

ON THE COMPRESSIBILITY EFFECTS IN MIXING LAYERS

by

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Previous studies of compressible flows carried out in the past few years have shown that the pressure-strain is the main indicator of the structural compressibility effects. Undoubtedly, these terms plays a key role toward strongly changing magnitude of the turbulent Reynolds stress anisotropy. On the other hand, the incompressible models of the pressure-strain correlation have not correctly predicted compressible turbulence at high speed shear flow. Consequently, a correction of these models is needed for precise prediction of compressibility effects. In the present work, a compressibility correction of the widely used incompressible Launder Reece and Rodi model making their standard coefficients dependent on the turbulent and convective Mach numbers is proposed. The ability of the model to predict the developed mixing layers in different cases from experiments of Goebel and Dutton is examined. The predicted results with the proposed model are compared with direct numerical simulation and experimental data and those obtained by the compressible model of Adumitroiaie et al. and the original Launder Reece and Rodi model. The results show that the essential compressibility effects on mixing layers are well captured by the proposed model.

Key words: *turbulence, compressible, pressure-strain, models of turbulence, mixing layer*

Introduction

Compressible turbulence flows intervene in many technological developments. Although better understanding compressibility effects are highly relevant to important applications in the design of advanced spacecraft, supersonic and hypersonic flights, combustion problems. The works recently developed by Goebel and Dutton [1], Vreman *et al.* [2], Pantano and Sarkar [3], Foysi and Sarkar [4] are of these types of research in which computational and analysis regarding growth rate, turbulence levels of anisotropy and pressure strain correlation of the compressible mixing layers between two streams are examined. From these studies, it has been found that at high speed fluid, the compressibility strongly affect the behavior of the mixing layers. To study these effects, many numerically and experimentally investigations [1, 3, 5] suggested the use of the convective Mach number which is defined by Bogdanoff [6] for two streams with equal ratio of specific heats as $M_c = (U_1 - U_2) / (a_1 - a_2)$, where U_1 , a_1 , and U_2 , a_2 , denoting the velocity and speed sound in the high speed stream and

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the low speed stream, respectively. The previously cited results show that compressibility effects are well described by M_c . One can see the dramatic reduction of the growth rate and of the Reynolds stress when increasing the convective Mach number. Obviously, M_c seems to be an appropriate parameter for studying the stabilization of the structural compressibility effects of the flow in the supersonic regime as it is shown in [7]. Also, this parameter may be useful to establish compressible turbulence models that are indispensable for a precise simulation of high-speed flows.

In the early past few years, previous studies being carried out show that the existent compressible models come from extension of its analogue incompressible models. A major challenge related to this extension is to take into account the compressibility effects in the classical scheme closures of turbulence. In the early 1990s, some compressible models for the dilatational terms: pressure dilatation correlation and the turbulent dilatation dissipation were developed by Zeman [8], Ristorcelli [9], and Sarkar *et al.* [10]. These models are used in conjunction with different incompressible models for the pressure strain correlation as a compressible modeling approach in within Reynolds stress closure [11]. Several studies conjectured that this approach does not reflect the structural compressibility effects on the turbulence. Thus, when the compressibility effects are more significant, the extended models do neither predict correctly the decrease in spreading rate of mixing layers as it is observed in the experiments of Goebel and Dutton [1] and Samimy and Elliot [5], nor the reduction in the growth rate of turbulent kinetic energy Sarkar [7]. The poor predictions of the changes in the magnitude of the Reynolds stress anisotropies show that the dilatational terms effects are much smaller than previously believed. According to the direct numerical simulation (DNS) of Blaisdell and Sarkar [12], the dilatational terms represent nearly 12 per cent of the turbulent kinetic energy production. Sarkar [7], Simone *et al.* [13], and Hamba [14] also performed DNS results of compressible homogeneous shear flow and reached similar conclusions concerning the roles of dilatational terms. They found out a notable decrease of the growth rate of the turbulent kinetic energy when the values of the turbulent Mach number increase, the reduction of the turbulence levels arising from compressibility effects is related to the inhibited turbulence production and not the explicit dilatational terms. These conclusions are confirmed by authors in [2] and [3], in their DNS results which show that compressibility terms do not affect the compressible mixing layer. In contrast to dilatational effects, the structural compressibility effects strongly affect the pressure field, so the pressure fluctuations were reduced. The consequent effects on the pressure-strain correlation may cause significant changes on turbulence structures. As a consequence, the Favre Reynolds stress closure using the standard models of the pressure strain correlation with the addition of the compressible dissipation and pressure-dilatation correlation models failed to predict compressible turbulence at high speeds.

It has been shown from previous works that new models of the pressure strain correlation taking into account structural compressibility effects are needed for precise prediction of hypersonic flows. We quote here the models in [3, 16-19]. The best approach of modelling the structural compressibility effects is to incorporate the extra-compressibility parameters as: turbulent Mach number $M_t = 2K / \bar{a}$, where $K = \overline{\rho u_i' u_i'} / 2\bar{\rho}$ is the turbulent kinetic energy and \bar{a} is the mean speed of sound, gradient Mach number which is defined by $M_g = Sl / \bar{a}$, where $S = (U_{i,j} U_{j,i})^{0.5}$ is the mean shear and l is an integral length scale, and convective Mach number M_c into closure of the pressure strain correlation. The present work focuses on this major issue. In this context, an extension of the Launder *et al.* [20] model in compressible flow making the standard coefficients C_i in function of the turbulent Mach number and the

convective Mach number is proposed. A comparison of our predictions with those obtained by Launder *et al.* [20] and Adumitroaie *et al.* [16] models and with DNS and experiment results is considered, as well.

Governing equations

The general equations governing the motion of a compressible fluid are the Navier-Stokes equations. They can be written as follows for mass, momentum, and energy conservation:

$$\frac{\partial}{\partial t} \rho + \frac{\partial}{\partial x_i} \rho u_i = 0 \quad (1)$$

$$\frac{\partial}{\partial t} \rho u_i + \frac{\partial}{\partial x_j} \rho u_i u_j = \frac{\partial}{\partial x_j} \sigma_{ij} \quad (2)$$

$$\frac{\partial}{\partial t} \rho e + \frac{\partial}{\partial x_j} \rho e u_j = \frac{\partial}{\partial x_j} \sigma_{ij} u_i - \frac{\partial}{\partial x_j} (\kappa T_{,j}) \quad (3)$$

where $e = c_v T$, $\sigma_{ij} = -p\delta_{ij} + \tau_{ij}$ and $\tau_{ij} = 2\mu(u_{i,j} + u_{j,i})$

The Favre averaged equations are:

$$\frac{\partial}{\partial t} \bar{\rho} + \frac{\partial}{\partial x_i} (\bar{\rho} \tilde{U}_i) = 0 \quad (4)$$

$$\frac{\partial}{\partial t} (\bar{\rho} \tilde{U}_i) + \frac{\partial}{\partial x_i} (\bar{\rho} \tilde{U}_i \tilde{U}_j) = \frac{\partial}{\partial x_j} (\bar{\sigma}_{ij} + \bar{\tau}_{ij}'' - \frac{\partial}{\partial x_j} \overline{\rho u_i'' u_j''}) \quad (5)$$

$$\frac{\partial}{\partial t} \bar{\rho} \tilde{e} + \frac{\partial}{\partial x_j} \bar{\rho} \tilde{e} \tilde{U}_j = -\phi_e + \pi_d - \frac{\partial}{\partial x_j} \overline{c_v \rho u_i'' T''} \quad (6)$$

where

$$\phi_e = \bar{p} \frac{\partial}{\partial x_i} (\tilde{U}_i + \overline{u_i''}) + \frac{\partial}{\partial x_i} \overline{(\kappa \frac{\partial}{\partial x_i} T)} + \overline{\tau_{ij} u_{i,j}} \quad \tilde{\tau}_{ij} = 2\mu \tilde{S}_{ij} - \frac{2}{3} \bar{\mu} \tilde{u}_{k,k} \delta_{ij}$$

and $\pi_d = \overline{p' u_{i,i}'}$.

The Favre averaged Reynolds stress $R_{ij} = \overline{\rho u_i'' u_j''} / \bar{\rho}$ are solutions of the transport equation, namely:

$$\frac{\partial}{\partial t} (\bar{\rho} R_{ij}) + \frac{\partial}{\partial x_m} (\bar{\rho} \tilde{U}_m R_{ij}) = P_{ij} + D_{ij} + \phi_{ij} + \varepsilon_{ij} + V_{ij} \quad (7)$$

where the symbols P_{ij} , D_{ijm} , ϕ_{ij} , ε_{ij} and V_{ij} represent turbulent production, turbulent diffusion, pressure strain correlation, turbulent dissipation and the mass flux variation, respectively.

$$P_{ij} = -\bar{\rho} R_{jm} \tilde{U}_{i,m} - \bar{\rho} R_{im} \tilde{U}_{j,m} \quad D_{ijm} = -(\overline{\rho u_i'' u_j'' u_m''} + \overline{p' u_j'' \delta_{im}} + \overline{p' u_i'' \delta_{jm}} - \overline{\tau_{im}'' u_j''} - \overline{\tau_{jm}'' u_i''})$$

$$\phi_{ij} = \overline{p'(u_{i,j}'' + u_{j,i}'')} = \phi_{ij}^* + \frac{2}{3} \overline{p' u_{k,k}'' \delta_{ij}} \quad \varepsilon_{ij} = \overline{\tau_{im}'' u_{j,m}''} - \overline{\tau_{jm}'' u_{i,m}''}$$

$$V_{ij} = -\bar{p}_{,j} \overline{u_i''} + -\bar{p}_{,i} \overline{u_j''} + \tilde{\tau}_{im,m} \overline{u_j''} + \tilde{\tau}_{jm,m} \overline{u_i''}$$

Classically, the second order closure suggests to determine the dissipation term ε_{ij} by using isotropic dissipation model:

$$\varepsilon_{ij} = \frac{2}{3} \varepsilon \delta_{ij} \quad (8)$$

Recently a concept of the dissipation in compressible turbulence was proposed by Sarkar [6, 13], and Zeman [19] as

$$\varepsilon = \varepsilon_s + \varepsilon_c \quad (9)$$

where for homogeneous shear flow turbulence $\overline{\rho \varepsilon_s} = \overline{\mu \omega_i' \omega_i'}$, ω_i' is the fluctuating vorticity, and

$$\varepsilon_c = \frac{4}{3} \overline{\mu u_{k,k}^2}$$

are the solenoidal and dilatational or (compressible) parts of the turbulent dissipation rate. The authors argued that the solenoidal part of the dissipation can be modeled by using the traditional incompressible equation model, namely:

$$\begin{aligned} \frac{\partial}{\partial t} (\overline{\rho \varepsilon_s}) + \frac{\partial}{\partial x_k} (\overline{\rho \varepsilon_s \tilde{U}_k}) = \overline{\rho} \frac{\varepsilon_s}{K} (C_{\varepsilon 1} R_{km} \frac{\partial}{\partial x_m} \tilde{U}_k - C_{\varepsilon 2} \varepsilon_s) - \\ - \frac{\partial}{\partial x_k} (C_{\varepsilon 3} \overline{\rho} \frac{K}{\varepsilon_s} R_{km} \frac{\partial}{\partial x_m} \varepsilon_s) \end{aligned} \quad (10)$$

The compressible dissipation ε_c is determined by the commonality used models as:

$$\varepsilon_c = g_c \varepsilon_s \quad (11)$$

where g_c is a function of the turbulent Mach number.

Compressible turbulence model for the pressure strain

The incompressible models of the pressure strain have shown a great success in simulating a variety of complex turbulent flows. The work developed by Yacine *et al.* [15] is one of these types of research. Many DNS and experiment results have been carried out on compressible turbulent flows, most of them show the significant compressibility effects on the pressure-strain correlation via the pressure field. Such effects induce reduction in the magnitude of the Reynolds shear stress anisotropy and increase in the magnitude of the normal stress anisotropy. Consequently, the pressure-strain correlation requires a careful modeling in the Reynolds stress turbulence model. With respect to the incompressible case, many compressible models have been developed for the pressure-strain correlation. Hereafter, most of all these models are generated from a simple extension of its incompressible counter-parts. In general, they perform well in the simulation of important turbulent flows evolving with low compressibility.

Model of Adumitroiaie et al. (16)

Adumitroiaie *et al.* assumed that incompressible modeling approach of the pressure-strain can be used to develop turbulent models taking into account compressibility effects. Considering a non zero divergence for the velocity fluctuations called the compressibility continuity constraint and using different models for the pressure dilatation which is proportional to the trace of the pressure-strain, their model for the pressure strain is written:

$$\begin{aligned} \phi_{ij}^* = & -C_1 \bar{\rho} \varepsilon_s b_{ij} + \left(\frac{4}{5} + \frac{2}{5} d_1 \right) \bar{\rho} K \left(\tilde{S}_{ij} - \frac{1}{3} \tilde{S}_{ll} \delta_{ij} \right) + 2 \bar{\rho} K (1 - C_3 + 2d_2) \cdot \\ & \cdot \left[b_{ik} \tilde{S}_{jk} + b_{jk} \tilde{S}_{ik} - \frac{2}{3} b_{ml} \tilde{S}_{ml} \delta_{ij} \right] - \\ & - \bar{\rho} K (1 - C_4 - 2d_2) \left(b_{ik} \tilde{\Omega}_{jk} + b_{jk} \tilde{\Omega}_{ik} - \frac{4}{3} d_2 \tilde{S}_{kk} b_{ij} \right) \end{aligned} \quad (12)$$

where

$$\tilde{S}_{ij} = 0.5(\tilde{U}_{i,j} + \tilde{U}_{j,i}), \quad \tilde{\Omega}_{ij} = 0.5(\tilde{U}_{i,j} - \tilde{U}_{j,i})$$

and

$$b_{ij} = R_{ij} / 2K - \frac{1}{3} \delta_{ij}$$

The compressible coefficients d_1 and d_2 are determined from some compressible closures for the pressure-dilatation correlation [16].

Model of Khelifi *et al.* (19)

Pantano *et al.* [3] pointed out that for compressible homogeneous turbulence with high mean shear, compressibility effects are closely linked with the turbulent Mach number and the gradient Mach number. They proposed a compressible model for the pressure strain by introducing a dumping function, the model reads:

$$\bar{\rho} \phi_{ij}^* = \left[1 - f(M_t, M_g) \right] \bar{\rho} \phi_{ij}^{*I} \quad (13)$$

where $f(M_t, M_g) = \alpha_1 M_t^2 + \alpha_2 M_t M_g + \alpha_3 M_g^2$, ϕ_{ij}^{*I} is the incompressible part of the pressure strain correlation and α_i are numerical coefficients.

As shown above the function $f(M_t, M_g)$ concerns all the coefficients of the pressure strain correlation for both slow and rapid parts. It seems that this approach is not suitable for the follow reasons: firstly, the slow part describes the return to isotropy process which is observed when the mean shear strain rate is removed. In fact, the C_1 coefficient (see LRR model [20]) is not related neither to the mean shear rate S nor to the gradient Mach number M_g . Secondly, even the slow part does not satisfy the realizability condition which is similar to that adopted for incompressible model ($C_1 \geq 1$). Marzougui *et al.* [18] used the concept of the turbulent kinetic energy growth rate to introduce compressibility correction on the LRR model coefficients [20] which became a polynomial functions of the turbulent Mach number. Application of this model on compressible homogeneous shear flow have shown predictions that are in disagreement with the DNS of Sarkar [7] for high compressibility. Thus, we have revised the model [18] by using Pantano *et al.* [3] to derive a new model [19] for the pressure strain correlation in which M_t and M_g are used to express compressibility effects as follow:

$$\begin{aligned} \phi_{ij}^* = & -C_1 \bar{\rho} \varepsilon_s b_{ij} + C_2 \bar{\rho} K \left(\tilde{S}_{ij} - \frac{1}{3} \tilde{S}_{ll} \delta_{ij} \right) + \\ & + C_3 \bar{\rho} K \left[b_{ik} \tilde{S}_{jk} + b_{jk} \tilde{S}_{ik} - \frac{2}{3} b_{ml} \tilde{S}_{ml} \delta_{ij} \right] + \\ & + C_4 \bar{\rho} K \left[b_{ik} \tilde{\Omega}_{jk} + b_{jk} \tilde{\Omega}_{ik} \right] \end{aligned} \quad (14)$$

$$C_1 = C_1^I(1 - 0.9M_t^2)$$

$$C_2 = C_2^I(1 - 0.4M_t^2)$$

$$C_3 = C_3^I(1 - 1.4M_t^2 - 0.012M_g^2)$$

$$C_4 = C_4^I(1 - 0.8M_t^2 - 0.005M_g^2)$$

where C_1^I , C_2^I , C_3^I , and C_4^I are the LRR standard model coefficients: $C_1^I = 3$, $C_2^I = 0.8$, $C_3^I = 1.31$, and $C_4^I = 1.75$.

Simulation of compressible mixing layers

Now we examine the performance of the proposed model for the pressure strain to simulate the fully developed stationary compressible mixing layers. This flow is governed by the averaged Favre equations deduced from eqs. 4 to 7, such equations can be written:

$$\frac{\partial}{\partial x_i} \rho \tilde{U}_i = 0 \quad (15)$$

$$\frac{\partial}{\partial x_i} (\bar{\rho} \tilde{U}_i \tilde{U}_j) = - \frac{\partial}{\partial x_j} (\overline{\rho u_i'' u_j''}) \quad (16)$$

$$\frac{\partial}{\partial x_j} (\bar{\rho} C_v \tilde{T} \tilde{U}_j) = - \frac{\partial}{\partial x_j} (C_v \overline{\rho u_i'' T''}) + \varepsilon_s + \varepsilon_s - \overline{\rho' u_{i,i}'} \quad (17)$$

The Reynolds stress are solutions of the equation:

$$\frac{\partial}{\partial x_m} (\bar{\rho} \tilde{U}_m R_{ij}) = -(R_{im} \tilde{U}_{j,m} + R_{jm} \tilde{U}_{i,m}) + \frac{\partial}{\partial x_m} (\overline{\rho u_i'' u_j'' u_m''}) + \phi_{ij}^* + \frac{2}{3} \overline{p' u_{i,i}'} \delta_{ij} - \frac{2}{3} \varepsilon \delta_{ij} \quad (18)$$

The turbulent solenoidal dissipation rate shall be calculated from the classical model equation:

$$\frac{\partial}{\partial x_k} (\bar{\rho} \varepsilon_s \tilde{U}_k) = \bar{\rho} \frac{\varepsilon_s}{K} (C_{\varepsilon 1} R_{km} \frac{\partial}{\partial x_m} \tilde{U}_k - C_{\varepsilon 2} \varepsilon_s) - \frac{\partial}{\partial x_k} (C_{\varepsilon 3} \bar{\rho} \frac{K}{\varepsilon_s} R_{km} \frac{\partial}{\partial x_m} \varepsilon_s) \quad (19)$$

In the mentioned transport equations, different terms should be modeled, the gradient diffusion hypothesis is used to represent:

– the turbulent heat flux [11]

$$\overline{\rho u_i'' T''} = -C_T \frac{K}{\varepsilon} \overline{\rho u_i'' u_m''} \frac{\partial}{\partial x_m} \tilde{T} \quad (20)$$

– the diffusion term [11]

$$\overline{\rho u_i'' u_j'' u_m''} = -C_s \frac{K}{\rho \varepsilon} \overline{\rho u_i'' u_m''} \frac{\partial}{\partial x_m} \overline{\rho u_j'' u_m''} \quad (21)$$

For the turbulent dilatational part of the dissipation and the correlation pressure-dilatation, we chose the models proposed by Sarkar [10, 21]:

$$\varepsilon_d = 0.5 M_t^2 \varepsilon_s \quad (22)$$

$$\overline{p' u_{i,i}'} = 0.15 M_t \bar{\rho} (R_{ij} - \frac{2}{3} K \delta_{ij}) + 0.2 \bar{\rho} M_t^2 \varepsilon_s \quad (23)$$

From many several researches concerning mixing layers flows, we argue that the pressure strain correlation is one the mean term contributing to the reduced growth rate and the changes of the Reynolds stress arising from compressibility effects. Modeling turbulent

pressure strain correlation occurs mainly at high speed for mixing layers which are known to be influenced by compressibility effects. The convective Mach number has been shown to be an appropriate parameter to characterize such effects on mixing layers. According to Sarkar [7] and Pantano *et al.* [3], the homogeneous shear flow is closely related to the mixing layers. This allows M_g to be connected to M_c as $M_g \approx 2.2M_c$. Thus, the coefficients C_i in the proposed model [19] became function of M_c and M_t as follow:

$$\begin{aligned} \phi_{ij}^* = & -C_1^1(1-4M_t^2)\bar{\rho}\varepsilon_s b_{ij} + C_2^1(1-0.4M_t^2)\bar{\rho}K(\tilde{S}_{ij} - \frac{1}{3}\tilde{S}_{ll}\delta_{ij}) + \\ & + C_3^1(1-1.4M_t^2 - 0.064M_c^2)\bar{\rho}K\left[b_{ik}\tilde{S}_{jk} + b_{jk}\tilde{S}_{ik} - \frac{2}{3}b_{ml}\tilde{S}_{ml}\delta_{ij}\right] + \\ & + C_4^1(1-0.8M_t^2 - 0.011M_c^2)\bar{\rho}K\left[b_{ik}\tilde{Q}_{jk} + b_{jk}\tilde{Q}_{ik}\right] \end{aligned} \quad (24)$$

Results and discussions

The basic equations 15 to 19 on which the second order model for the stationary compressible mixing layers is based are solved using a finite difference scheme. We have calculated two free streams of a fully developed compressible mixing layers (fig.1) which are characterized typically by the convective Mach number M_c and the parameters $s = \rho_2/\rho_1$ and $r = U_2/U_1$, are respectively the density and velocity ratios, the experiment conditions of Goebel *et al.* [1] are listed in tab. 1.

The values of the constants models used in the present simulation are:

$$\begin{aligned} C_{\varepsilon 1} = 1,4, \quad C_{\varepsilon 2} = 1,8, \quad C_{\mu} = 0,09, \\ C_{\varepsilon} = 0,25, \quad C_T = 0,26. \end{aligned}$$

To evaluate the proposed model for the pressure strain correlation which is a compressibility correlation of the LRR model [20], we compare the computational results to what is expected from laboratory and numerical experiments. Results obtained by using the incompressible LRR model [20] and the compressible model of Adumitroaie *et al.* [16] are also included for comparison. The fundamental parameter characterizing the compressibility effects on the mixing layer is the growth rate ($\dot{\delta} = d\delta/dx$), δ denotes the momentum thickness of the mixing layer. Figure 2 shows the comparison between the computed normalized growth rate by its incompressible counterpart $G = (d\delta/dx)/(d\delta/dx)_{M_c=0}$ with different experiment and numerical results available in the literature and with those obtained by empirical formula of Dimotakis [22]:

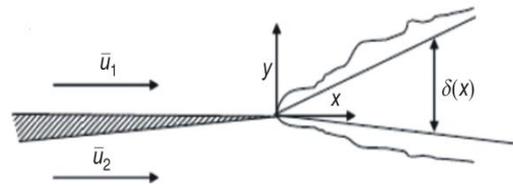


Figure 1. Turbulent mixing layers

Table 1. Experience of Goebel and Dutton [1]

M_c	0.2	0.46	0.69	0.86	1
$r = \frac{U_2}{U_1}$	0.78	0.57	0.18	0.16	0.16
$s = \frac{\rho_2}{\rho_1}$	0.76	1.55	0.57	0.6	1.14

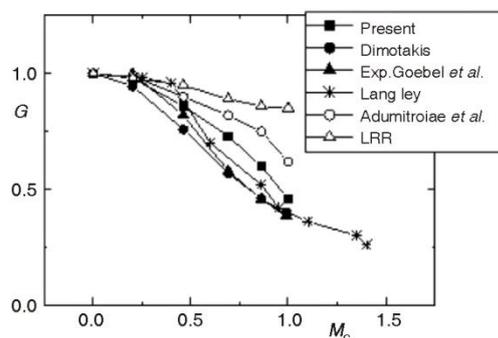


Figure 2. The growth rate
 $G = (d\delta / dx) / (d\delta / dx)_{M_c=0}$ v.s M_c

$$G = 0.8\exp(-M_c^2) + 0.2 \tag{25}$$

The calculated growth rate G decreases with increasing convective Mach number, this phenomenon which has often been observed in experimental studies of compressible mixing layers is well captured by the proposed model.

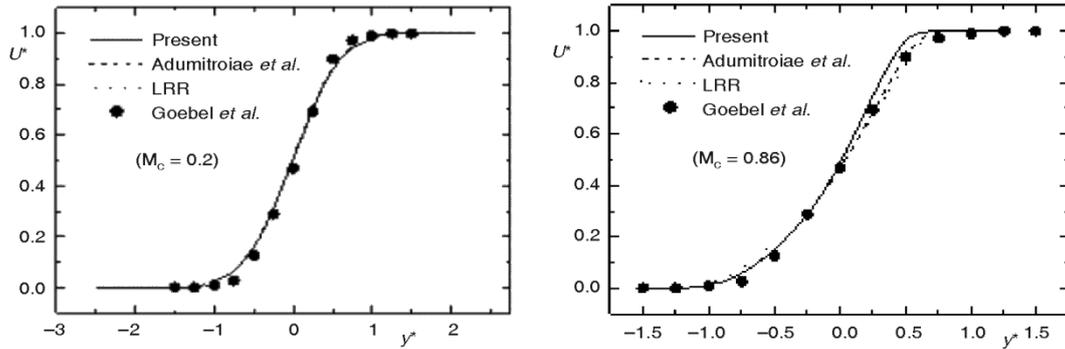


Figure 3. Similarity profiles of the mean velocity $U^* = \tilde{U} - U_2/U_1 - U_2$

The LRR and Adumitroiaie *et al.* models over-predict the growth rate G , the reduction of G with M_c is slightly smaller than in reference results.

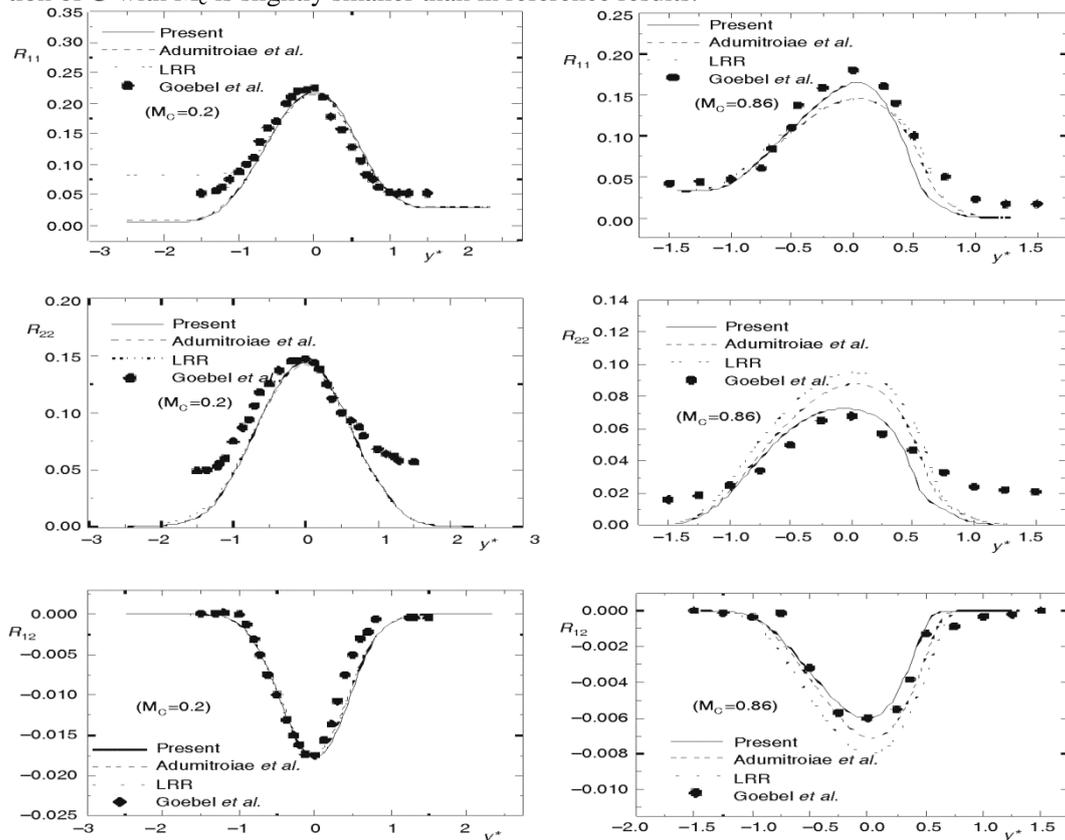


Figure 4. Similarity profiles of Reynolds intensities

The normalized stream mean velocity $U^* = \tilde{U} - U_2/U_1 - U_2$ is represented in relation to the similarity variable $y^* = (y - y_c)/\delta$ in fig. 3, where y is the local cross-stream co-ordinate and y_c is the cross-stream co-ordinates corresponding to $U^* = 0.5$. The calculated velocity profiles with all the models are in reasonable agreement with experimental results from the low convective Mach number ($M_c = 0.2$) and the high convective Mach number ($M_c = 0.86$).

In fig. 4, the Reynolds similarity intensity profiles:

– the streamwise intensity

$$R_{11} = \left(\overline{\rho u'^2} / \bar{\rho}(U_1 - U_2)^2 \right)^{1/2},$$

– the transverse intensity

$$R_{22} = \left(\overline{\rho v'^2} / \bar{\rho}(U_1 - U_2)^2 \right)^{1/2},$$

– and the shear stress

$$R_{12} = \overline{\rho u'v'} / \bar{\rho}(U_1 - U_2)^2$$

obtained from the proposed and [16, 20] models are compared with experiment results of Goebel and Dutton [1]. It is clear that all the models lead to similar results which are in good accordance with experiment results [1] for small value of convective Mach number ($M_c = 0.2$). When the compressibility effects are more significant $M_c = 0.86$, it is found that the computed results of the proposed model are in good agreement with the experimental data [1] than those offered by the models [16, 20]. The variation of the maximum values of the Reynolds stresses are plotted as function of the convective Mach number in fig. 5. One can see that the computed maximum values of the transverse $\sigma_{uu}/\Delta U = (R_{11})_{\max}$, the streamwise $\sigma_{vv}/\Delta U = (R_{22})_{\max}$ normal stress and the shear $\sigma_{uv}/\Delta U = (R_{12})_{\max}$ components of Reynolds stresses with the proposed model decrease as convective Mach number increases in accordance with the DNS [3] and experiment results [1, 5]. At high compressibility, one can see that Adumitroaie *et al.* model [16] gives better results than from LRR model [20], but, it is still unable to accurately predict the peaks of turbulent Reynolds intensities.

Figure 6 show the convective Mach number M_c -variation of the peak values of the Reynolds stress anisotropies b_{11} , b_{22} , and b_{12} . From these figures, it is clear that the proposed model appears to be able to correctly predict the significant decrease of the shear stress b_{12} and the increase of the normal

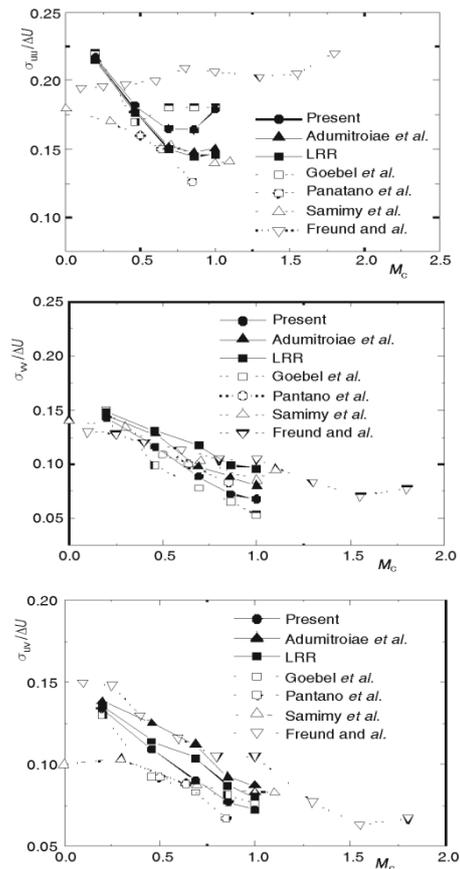


Figure 5. Variation of the maximum Reynolds stresses with the convective Mach number

stresses anisotropies (the streamwise b_{11} and the transverse b_{22}) with increasing the convective Mach number. As can be seen, the incompressible LRR model results are in disagreement with DNS results [3, 23]. Adumitroaie *et al.* [16] gives results that are much better than from the original model of LRR [20] but this model is also still unable to predict compressibility effects on the anisotropy at high M_c .

From the above results, it is clearly seen that all the models are similar for low convective Mach number. But at high compressibility (the convective Mach number is higher), there is substantial differences between these models in their predictions. To find the cause of this discrepancy, several studies pointed out on the mechanisms that lead to the dramatic changes of the Reynolds stresses when compressibility increases. It is found that the most important term in the Reynolds stress transport equations is the pressure strain correlation which governs the level of the structural compressibility effects. The maximum values of

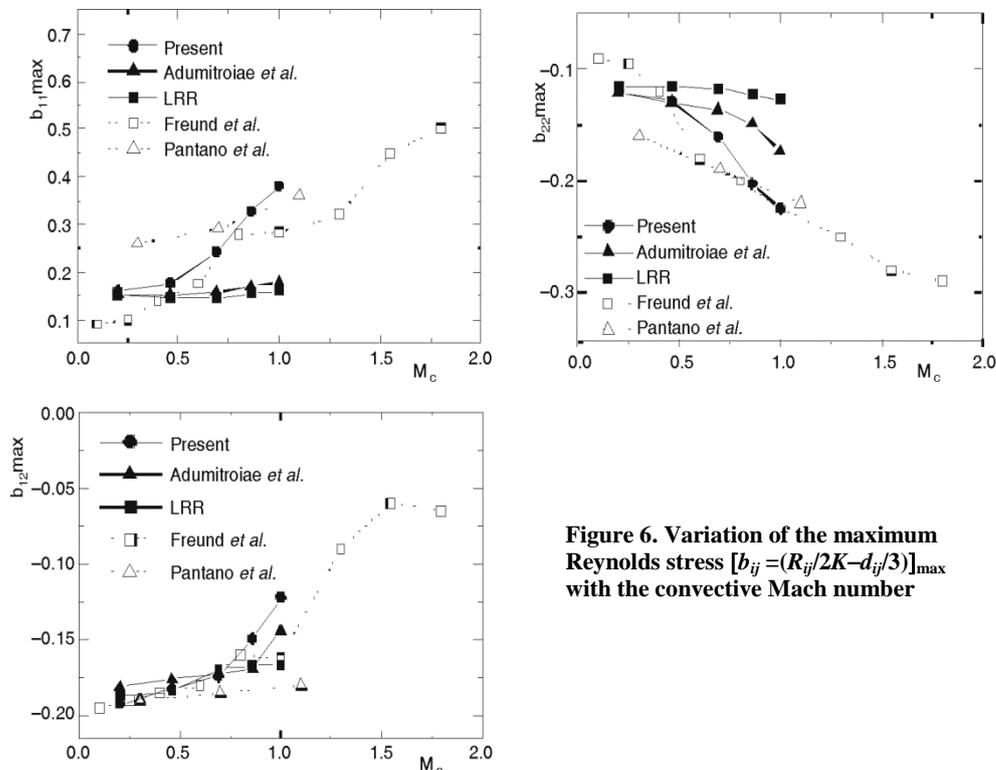
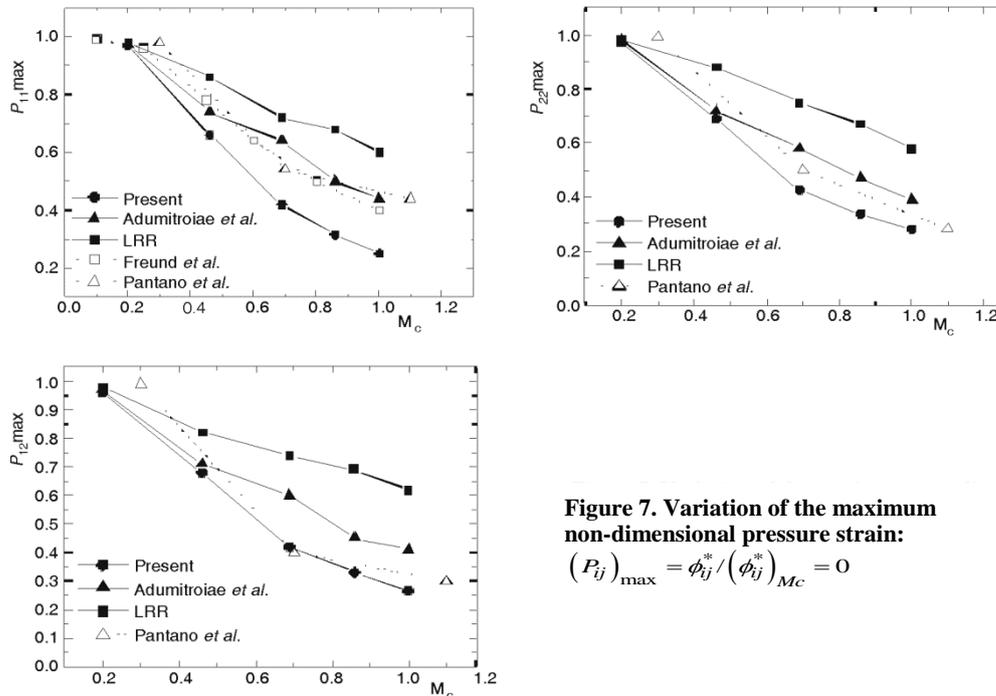


Figure 6. Variation of the maximum Reynolds stress $[b_{ij}=(R_{ij}/2K-d_{ij}/3)]_{max}$ with the convective Mach number

diverse compressible pressure strain components normalized by its incompressible counterparts are plotted as a function of the convective Mach number in fig. 7. It can be seen that the original LRR model [20] does not reproduce the decrease of these turbulent quantities. The compressibility correction model proposed by Adumitroaie *et al.* [16] induces a certain over-prediction of the pressure strain correlation, the reduction of this term with increasing M_c is slightly than in DNS results [3, 23]. However, the pressure strain reduction which is the main responsible for the reduction of production term and of the shear layer growth rate appear to be accurately captured by the proposal model. Therefore, the convective Mach number is concluded to be important in addition with the turbulent Mach number for modeling the pressure strain in turbulent mixing layers.



Conclusions

In this paper, the Favre second order closure has been used for the prediction of spatially developing compressible mixing layers. The standard Reynolds stress turbulence closure with addition of the pressure-dilatation and compressible dissipation models yields very poor predictions of the changes in the behaviors of different fundamental parameters characterizing compressibility effects on turbulent mixing layers. Certainly, the deficiency of this closure is due to the use of the incompressible LRR model of the pressure strain correlation. A compressibility correction of the LRR model involving the commonly used turbulent Mach number with the convective Mach number has been proposed in order to reflect compressibility effects. A comparison has been made for the behavior of the proposed model, the LRR model and the compressible model of Adumitroiaie *et al.* for the pressure strain correlation. References have been made to DNS and experiment results. The model of Adumitroiaie *et al.* which is constructed using pressure dilatation model is found to be accurate for low convective Mach number. For high convective Mach numbers, this model cannot be considered as essential in reproducing structural compressibility effects. The results obtained with the proposed model are in better agreement with experimental and DNS data than the results obtained from the two other models, especially at high convective Mach number. The proposed model successfully predicts the reduced growth rate, the decrease of the shear stress and the increase of the normal Reynolds stresses anisotropies with increasing the convective Mach number. Also, the reduction of the pressure strain correlation found in different works as the most important physical phenomenon in compressible mixing layers is well predicted by the proposed model. Therefore, the convective Mach number is found out to be an important parameter in addition to the turbulent Mach number in the modeling of the compressible pressure strain correlation.

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