NEW MULTI-SOLITON SOLUTIONS FOR GENERALIZED BURGERS-HUXLEY EQUATION

by

Jun LIU*, Hong-Ying LUO*, Gui MU*, Zhengde DAI*, and Xi LIU*

* Corresponding author; e-mail: liujunxie@126.com

a College of Mathematics and Information Science, Qujing Normal University, Qujing, China
b School of Mathematics and Statics, Yunnan University, Kunming, China
c School of Information Science and Engineering, Yunnan University, Kunming, China

Short paper

DOI: 10.2298/TSCI1305486L

The double exp-function method is used to obtain a two-soliton solution of the generalized Burgers-Huxley equation. The wave has two different velocities and two different frequencies.

Key words: solitary solution, non-linear evolving equation, double exp-function method

Introduction

The Burgers-Huxley equation is encountered in the description of many non-linear wave phenomena [1]. It can be written [1]:

$$u_t + c u_n u_x - u_{xx} - \beta u(u^n - \gamma)(1 - u^n) = 0$$  \hspace{1cm} (1)

where \(\alpha, \beta, \gamma\) and \(n\) are constants.

This equation can be solved by various analytical methods, such as the variational iteration method [2], the homotopy perturbation method [3-5], and the exp-function method [6, 7]. A complete review on various analytical method is available in [8, 9]. In this paper the double exp-function method [10] is adopted to elucidate the different velocities and different frequencies in the travelling wave.

Double exp-function method

The multiple exp-function method was first proposed in [10], and the double exp-function method was used to search for double-soliton solutions in [11]. Assume that the solution of eq. (1) can be expressed in the form:

$$u = \frac{a_1 e^\xi + a_2 e^{-\xi} + a_3 + a_4 e^\eta + a_5 e^{-\eta}}{k_1 e^\xi + k_2 e^{-\xi} + k_3 + k_4 e^\eta + k_5 e^{-\eta}}$$  \hspace{1cm} (2)

where \(\xi = c_1 x + c_2 t\), and \(\eta = c_3 x + c_4 t\).
Substituting eq. (2) into eq. (1) and equating all coefficients of $e^{i(\xi + \eta)}$ to zero, we have a set of algebraic equations. Solving the resulting system with the aid of some mathematical software, we can identify the constants in eq. (2).

**Case 1. One-soliton solution**

$$u(x,t) = \frac{a_5}{k_1e^{\xi} + a_5 + k_4e^{-\xi}}$$  \hspace{1cm} (3)

where $k_1$, $k_4$, and $a_5$ are free parameters, and:

$$\xi = \frac{\alpha}{4} - \frac{\sqrt{\alpha^2 + 8\beta}}{4}x + c_2t$$

**Case 2. Two-soliton solution**

$$u(x,t) = \frac{sk_3k_4}{k_2}e^{\xi} + k_2e^{-\xi} + sk_3e^{\eta} + k_4e^{-\eta}$$

$$\xi = \frac{\alpha}{4} - c_3 - \frac{\alpha\gamma}{4} + \frac{\delta}{4}x + (2c_3 - \alpha)\left(\frac{\alpha}{4} - c_3 - \frac{\alpha\gamma}{4} + \frac{\delta}{4}\right)t$$

$$\eta = c_3x + \left\{-\frac{1}{2}\alpha c_3 - \frac{\beta}{2} - \frac{1}{2}\alpha\gamma c_3 + \frac{\beta\gamma^2}{2} + \left(\frac{1}{2}\alpha - 2c_3 - \frac{1}{2}\alpha\gamma\right)\left(\frac{\alpha}{4} - c_3 - \frac{\alpha\gamma}{4} + \frac{\delta}{4}\right)t\right\}$$

$$\delta = \sqrt{8\beta\gamma^2 - 16\beta\gamma + 8\beta + \alpha^2\gamma^2 - 2\alpha^2\gamma + \alpha^2}$$

Alternatively by the following transformation:

$$u = \nu^n$$  \hspace{1cm} (5)

eq (1) becomes:

$$\nu \nu_t + \alpha n^2 \nu_x + \left(1 - \frac{1}{n}\right)\nu_x^2 - \nu \nu_{xx} + \beta n^2 (\nu - 1)(\nu - \gamma) = 0$$  \hspace{1cm} (6)

By the similar solution process as above, we have:

**Case 1. One-soliton solution**

$$u(x,t) = \frac{\gamma k_5}{k_3e^{\xi} + k_5}$$  \hspace{1cm} (7)
where $k_3$, and $k_5$ are some free parameters, and:

$$\xi = c_3 x + c_4 t$$

$$c_3 = \frac{1}{2} \left[ a_\gamma n - \sqrt{(a_\gamma n)^2 + 4b_\gamma^2 (1+n)} \right] (1+n)^{-1}$$

$$c_4 = \frac{-n\lambda [b_\gamma + c_3 \alpha - \beta (1+n)]}{1+n}$$

**Case 2. One-soliton solution**

$$u(x,t) = \frac{\gamma^3 k_2^2 c^n + a_\gamma^2 k_2}{k_2^2 \gamma^2 e^n - a_5 (a_5 - \gamma k_3) e^{-n} + k_5 \gamma^2 k_2}$$

$$\eta = c_1 x + c_2 t$$

$$c_1 = \frac{1}{2} \left[ a_\gamma n - \sqrt{(a_\gamma n)^2 + 4b_\gamma^2 (1+n)} \right] (1+n)^{-1}$$

$$c_2 = \frac{-n\lambda [b_\gamma + c_3 \alpha - \beta (1+n)]}{1+n}$$

**Case 3. Two-soliton solution**

$$u(x,t) = \left[ \frac{\gamma k_4 (k_2^2 e^{(c_1+c_2)t} + k_4 e^{-c_3+c_4 t})}{k_1 k_2 k_4 e^{c_1+c_2 t} + k_2 k_4 (c_1+c_2 t) + k_2^2 k_4 e^{c_3+c_3 t} + k_4^2 e^{-c_3+c_4 t}} \right]^{1/n}$$

where $k_2$, $k_4$, and $c_3$ are free parameters

$$c_1 = \frac{-2c_3 (1+n) + a_\gamma n^2 + \sqrt{\delta}}{2(1+n)}$$

$$c_2 = -c_4 + \beta n \gamma - \frac{1}{2} a_\gamma n \left[ a_\gamma n^2 - 2c_3 (1+n) + \sqrt{\delta} \right] + (-\beta \lambda^2 n - \gamma n c_3 (1+n)^{-1}$$

$$\delta = \gamma^2 n^2 \left[ a^2 \gamma^2 n^2 + 4\beta (1+n) \right]$$

**Conclusions**

Using the double exp-function method, new two-solition solutions are obtained for generalized Burgers-Huxley equation. This method can also be applied to solve other types of non-linear evolution equations.

**Acknowledgments**

The work was supported by Chinese Natural Science Foundation Grant No. 11061028, 11361048, Yunnan NSF Grant No. 2010CD086, 2011Y012 and Qujin Normal University NSF Grant No. 2012QN016, 2010QN018.
References


