NEW MULTI-SOLITON SOLUTIONS FOR GENERALIZED BURGERS-HUXLEY EQUATION

by

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The double exp-function method is used to obtain a two-soliton solution of the generalized Burgers-Huxley equation. The wave has two different velocities and two different frequencies.

Key words: *solitary solution, non-linear evolving equation, double exp-function method*

Introduction

The Burgers-Huxley equation is encountered in the description of many non-linear wave phenomena [1]. It can be written [1]:

$$u_t + \alpha u^n u_x - u_{xx} - \beta u(u^n - \gamma)(1 - u^n) = 0 \tag{1}$$

where α, β, γ, and n are constants.

This equation can be solved by various analytical methods, such as the variational iteration method [2], the homotopy perturbation method [3-5], and the exp-function method [6, 7]. A complete review on various analytical method is available in [8, 9]. In this paper the double exp-function method [10] is adopted to elucidate the different velocities and different frequencies in the travelling wave.

Double exp-function method

The multiple exp-function method was first proposed in [10], and the double exp-function method was used to search for double-soliton solutions in [11]. Assume that the solution of eq. (1) can be expressed in the form:

$$u = \frac{a_1 e^{\xi} + a_2 e^{-\xi} + a_5 + a_3 e^{\eta} + a_4 e^{-\eta}}{k_1 e^{\xi} + k_2 e^{-\xi} + k_5 + k_3 e^{\eta} + k_4 e^{-\eta}} \tag{2}$$

where $\xi = c_1 x + c_2 t$, and $\eta = c_3 x + c_4 t$.

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Substituting eq. (2) into eq. (1) and equating all coefficients of $\mathrm{e}^{(i\xi+j\eta)}$ to zero, we have a set of algebraic equations. Solving the resulting system with the aid of some a mathematical software, we can identify the constants in eq. (2).

Case 1. One-soliton solution

$$u(x,t)=\frac{a_5}{k_1\mathrm{e}^{\xi}+a_5+k_4\mathrm{e}^{-\xi}} \tag{3}$$

where k_1, k_4, and a_5 are free parameters, and:

$$\xi=\frac{\alpha}{4}-\frac{\sqrt{\alpha^2+8\beta}}{4}x+c_2t$$

Case 2. Two-soliton solution

$$u(x,t)=\frac{\frac{sk_3k_4}{k_2}\mathrm{e}^{\xi}+k_2\mathrm{e}^{-\xi}+sk_3\mathrm{e}^{\eta}+k_4\mathrm{e}^{-\eta}}{k_1\mathrm{e}^{\xi}+k_2\mathrm{e}^{-\xi}+k_3\mathrm{e}^{\eta}+k_4\mathrm{e}^{-\eta}} \tag{4}$$

where

$$\xi=\left(\frac{\alpha}{4}-c_3-\frac{\alpha\gamma}{4}+\frac{\delta}{4}\right)x+(2c_3-\alpha)\left(\frac{\alpha}{4}-c_3-\frac{\alpha\gamma}{4}+\frac{\delta}{4}\right)t$$

$$\eta=c_3x+\left\{-\frac{1}{2}\alpha c_3-\frac{\beta}{2}-\frac{1}{2}\alpha\gamma c_3+\frac{\beta\gamma^2}{2}+\left(\frac{1}{2}\alpha-2c_3-\frac{1}{2}\alpha\gamma\right)\left(\frac{\alpha}{4}-c_3-\frac{\alpha\gamma}{4}+\frac{\delta}{4}\right)\right\}t$$

$$\delta=\sqrt{8\beta\gamma^2-16\beta\gamma+8\beta+\alpha^2\gamma^2-2\alpha^2\gamma+\alpha^2}$$

Alternatively by the following transformation:

$$u=v^{\frac{1}{n}} \tag{5}$$

eq. (1) becomes:

$$vv_t+\alpha nv^2v_x+\left(1-\frac{1}{n}\right)v_x^2-vv_{xx}+\beta nv^2(v-1)(v-\gamma)=0 \tag{6}$$

By the similar solution process as above, we have:

Case 1. One-soliton solution

$$u(x,t)=\frac{\gamma k_5}{k_3\mathrm{e}^{\xi}+k_5} \tag{7}$$

where k_3, and k_5 are some free parameters, and:

$$\xi = c_3 x + c_4 t$$

$$c_3 = \frac{1}{2}\left[\alpha\gamma n - \sqrt{(\alpha\gamma n)^2 + 4\beta\gamma^2(1+n)}\right](1+n)^{-1}$$

$$c_4 = \frac{-n\lambda[\beta\gamma + c_3\alpha - \beta(1+n)]}{1+n}$$

Case 2. One-soliton solution

$$u(x,t) = \frac{\gamma^3 k_2{}^2 e^{\eta} + a_5\gamma^2 k_2}{k_2{}^2\gamma^2 e^{\eta} - a_5(a_5 - \gamma k_5)e^{-\eta} + k_5\gamma^2 k_2} \tag{8}$$

$$\eta = c_1 x + c_2 t$$

$$c_1 = \frac{1}{2}\left[\alpha\gamma n - \sqrt{(\alpha\gamma n)^2 + 4\beta\gamma^2(1+n)}\right](1+n)^{-1}$$

$$c_2 = \frac{-n\lambda[\beta\gamma + c_3\alpha - \beta(1+n)]}{1+n}$$

Case 3. Two-soliton solution

$$u(x,t) = \left[\frac{\gamma k_4(k_2 e^{-c_1 x - c_2 t} + k_4 e^{-c_3 x - c_4 t})}{k_1 k_4 e^{c_1 x + c_2 t} + k_2 k_4 e^{-c_1 x - c_2 t} + k_2 k_1 e^{c_3 x + c_4 t} + k_4{}^2 e^{-c_3 x - c_4 t}}\right]^{\frac{1}{n}} \tag{9}$$

where k_2, k_4, and c_3 are free parameters

$$c_1 = \frac{-2c_3(1+n) + \alpha\gamma n^2 + \sqrt{\delta}}{2(1+n)}$$

$$c_2 = -c_4 + \beta n\gamma - \frac{1}{2}\alpha\gamma n\left[\alpha\gamma n^2 - 2c_3(1+n) + \sqrt{\delta}\right] + (-\beta\lambda^2 n - \gamma n\alpha c_3)(1+n)^{-1}$$

$$\delta = \gamma^2 n^2\left[\alpha^2\gamma^2 n^2 + 4\beta(1+\gamma n)\right]$$

Conclusions

Using the double exp-function method, new two-solition solutions are obtained for generalized Burgers-Huxley equation. This method can also be applied to solve other types of non-linear evolution equations.

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