NEW SIMILARITY SOLUTION OF BOUNDARY LAYER FLOW ALONG A CONTINUOUSLY MOVING CONVECTIVELY HEATED HORIZONTAL PLATE BY DEDUCTIVE GROUP METHOD

by

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A mathematical model is presented and analyzed for steady 2-D non-isothermal laminar free convective boundary layer flow along a convectively heated moving horizontal plate. New similarity transformations are developed using one parameter deductive group transformations and hence the governing transport equations are reduced to a system of coupled, non-linear ordinary differential equations with associated boundary conditions. The reduced equations are then solved numerically by an implicit finite difference numerical method. The effects of pertinent parameters on the non-dimensional velocity, temperature, friction factor, and heat transfer rates are investigated and presented graphically. It is found that friction factor decreases with the free convective parameter and rate of heat transfer increases with the convection-conduction parameter.

Key words: *convective boundary condition, deductive group method, heat transfer, horizontal moving plate, similarity solutions*

Introduction

In many engineering processes such as extrusion, glass fiber drawing, hot rolling and casting, the material being manufactured is processed thermally by allowing it to exchange heat with the surrounding [1]. The objective of the thermal treatment is to cool the material to a desirable temperature before it is spooled or removed. As the material at high temperature emerges from a furnace or a die, it is exposed to the colder ambient and a transient conduction process accompanied by surface heat loss is initiated. Momentum and heat transfer in boundary layer on a continuous moving surface have been studied by many researchers. Sakiadis [2] studied boundary layer flow and heat transfer along a moving plate. A considerable amount of research has been reported on this topic (see [3-5]). Similarity solutions for moving surface were investigated also by Ferdows *et al*. [6], Weidman *et al*. [7], and Fang and Lee [8]. Ishak *et al*. [9] studied the boundary layer flow on a moving permeable plate parallel to a moving stream.

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Free convection flow occurs in atmospheric and oceanic circulation, electronic machinery, heated or cooled enclosures, electronic power supplies, *etc*. It has many applications such as its influence on operating temperatures of power generating and electronic devices. It plays a great role in thermal manufacturing applications and is important in establishing the temperature distribution within buildings as well as heat losses or heat loads for heating, ventilating, and air conditioning systems. Free convection problems due to a heated/cooled horizontal flat plate has been studied by authors since it appears in several science and engineering applications, as well as in a wide variety of natural circumstances. In this regard [10] studied similarity analysis for the free and forced convection hydromagnetic flow over a horizontal semi-infinite flat plate through a non-homogeneous medium which is porous. Fan *et al*. [11] investigated the mixed convective heat and mass transfer over a horizontal plate. Plane and axisymmetric horizontal laminar jet flows produced by natural convection on horizontal finite plate acting as a heat dipole at large distances from the plate was studied by Noshadi and Schneider [12]. A numerical and experimental study of the natural convection flow above an upward facing horizontal heated plate placed in a semi-infinite medium by control volume was conducted by Pretot *et al.* [13]. Datta *et al*. [14] studied the non-similar solution of a steady mixed convection flow over a horizontal flat plate in the presence of surface mass transfer. Convective boundary condition has been applied by Aziz [15] and Ishak [16]. Khan *et al*. [17] studied the mixed convection of Newtonian fluid flow along a moving horizontal plate with higher order chemical reaction, variable concentration reactant and variable wall temperature and concentration. Aziz *et al*. [18] studied the steady boundary layer free convection flow past a horizontal flat plate located in a porous medium filled by a water-based Newtonian nanofluid with gyrotactic microorganisms. Uddin *et al.* [19] presented a mathematical model for steady 2-D boundary layer flow from a heated horizontal surface which is embedded in a porous medium saturated with a non-Newtonian power-law nanofluid. Most of the investigators transform the partial differential equations to ordinary differential equations via a similarity transformation found by *ad hoc* methods.

Group methods determine similarity transformations (the symmetries of differential equations) of the governing partial differential equations with initial and boundary conditions [20-23]). The number of independent variables of the partial differential equations is reduced and the independent variables are converted into a single independent variable (called similarity variable). Also, the original initial and boundary conditions become two boundary conditions in the new combined variable. The primary advantage of the group method is that it can be successfully applied to non-linear differential equations. This method has been extensively adopted by many authors. Abd-el-Malek *et al.* [24] analyzed unsteady free convection flow over a continuous moving vertical sheet. Kassem and Rashed [25] investigated progress of a time dependent chemical reaction over a flat vertical plate. Akgu *et al.* [26] investigated the problem of a 2-D, unsteady flow and a heat transfer of a viscous fluid past a surface in the presence of variable suction/injection using Lie group theory. Uddin *et al*. [27] studied MHD free convective boundary layer flow of a nanofluid past a flat vertical plate with Newtonian heating boundary condition.

Most of the above-mentioned researchers applied conventional isothermal, or iso-flux thermal boundary condition. The use of convective boundary condition and investigation of the complete symmetries for free convective boundary layer flow and heat transfer equations are lacking in the literature. The present study aims to develop and investigate new similarity transformations and hence the corresponding similarity solutions for free convection flow with heat transfer adjacent to a horizontal moving flat plate with convective boundary condition using deductive group method as used by Moran and Gaggioli [20, 21]. The velocity and temperature distributions are determined by solving non-linear similarity ordinary differential equations subject to boundary conditions by an implicit finite difference numerical method. Influence of index parameter n, free convection parameter λ, convection-conduction parameter b and Prandtl number Pr on the dimensionless velocity, temperature as well as friction factor, and the rate of heat transfer at the wall are investigated numerically and presented graphically.

Problem formulation

The geometry assumed in this study, along with the rectangular co-ordinates, $\bar{x}$ and $\bar{y}$, and the corresponding velocity components, $\bar{u}$ and $\bar{v}$ is depicted in fig. 1.

It is assumed that the temperature of the ambient fluid is T_∞, the unknown temperature of the plate is T_w and the bottom of the plate is heated from a hot fluid of temperature $T_f(> T_\infty)$ by the process of convection. This then yields a heat transfer variable coefficient $h_f(x)$. Fluid properties are assumed to be uniform except density which is assumed to vary with temperature (*i. e.*, the Boussinesq approximation). We neglect the viscous dissipation term in the energy equation. With these assumptions, the governing basic boundary layer equations in dimensional form for momentum and energy conservation, can be written as (Fan *et al.* [11]):

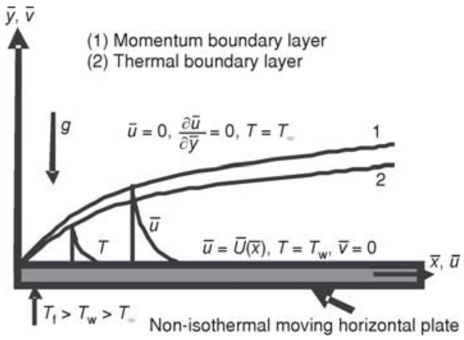

Figure 1. Schematic representation of the boundary layer flow of a continuously moving plate

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\mu}{\rho}\frac{\partial^2 \bar{u}}{\partial \bar{y}} \tag{2}$$

$$-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}} + g\beta(T - T_\infty) = 0 \tag{3}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 T}{\partial \bar{y}^2} \tag{4}$$

The boundary conditions are taken as:

$$\begin{aligned} &\bar{u} = \bar{U}(\bar{x}), \quad \bar{v} = 0, \quad \frac{\partial T}{\partial \bar{y}} = -\frac{h_f(\bar{x})}{k}(T_f - T_w) \quad \text{if} \quad \bar{y} = 0 \\ &\bar{u} \to 0, \quad \frac{\partial \bar{u}}{\partial \bar{y}} = 0, \quad T \to T_\infty \quad \text{if} \quad \bar{y} \to \infty \end{aligned} \tag{5}$$

where symbols are defined in the nomenclature. An additional boundary condition ($\partial \bar{u}/\partial \bar{y} = 0$ as $\bar{y} \to \infty$ is considered, as the boundary conditions in eq. (5) are not sufficient to completely determine the solution. The extra boundary condition is fully valid from physical considerations, as there is no shear stress in free stream [28]. Eliminating pressure gradient terms from eqs. (2) and (3) by cross differentiation, we get,

$$\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial \bar{u}}{\partial \bar{y}} + \bar{v}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} = \frac{\mu}{\rho}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - g\beta\frac{\partial(T - T_\infty)}{\partial \bar{x}} \tag{6}$$

Non-dimensionalization of the system of governing transport equations

We introduce the dimensionless variables:

$$x=\frac{\bar{x}}{L},\quad y=\frac{\bar{y}}{L}\sqrt[4]{\mathrm{Gr}},\quad u=\frac{\bar{u}}{U_{\mathrm{F}}},\quad v=\frac{\bar{v}}{U_{\mathrm{F}}}\sqrt[4]{\mathrm{Gr}},\quad \theta=\frac{T-T_{\infty}}{T_{\mathrm{f}}-T_{\infty}},\quad U=\frac{\bar{U}}{U_{\mathrm{F}}} \tag{7}$$

with L being characteristic length, $U_{\mathrm{F}} = [g\beta(T_{\mathrm{f}} - T_{\infty})L]^{1/2}$ is the reference velocity, and Gr = $= (\rho^2 g\beta\Delta TL^3)/\mu^2$ is the Grashof number. Define $\Delta T = T_{\mathrm{f}} - T_{\infty} = \Delta T_0\mathrm{Gr}^{1/4}g(x)$ and hence introducing stream function $\psi(x, y)$ so that $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$ we obtain, from eqs. (4) and (6):

$$\frac{\partial^4\psi}{\partial y^4}-\lambda\frac{\partial g(x)\theta}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial^3\psi}{\partial y^3}-\frac{\partial\psi}{\partial y}\frac{\partial^3\psi}{\partial y^2}=0 \tag{8}$$

$$\frac{1}{\mathrm{Pr}}g(x)\frac{\partial^2\theta}{\partial y^2}+g(x)\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}-g(x)\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\theta\frac{\partial\psi}{\partial y}\frac{\partial g(x)}{\partial x}=0 \tag{9}$$

The corresponding boundary conditions in eq. (5) become:

$$\frac{\partial\psi}{\partial y}\to 0,\quad \frac{\partial^2\psi}{\partial y^2}\to 0,\quad \theta\to 0 \quad \text{as} \quad y\to\infty \tag{10}$$

here $\lambda = (g\beta\Delta T_0 L)/U_F^2$ is the buoyancy parameter. Using one parameter transformation group analysis as used by Moran and Gaggioli [20], we have the following new similarity transformations:

$$\eta=y(x+\beta_1)^{-n},\quad \psi=(x+\beta_1)^{1-n}f(\eta),\quad \theta=\theta(\eta),\quad h_{\mathrm{f}}=(h_{\mathrm{f}})_0(x+\beta_1)^{-n}$$
$$U(x)=(x+\beta_1)^{1-2n},\quad g(x)=(x+\beta_1)^{2-5n} \tag{11}$$

Using the similarity transformations (11), eqs. (8) and (9) can be written as:

$$f'''' + (3n-1)f'f'' + (1-n)ff''' + (5n-2)\lambda\theta + n\lambda\eta\theta' = 0 \tag{12}$$

$$\frac{1}{\mathrm{Pr}}\theta'' + (5n-2)f'\theta + (1-n)f\theta' = 0 \tag{13}$$

The boundary conditions (10) become:

$$f'(0)=1,\quad f(0)=0,\quad \theta'(0)=-b[1-\theta(0)],\quad f''(\infty)=f(\infty)=\theta(\infty)=0 \tag{14}$$

Here $b = (L/k\mathrm{Gr}^{1/4})(h_{\mathrm{f}})_0$ is the convection-conduction parameter and primes denote ordinary derivatives with respect to the similarity independent variable η. It is interesting to mention that the convective boundary condition in eq. (14) can be written as:

$$\theta(0)=1+\gamma\theta'(0) \tag{15}$$

where $\gamma = 1/b$, $(b \neq 0)$ is the thermal slip parameter, so we can say that thermal slip is a special case of convective surface boundary condition. For $\gamma = 0$ this boundary condition becomes $\theta(0) = 1$ which is the isothermal case. Convective boundary condition occurs in a variety of real situations such as fluid flow around micro-electromechanical system (MEMS) [29].

It is interesting to note that when $b_1 = 0$, $n = 1/2$, the similarity transformations in eq. (11) reduce to those reported by Fan *et al.* [11] and yield an excellent benchmark for validation of our one parameter deductive group analysis.

Numerical scheme

Equations (12) and(13) with the boundary conditions (14) were solved numerically using a finite difference method. The step size was assumed to be 0.001 and the convergence criteria was taken as 10^{-6}. The asymptotic boundary conditions, given by eq. (14), were replaced by using a value of 10 for the similarity variable $\eta_{\max}$ as:

$$\eta_{\max} = 10, \quad f'(10) = f''(10) = \theta(10) = 0 \qquad (16)$$

The choice of $\eta_{\max} = 10$ ensures that all numerical solutions approached the asymptotic values correctly.

Discussion of results

Equations (12) and (13) subject to the boundary conditions (14) have been solved numerically for several values of the free convection parameter λ, index parameter n, convection-conduction (essentially the Biot number) parameter b, and Prandtl number Pr. The influences of these parameters on the flow, skin friction factor and heat transfer characteristics are investigated. The effects of index parameter n = 1/2, 3/4, and free convection parameter λ = 0, 0.5, and 1.0 on the velocity is shown in fig. 2. It is clear from fig. 2 that, as the index parameter increases, the velocity increases. The velocity decreases with free convection parameter for n = 3/4 whilst for n = = 1/2, the behavior of velocity is opposite.

Influence of the index parameter n = 0.5, 0.45, and free convection parameter λ = 0, 0.5, and 1.0 on the temperature is shown in fig. 3. It is obvious from fig. 3 that the thermal boundary layer thickness reduces as free convection parameter increases. We also observed that the thickness of thermal boundary layer increases as the index parameter increases. Note that values of the Biot number smaller than or equal to 0.1 imply that the heat conduction inside the body is much quicker than the heat convection away from its surface, and temperature gradients are negligible inside of it. Having a Biot number smaller than or equal to 0.1 labels a substance as thermally thin, and temperature can be assumed to be constant throughout the materials volume. The opposite is also true. A Biot number greater than 0.1 (a "thermally thick" substance) indicates that one cannot make this assumption, and more complicated heat transfer equations for "transient heat conduction" will be required to describe the time-varying and non-spatially-uniform temperature field within the material body. Note that in figs. 2 and 3 we have taken Pr = 1 which means that momentum and heat diffuses in an equal rate.

Figure 4 shows the influence of Prandtl number and convection-conduction parameter on the dimensionless temperature. A rise in Pr from 1 through 5, decreases in

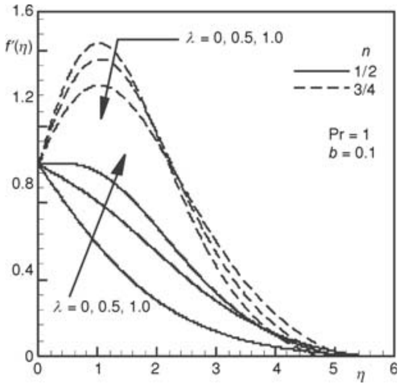

Figure 2. Effects of index parameter *n* and buoyancy parameter λ on velocity profiles

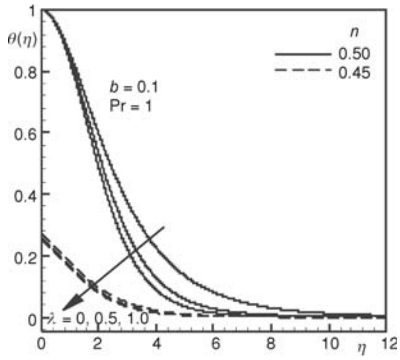

Figure 3. Effects of index parameter *n* and buoyancy parameter λ on temperature profiles

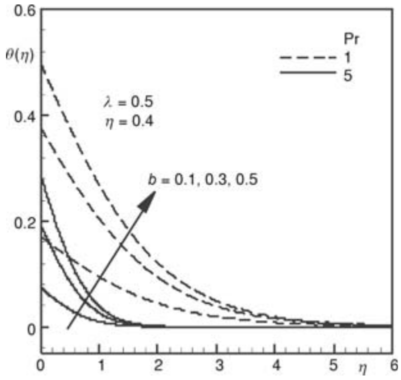

Figure 4. Effects of Prandtl number and convection-conduction parameter *b* on temperature profiles

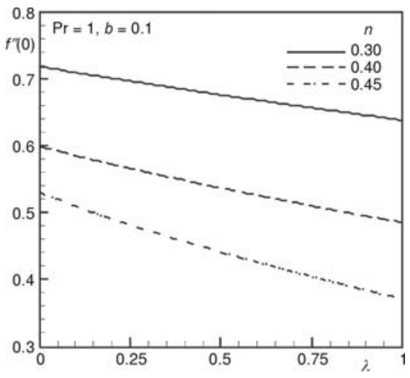

Figure 5. Variation of friction factor *vs.* buoyancy parameter λ for different *n*

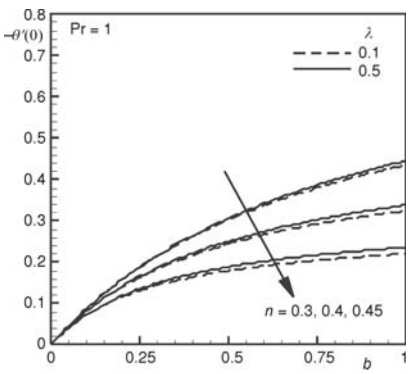

Figure 6. Variation of dimensionless heat transfer rates *vs.* convection-conduction parameter *b* for different buoyancy parameter λ

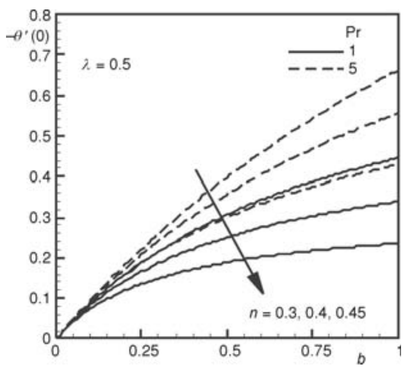

Figure 7. Variation of dimensionless heat transfer rates *vs.* convection-conduction parameter *b* for different Prandtl number Pr

thermal conductivity of the fluid (*i. e.* an increase in Prandtl number for constant values of dynamic viscosity and specific heat capacity). Temperatures are therefore dramatically decreased. This is consistent with the well--known behaviour associated with lower thermal conductivity fluids (high Prandtl numbers) compared with higher thermal conductivity fluids (low Prandtl numbers), which is verified by many experiments described in the classical exposition by Schlichting [30]. We notice from fig. 4 that temperature is a maximum at the wall and decreases to free stream temperature for all values of convection-conduction parameter.

The dimensionless friction factor as a function of free convection parameter λ and index parameter $n =$ $= 0.3$, 0.4, and 0.45 is shown in fig. 5. Due to the rise in index parameter, the friction factor falls. Also, the friction factor decreases with an increase in the free convection parameter.

The effects of the free convection parameter λ, convective heat transfer parameter *b*, index parameter *n,* and Prandtl number on the rate of wall heat transfer are illustrated in figs. 6 and 7 for $n = 0.3$, 0.4, and 0.45, $\lambda =$ $= 0.1$ and 0.5, and Pr = 1 (fig. 6), Pr = 1.5 (fig.7). The rate of wall heat transfer decrease with an increase in index parameter *n* whilst it increases with an increase in *b*. We noticed from fig. 6 that as free convective parameter λ increases, the rate of wall heat transfer increase.

The rate of dimensionless wall heat transfer increase with the Prandtl number, as expected. This trend is clear from fig. 7.

Conclusions

The 2-D steady laminar boundary layer flow along a moving horizontal plate with convective boundary conditions has been studied theoretically as well as numerically. The governing conservation equations have been reduced to a system of coupled non-linear ordinary differential equations with the associated boundary conditions using new similarity transformations generated by one parameter deductive group transformations method. The main findings are as follows.

- The free convection parameter reduces the thermal boundary layer thickness but increases the heat transfer rates.
- The index parameter increase velocity and temperature while it decreases wall friction factor and rate of wall heat transfer.

- Convection-conduction parameter increases both the temperature and rate of wall temperature.
- The Prandtl number decrease temperature whilst it increases the rate of wall heat transfer.

In future, this paper may be extended for steady/unsteady double diffusive Newtonoian/non-Newtonian nanofluid and may be investigated for multiple solutions.

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Nomenclature

b – convective heat transfer parameter, [–]
$f(\eta)$ – dimensionless stream function, [–]
g – acceleration due to gravity, [ms^{-2}]
Gr – Grashof number, [–]
h_f – heat transfer coefficient, [Wm^{2}K^{-1}]
k – thermal conductivity, [m^{2}s^{-1}]
L – characteristic length, [m]
m – power law index, [–]
Pr – Prandtl number, [–]
$\bar{p}$ – pressure, [Pa]
T – temperature, [K]
$\bar{u}, \bar{v}$ – velocity components along axes, [ms^{-1}]
$\bar{x}, \bar{y}$ – Cartesian co-ordinates along and normal to the plate, [m]

Greek symbols

α – thermal diffusivity, [m^{2}s^{-1}]
β – volumetric thermal expansion, [K^{-1}]
η – similarity variable, [–]
$\theta(\eta)$ – dimensionless temperature, [–]
λ – buoyancy parameter
μ – dynamic viscosity of the fluid, [kgm^{-1}s^{-1}]
ρ – density of fluid, [kgm^{-3}]
ϕ – stream function

Subscripts/superscripts

w – condition at the wall
∞ – free stream condition
' – differentiation with respect to η

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