NUMERICAL SIMULATION AND PARAMETRIC STUDY OF LAMINAR MIXED CONVECTION NANOFLUID FLOW IN FLAT TUBES USING TWO PHASE MIXTURE MODEL

by

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In this article, the laminar mixed convection of Al_2O_3 -water nanofluid flow in a horizontal flat tube has been numerically simulated. The two-phase mixture model has been employed to solve the nanofluid flow, and constant heat flux has been considered as the wall boundary condition. The effects of different and important parameters such as the Reynolds number, Grashof number, nanoparticles volume fraction, and nanoparticle diameter on the thermal and hydrodynamic performances of nanofluid flow have been analyzed. The results of numerical simulation were compared with similar existing data and good agreement is observed between them. It will be demonstrated that the Nusselt number and the friction factor are different for each of the upper, lower, left, and right walls of the flat tube. The increase of Reynolds and Grashof numbers, nanoparticles volume fraction, and the reduction of nanoparticle diameter lead to the increase of Nusselt number. Similarly, the increase of Reynolds number and nanoparticles volume fraction results in the increase of friction factor. Therefore, the best way to increase the amount of heat transfer in flat tubes using nanofluids is to increase the Grashof number and reduce the nanoparticle diameter.

Key words: mixed convection, nanofluid, flat tube, parametric study, mixture model

Introduction

The use of nanofluids is one of the most effective mechanisms of increasing the amount of heat transfer in heat exchangers. The use of flat tubes, in which the fluid flow has a lower thermal resistance, is another way of improving the rate of heat transfer in tubes. The subject of the present paper is combining of the two mentioned methods for increase of heat transfer and parametric study of thermal and hydrodynamic performances of flow field.

The word nanofluid refers to a mixture in which solid particles of nanosize (generally less than 100 nm) are added to a base fluid and cause the increase of heat transfer in that mixture. Different mechanisms are presented to explain this enhancement in heat transfer. Xuan and Li [1] and Xuan and Roetzel [2] have identified two reasons of improved heat transfer by nanofluids: the increased thermal dispersion due to the chaotic movement of nanoparticles that accelerates energy exchanges in the fluid and the enhanced thermal conductivity of nanofluids [3]. Similarly, Keblinski *et al.* [4] have studied four possible mechanisms that contribute to the increase in nanofluid heat transfer: Brownian motion of the nanoparticles, mo-

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lecular-level layering of the base fluid/nanoparticle interface, heat transport in the nanoparticles, and nanoparticles clustering. Due to the many advantages of using nanofluids, researchers have conducted numerous experimental and numerical studies on nanofluids in recent years [5-7]. Kumar Das et al. [5] experimentally investigated the effects of different parameters (e. g., temperature, nanoparticle volume fraction, etc.) on the thermal conductivity of nanofluids. They ultimately presented a relation for thermal conductivity of nanofluids as a function of temperature, nanoparticle volume fraction, etc. In addition to costly experimental studies, the numerical simulation of nanofluids using CFD techniques is another effective approach in analyzing the performance of nanofluids [8, 9]. In general, for the numerical simulation of nanofluids, there are two methods called the single-phase and two-phase, with the twophase method being much more exact [10]. The two-phase method itself has different categories, including the Eulerian-Eulerian method, mixture method, etc. Using the three methods of two-phase Eulerian-Eulerian, two-phase mixture, and single-phase homogeneous, Lotfi et al. [11] simulated the Al₂O₃-water nanofluid flow in circular tubes. By comparing the simulation results with the experimental data, they came to the conclusion that the two-phase mixture method is the most exact method among the existing approaches. In this article also, the twophase mixture method is used for the numerical simulation of nanofluid flow in flat tubes.

Another effective method for increasing the amount of heat transfer in tubes is the use of flat tubes instead of circular ones. In these tubes, the ratio of cross-sectional area to lateral surface area is less than that of circular tubes, and this factor reduces the thermal resistance of fluid flow and thus increases the amount of heat transfer. Of course, next to increasing the degree of heat transfer, which is a useful aspect, the use of flat tubes also causes the pressure drop and friction factor to increase in these tubes, which is a negative point. Therefore, the behaviors of heat transfer and friction in tubes should be investigated simultaneously. Based on our knowledge, up to now, no numerical simulation of two-phase nanofluid flow in flat tubes has been performed. Altogether, few research works have been carried out on flat tubes, which most of them have concerned the multiphase flows [12-14] and very few of them have dealt with nanofluids [15, 16]. Razi et al. [15] considered CuO-oil nanofluid experimentally in various flat tubes and finally presented relations for Nusselt number and pressure drop of nanofluid flow in horizontal flat tubes. Vajjha et al. [16] investigated nanofluid flow in a flat tube of an automobile radiator using the single phase numerical method. They used convection heat transfer coefficient for the wall boundary condition and finally presented the correlation for local Nusselt number and friction factor of the automobile flat tube.

In this article, the simultaneous use of nanofluids and flat tubes to increase the heat transfer in heat exchangers has been numerically investigated by means of the two-phase mixture method. The effects of different and important parameters including the Reynolds number, Grashof number, nanoparticle volume fraction (φ), and nanoparticle diameter (d_p) on the thermal and hydrodynamic performances of the nanofluid have been evaluated.



Figure 1. Geometry cross-section of a flat tube

Mathematical modeling

Geometry

Figure 1 shows the geometry cross-section of the considered problem in this paper. The computation domain is a straight horizontal flat tube with length of L, width and height of W and H, respectively, at the flat parts.

Mixture model

In the present study, numerical simulation of nanofluid flow is performed using twophase mixture model. The mixture model is a two phase numerical method that assumes local equilibrium over short spatial length scales. The two phases are treated to be interpenetrating continua, meaning that each phase has its own velocity vector field, and within a given control volume there is a certain fraction of each phase. Instead of utilizing the governing equations of each phase separately, it solves the continuity, momentum and energy equations for the mixture, and the volume fraction equation for the secondary phases, as well as algebraic expressions for the relative velocities. The equations for the steady-state conditions and mean flow are:

continuity equation

$$\nabla(\rho_{\rm m}V_{\rm m}) = 0 \tag{1}$$

– momentum equation [17]

$$\nabla(\rho_{\rm m}V_{\rm m}V_{\rm m}) = -\nabla P + \nabla(\mu_{\rm m}\nabla V_{\rm m}) + \nabla\left(\sum_{k=1}^{n}\varphi_{k}\rho_{k}V_{{\rm dr},k}V_{{\rm dr},k}\right) - \rho_{{\rm m},{\rm i}}\beta_{\rm m}g(T-T_{\rm i})$$
(2)

energy equation [17]

$$\nabla \left[\sum_{k=1}^{n} \varphi_k V_k (\rho_k H_k + P) \right] = \nabla (k_{\rm m} \nabla T)$$
(3)

volume fraction [17]

$$\nabla(\varphi_{\rm P}\rho_{\rm P}V_{\rm m}) = -\nabla(\varphi_{\rm P}\rho_{\rm P}V_{\rm dr,P}) \tag{4}$$

where $V_{\rm m}$ is the mass average velocity and defined:

$$V_{\rm m} = \frac{\sum_{k=1}^{n} \varphi_k \rho_k V_k}{\rho_{\rm m}}$$
(5)

In eq. (2), $V_{dr,k}$ is the drift velocity for the secondary phase k, i. e. the nanoparticles in the present paper and defined:

$$V_{\mathrm{dr},k} = V_k - V_\mathrm{m} \tag{6}$$

The slip velocity (relative velocity) is defined as the velocity of nanoparticles relative to the velocity of base fluid:

$$V_{\rm pf} = V_{\rm p} - V_{\rm f} \tag{7}$$

The relation between drift velocity and relative velocity is:

$$V_{\rm dr,p} = V_{\rm pf} - \sum_{k=1}^{n} \frac{\varphi_k \rho_k}{\rho_{\rm m}} V_{\rm fk}$$
(8)

The relative velocity and drag function are calculated using Manninen *et al.* [18] and Schiller and Naumann [19] relations, respectively:

$$V_{\rm pf} = \frac{\rho_{\rm P} d_{\rm p}^2}{18\mu_{\rm f} f_{\rm drag}} \frac{(\rho_{\rm P} - \rho_{\rm m})}{\rho_{\rm P}} a$$
(9)

$$f_{\rm drag} = \begin{cases} 1 + 0.15 \ {\rm Re}_{\rm p}^{0.687} & \text{for} & {\rm Re}_{\rm p} \le 1000 \\ 0.0183 \ {\rm Re}_{\rm p} & \text{for} & {\rm Re}_{\rm p} > 1000 \end{cases}$$
(10)

The acceleration a in eq. (9) is:

$$a = g - (V_{\rm m} \nabla) V_{\rm m} \tag{11}$$

Nanofluid mixture properties

The mixture properties for Al₂O₃-water nanofluid are calculated based on expressions:

- density [20]

$$\rho_{\rm m} = \varphi \rho_{\rm p} + (1 - \varphi) \rho_{\rm f} \tag{12}$$

- specific heat capacity [2]

$$(\rho C_p)_{\rm m} = \varphi(\rho C_p)_p + (1 - \varphi)(\rho C_p)_{\rm f}$$
⁽¹³⁾

- dynamic viscosity [21]

$$\mu_{\rm m} = \mu_{\rm f} + \frac{\rho_{\rm p} V_{\rm B} d_{\rm p}^2}{72C\delta} \tag{14}$$

where $V_{\rm B}$ and δ are Brownian velocity of nanoparticles and distance between particles, respectively, which can be calculated from:

$$V_{\rm B} = \frac{1}{d_{\rm p}} \sqrt{\frac{18k_{\rm B}T}{\pi\rho_{\rm p}d_{\rm p}}} \tag{15}$$

$$\delta = \sqrt[3]{\frac{\pi}{6\varphi}} d_{\rm p} \tag{16}$$

The C in eq. (14) is defined:

$$C = \frac{(C_1 d_p + C_2)\varphi + (C_3 d_p + C_4)}{\mu_f}$$
(17)

where C_1 , C_2 , C_3 , and C_4 are given:

 $C_1 = -0.000001133, C_2 = -0.000002771, C_3 = 0.00000009, C_4 = -0.000000393$ (18) - thermal conductivity [22]

$$\frac{k_{\rm m}}{k_{\rm f}} = 1 + 64.7\varphi^{0.7460} \left(\frac{d_{\rm f}}{d_{\rm p}}\right)^{0.3690} \left(\frac{k_{\rm p}}{k_{\rm f}}\right)^{0.7476} \Pr_{\rm f}^{0.9955} \operatorname{Re}_{\rm f}^{1.2321}$$
(19)

where Pr_f and Re_f are defined:

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$$\Pr_{\rm f} = \frac{\eta}{\rho_{\rm f} \alpha_{\rm f}} \tag{20}$$

$$\operatorname{Re}_{\mathrm{f}} = \frac{\rho k_{\mathrm{B}} T}{3\pi \eta^2 \lambda_{\mathrm{f}}}$$
(21)

where λ_f is the mean free path of water molecular ($\lambda_f = 0.17 \text{ nm}$), k_B – the Boltzmann constant ($k_B = 1.3807 \cdot 10^{-23} \text{ J/K}$), and η has been calculated by the equation:

$$\eta = A \, 10^{\frac{B}{T-C}}, \qquad A = 2.414 \cdot 10^{-5}, \qquad B = 247.8, \qquad C = 140$$
 (22)

- thermal expansion coefficient [23]

$$\beta_{\rm m} = \left[\frac{1}{1 + \frac{(1-\varphi)\rho_{\rm f}}{\varphi\rho_{\rm p}}} \frac{\beta_{\rm p}}{\beta_{\rm f}} + \frac{1}{1 + \frac{\varphi}{1-\varphi} \frac{\rho_{\rm p}}{\rho_{\rm f}}} \right] \beta_{\rm f}$$
(23)

Boundary conditions

For numerical simulation, the equations of previous sections should be solved subject to the following boundary conditions: – tubes inlet

$$V_{m,z} = V_i, \qquad V_{m,x} = V_{m,y} = 0$$
 (24a)

$$T = T_i \tag{24b}$$

$$\varphi = \varphi_i \tag{24c}$$

- fluid-wall interface

$$V_{m,x} = V_{m,y} = V_{m,z} = 0$$
(25a)

$$q''_{\rm w} = -k_{\rm m} \left. \frac{\partial T}{\partial n} \right|_{\rm w} \tag{25b}$$

- tubes outlet: zero gradient is applied to hydrodynamic variables and constant gradient is applied to temperature [24, 25]:

$$\frac{\mathrm{d}V_z}{\mathrm{d}z} = 0 \tag{26a}$$

$$\frac{\mathrm{d}T}{\mathrm{d}z} = cte = \frac{P}{A} \frac{q''}{\rho U C_p} \tag{26b}$$

Numerical methods

The numerical simulation is performed using the finite volume method by FORTRAN programming language. A second order upwind method is used for the convec-

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tive and diffusive terms and the SIMPLE algorithm is employed to solve the coupling between the velocity and pressure fields. For grid generation, first, the system boundaries are determined and then by interpolating these boundaries, the inner points are generated. For this purpose, the lower and upper boundaries have been considered as $\vec{r}_L(\zeta, \eta_{\min})$ and $\vec{r}_U(\zeta, \eta_{\max})$, respectively. In this case, a linear one directional interpolation for finding the inner points of the field is defined:

$$\vec{\mathbf{r}}(\xi,\eta) = (1-\bar{\eta})\vec{\mathbf{r}}_L(\zeta,\eta_{\min}) + \bar{\eta}\vec{\mathbf{r}}_U(\xi,\eta_{\max})$$
(27)

$$\bar{\eta} = \frac{\eta - \eta_{\min}}{\eta_{\max} - \eta_{\min}}$$
(28)

In the generated grid, because of the large hydrodynamic and thermal gradients, an exponential function is used near the walls and also at the tube inlet. This function is expressed:

$$f(\overline{\eta}) = 1 + \beta \frac{1 - \alpha^{1 - \eta}}{1 + \alpha^{1 - \eta}}, \qquad \alpha = \frac{\beta + 1}{\beta - 1}$$
(29)

where, β is the clustering ratio.



Figure 2. Grid independency test: (a) θ distribution at x-direction, (b) θ distribution at y-direction, (c) axial velocity at center line, and (d) local Nusselt number

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To make sure that the results are independent of the generated grid, four different grids with $60 \times 40 \times 80$, $75 \times 50 \times 120$, $90 \times 60 \times 100$ \times 160, and 105 \times 70 \times 200 elements along the x-, y-, and z-axes have been generated. Different parameters such as dimensionless temperature (θ) along the x and y, dimensionless axial velocity along the centerline, and the local Nusselt number have been calculated for these four grids and their values have been compared with one another in fig. 2. As this figure shows, all the four generated grids successfully pass the grid independency test for the temperature along the x-axis and axial velocity, but the $60 \times 40 \times 80$ grid fails the grid independency test for the other two parameters. Therefore, the $75 \times 50 \times 120$ grid will be used in all the simulations. Figure 3 shows the structure non-uniform generated grid for the flat tube of present paper. As shown in this figure, the grids are finer near the tubes entrance and near the wall where the velocity and temperature gradients are high.

Validations of the numerical simulations

To attain the confidence about the CFD results, it is necessary to compare the simulation results with the available data. Figure 4 com-



Figure 3. Structure non-uniform grid generation for the flat tube

pares the \overline{Nu} and $\overline{C_f}$ Re vs. flattening, predicted in the present study with analytical data of Shah and London [26] and numerical simulations of Vajjha *et al.* [16].



Figure 4. Comparison of: (a) Nu and (b) C_f Re of the present numerical calculation with the Shah and London [26] data and numerical results of Vajjha *et al.* [16] at Re = 100

The simulations of Vajjha *et al.* [16] are performed for specified convection coefficient wall boundary condition whereas present simulations and Shah and London [26] data are

related to constant heat flux wall boundary condition. As evident from fig. 4 the present CFD simulations agree well with the available numerical and analytical data.

Results and discussion

The laminar mixed convection of Al_2O_3 -water nanofluid flow in a horizontal flat tube has been simulated by using the two-phase mixture model, and the results have been presented in this section.

The Nusselt number is one of the most important thermal parameters of the flow field inside a tube and is defined:

$$Nu = \frac{hD_{h}}{k}$$
(30)

where

$$h = \frac{q''}{T_{\rm w} - T_{\rm b}} \tag{31}$$

In fig. 5, the local Nusselt number has been presented for each of the upper, lower, left, and right walls of the flat tube. As this figure shows, the Nusselt number values vary for different walls of the tube, and the lower wall has the highest Nusselt number, followed by the upper wall. Also, the values of Nusselt number for the left and right walls are equal.

The reason for a higher Nusselt number at the lower wall is that due to the laminar flow regime in this region, gravity plays a significant role and causes the increase of the Nusselt number at the lower wall of tube. From now on, to present the Nusselt number under different conditions, the Nusselt number values are averaged along the perimeter and the mean values are presented for various walls. In order to study the effects of important flow parameters



Figure 5. Local Nusselt number at top, bottom, left and right walls of the flat tube using Al_2O_3 -water nanofluid, Gr = 23000

on the heat transfer value in flat tubes, the local heat transfer coefficient has been presented in fig. 6 for different Reynolds and Grashof numbers, nanoparticle volume fractions (φ), and nanoparticle diameters (d_p). As is illustrated in this figure, the increase of Reynolds number, Grashof number, φ , and the reduction of d_p causes the heat transfer coefficient to increase. Although, it seems that the effect of Grashof number and φ on the increase of heat transfer coefficient is higher than that of the other two parameters.

Another important parameter for the flow field inside a heat exchanger tubes is the value of pressure drop, which has a direct relationship with the friction factor (C_f) and wall shear stress (τ_w) with the expression:

$$C_{\rm f} = \frac{\tau_{\rm w}}{\frac{1}{2}\,\rho_{\rm m}u_i^2}\tag{32}$$



Figure 6. Effects of changing: (a) Reynolds number, (b) Grashof number, (c) and (d) d_p on peripherally averaged local heat transfer coefficient

where

$$\tau_{\rm w} = \mu_{\rm m} \left. \frac{\partial u}{\partial n} \right|_{\rm w} \tag{33}$$

The $C_{\rm f}$ is shown in fig. 7 for each wall of the flat tube. Similar to the Nusselt number, $C_{\rm f}$ also has the highest value at the lower wall, followed by the upper wall.

Effects of Reynolds number, Grashof number, φ , and d_p changes on the local wall shear stress averaged along the perimeter have been illustrated in fig. 8. According to this figure, the increase of Reynolds number and φ leads to the increase of wall stress; while the changes of Grashof number and d_p have no impact on wall stress.



Figure 7. Local $C_{\rm f}$ at top, bottom, left, and right walls of the flat tube

According to figs. 6 and 8, although the increase of Reynolds number and φ causes the increase of the *h*, but they increase the friction factor as well. While the increase of Grashof number and the reduction of d_p leads to the increase of *h* without any increase in the

friction factor therefore, the best way of increasing the heat transfer of nanofluid flow in flat tubes is to increase the Grashof number and decrease the d_{p} .



Figure 8. Effects of changing; (a) Reynolds number, (b) Grashof number, (c) and (d) on peripherally averaged local wall shear stress



Figure 9. Effects of changing d_p and Grashof number on secondary flow vectors in the flat tube

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In this article, the effect of changing the flow parameters on the secondary vectors has also been investigated. Secondary vectors play a very important role in the thermal and hydrodynamic characteristics of flow. These vectors are formed in a tube due to the existence of a buoyancy force and different densities of nanofluid in the upper and lower regions of tube. The effects of changes of d_p , Grashof number, φ , and Reynolds number on the quality of secondary vectors have been reviewed in figs. 9 and 10. It is observed that under all conditions, two vortices are created inside the tube. Figures 9 and 10 also indicate that the changes of d_p and Reynolds number have no effect on secondary vectors. Whereas the changes of Grashof number and φ have considerable effects on the quality of secondary vectors, and improving the mentioned parameters leads to the strengthening of these secondary vectors.



Figure 10. Effects of changing φ and Reynolds number on secondary flow vectors in the flat tube

Knowing the details of velocity profiles in the flat tubes, helps better understanding the nature of the flow field. Effects of changing d_P , on velocity profiles at x-, y-, and z- directions are shown in fig. 11. As shown the changes of d_P , do not lead to change of velocity field and considering the previous discussions about C_f and Nusselt number it can be concluded that the thermal flow field is more sensitive than the hydrodynamic flow field with respect to changing the parameters such as d_P and Grashof number.

Since in this article, the flow of nanofluid has been simulated by means of a twophase method, the distribution of nanoparticles in the flat tube is known. The distribution of nanoparticles along the vertical direction in the flat tube has been illustrated in fig. 12 for various parameters. As is shown in this figure, the changes of Reynolds number and Grashof number have a negligible effect on the distribution of nanoparticles. But the increase of nanoparticle diameter causes severe non-uniformity in the distribution of nanoparticles.

Conclusions

In this article, the laminar mixed convection of Al₂O₃-water nanofluid flow in a horizontal flat tube was numerically simulated. The two-phase mixture model was used to solve the nanofluid flow. Constant heat flux was assumed as the wall boundary condition. The effects of different and important parameters such as Reynolds number, Grashof number, φ , and d_p on the thermal and hydrodynamic performances of flat tubes containing nanofluids were discussed, and ultimately the following results were obtained:



Figure 11. Effect of changing d_p on dimensionless velocity profile at: (a) horizontal, (b) vertical, and (c) axial directions

Figure 12. Effects of changing: (a) Reynolds number, (b) Grashof number, and (c) on vertical nanoparticle distributions

- The values of Nusselt number and C_f are not equal for each wall of the flat tube, and the flat tube's lower surface has higher values of Nusselt number and C_f relative to the other surfaces.
- The increase of Reynolds number, Grashof number, and φ , and the reduction of d_p lead to increase of Nusselt number. Similarly, increase of Reynolds number and φ leads to in-

crease of $C_{\rm f}$, and the changes of Grashof number and $d_{\rm p}$ have no effect on $C_{\rm f}$. Therefore, the best way of increasing the heat transfer of nanofluid flow in flat tubes is to increase the Grashof number and decrease the $d_{\rm p}$.

- The increase of Grashof number and φ lead to the strengthening of secondary flows, and the changes of Reynolds number and d_p in a mixed laminar flow have negligible effects on secondary flows.
- The changes of Reynolds number and Grashof number does not have a significant effect on the distribution of nanoparticles. However, the changes of d_p has a considerable impact on the distribution of nanoparticles, and the increase of d_p can lead to a severe nonuniformity of nanoparticles in flat tubes.

Nomenclature

- a acceleration, [ms⁻²]
- C constant in eq. (14), [–]
- $C_{\rm f}$ skin friction coefficient, [–]
- C_p specific heat, [Jkg⁻¹K⁻¹] D_h hydraulic diameter of tubes, [m]
- $d_{\rm p}$ diameter of nanoparticles, [m]
- g gravitational acceleration, [ms⁻²] H flat tube internal height, [mm]
- *h* local heat transfer coefficient, $[Wm^{-2}K^{-1}]$ Gr Grashof number (= $g\beta_m g'' D_h^4 / k_m v_m^2$), [–] *k* thermal conductivity, $[Wm^{-1}K^{-1}]$

- $k_{\rm B}$ Boltzmann constant (=1.3807 $\cdot 10^{-23} \, {\rm JK}^{-1}$)
- L length of tubes, [m]
- Р - pressure, [Pa]
- Pr Prandtl number (= $\alpha_{\rm m}/\nu_{\rm m}$), [–] q'' heat flux, [Wm⁻²]
- Re Reynolds number (= VD_h/v_m), [–] Ri Richardson number (= Gr/Re^2), [–]
- temperature, [K] Т
- V velocity, [ms⁻¹]
- W width of flat area in tubes, [mm]

Greek symbol

- α thermal diffusivity, $[m^2 s^{-1}]$
- volumetric expansion coefficient, [K⁻¹] ß
- δ distance between particles, [m]
- nanoparticles volume fraction, [%] Ø
- θ - dimensionless temperature
- $[(=T T_i)/(q''D_h/k)], [-]$
- $\lambda_{\rm f}$ mean free path of water molecular, [m]
- μ dynamic viscosity, [Nsm⁻²]
- v kinematic viscosity, $[m^2s^{-1}]$
- ρ density, [kgm⁻³]
- $\tau_{\rm w}$ wall shear stress, [Pa]

Subscripts

- dr drift
- f fluid
- inlet conditions i
- k indices
- m mixture
- P particle
- p nanoparticle phase
- w – wall

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