient were larger than those of albedo for most of measurement errors, which indicated that extinction coefficient reconstruction was more sensitive to measurement errors than scattering albedo reconstruction.

3.1.5. Effect of measurement errors on the reconstruction accuracy of temperature field

Swarm size 50 and generation 15000 was used here. From Fig. 6, it was noticed that the temperature field can be reconstructed well under different measurement errors. The reconstruction relative errors can be maintained at a low level and did not increase fast with the measurement errors. The reconstruction accuracy of temperature field was better than that of radiative properties, which demonstrated the temperature field was easier to be reconstructed than radiative properties.
3.2. Case 2

In this case, the scattering albedo and extinction coefficient are assumed to be 0.5 and 0.6 m⁻¹. The optical thickness is 6.0. The searching spaces are [1576K, 1800K] for temperature, [0.1, 0.9] for scattering albedo, and [0.1 m⁻¹, 0.8 m⁻¹] for extinction coefficient.

3.2.1. Effects of measurement errors and generation on the reconstruction accuracy of radiative properties

During calculations, it was found that accurate radiative properties can be obtained after generation 50000, so from the discussion of Section 3.1.1, particle swarm size 30 was used here for saving computing time. The reconstructed radiative properties with generation 70000 under different measurement errors were shown in Fig. 7. Comparing Fig. 7 with Fig. 3, as the measurement errors are zero, the reconstructed results with optical thickness 6.0 were much worse than those with optical thickness 2.0, which may be due to the less radiative information received by CCD cameras for larger optical thickness. With measurement errors of σ=0.01, from optical thickness 2.0 to 6.0 the extinction coefficient reconstruction relative error increases largely, but the albedo reconstruction relative error
was similar.

The variations of reconstructed albedo and extinction coefficient with generation under measurement error $\sigma=0.01$ were shown in Fig. 8. It was found that the albedo can be reconstructed well in and after generation 5000, but extinction coefficient can only be reconstructed with reasonable error in generation 70000, which implied that the extinction coefficient was more difficult to reconstruct than the scattering albedo.

The computing time is 726s of generation 70000 and much longer than that for optical thickness 2.0. More time is needed to search for accurate radiative properties (extinction coefficient) when the optical thickness increases.

### 3.2.2. Effects of measurement errors and generation on the reconstruction accuracy of temperature field

The relative errors of reconstructed temperature fields with generation 70000 under different measurement errors with optical thickness 6.0 were shown in Fig. 9. It can be seen that temperature fields can be reconstructed well under different measurement errors. The maximum and mean relative errors do not vary much for different measurement errors, which indicates that the stochastic PSO has anti-error characteristic.

Fig. 10 shows the variations of relative errors of reconstructed temperature fields with generation under measurement error $\sigma=0.01$ for optical thickness 6.0. The maximum and mean relative errors decrease with the generation, and the minimum relative error does not vary much with the generation. As the generation is 30000, the maximum reconstruction relative error of temperature field is 1.88% which is satisfying, and also from Fig. 8 the albedo is reconstructed well. But the extinction coefficient can not be reconstructed well in generation 30000. The computing time is about 313s for generation 30000. All the calculations were carried out on the laptop with an Intel Core 2 Duo CPU 2.40 GHz processor and 2 GByte memory.

### 4. Conclusions

This paper presents simultaneous reconstruction of temperature field and radiative properties including scattering albedo and extinction coefficient in a 2-D rectangular, absorbing, emitting and
scattering gray medium from the knowledge of the exit radiative energy received by CCD cameras at boundary surfaces. The inverse problem is formulated as a non-linear optimization problem and solved by stochastic PSO. Two kinds of optical thicknesses were tested. The effects of particle swarm size, generation number, measurement errors on the accuracy of the estimation were examined. The results show that the temperature field and radiative properties can be reconstructed well for the exact and noisy data. Radiative properties are harder to obtain than temperature field. Moreover, the extinction coefficient is more difficult to reconstruct than scattering albedo.

Fig. 9 Effect of measurement errors on the reconstruction accuracy of temperature field for optical thickness 6.0.

Fig. 10 Variations of relative errors of reconstructed temperatures with generation under $\sigma=0.01$ for optical thickness 6.0.

Nomenclature

$I$ – radiation intensity, $[\text{W/(m}^3\text{sr)}]$  
$s$ – $s$th volume element of ray traversing, [-]  
$S$ – total volume element number of ray traversing, [-]  
$t$ – generation, [-]  
$x$ – position of particle, [-]  
$v$ – velocity of particle, [-]  
$p$ – local best position of particle, [-]  
$p_g$ – best of all positions, [-]  
$n$ – number of optimization variables, [-]  
$\Psi$ – smoothing function of temperature field, [-]  
$T$ – temperature, [K]  
$M$ – discrete direction number [-]  
$A$ – coefficient matrix, [-]  
$I_b$ – blackbody intensity vector, [-]  
$I_{CCD}$ – monochromatic intensity vector, [-]

Greek symbols

$\xi$ – random variable of standard normal distribution, [-]  
$\omega$ – local scattering albedo, [-]  
$\tau$ – optical thickness, [-]  
$\Phi$ – scattering phase function, $[\text{sr}^{-1}]$  
$\Omega$ – solid angle, [sr]

Subscripts

$in$ – entry point of volume boundary  
$out$ – out point of volume boundary  
$\lambda$ – wavelength, [m]  
$ext$ – extinction  
$b$ – blackbody value

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References


