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ANALYSIS MODEL FOR FORECASTING EXTREME TEMPERATURE USING REFINED RANK SET PAIR

by

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In order to improve the precision of forecasting extreme temperature time series, a refined rank set pair analysis model with a refined rank transformation function is proposed to improve precision of its prediction. The measured values of the annual highest temperature of two China's cities, Taiyuan and Shijiazhuang, in July are taken to examine the performance of a refined rank set pair model.

Key words: *extreme temperature, prediction, time series, refined rank set pair analysis model, robust analysis, error analysis*

Introduction

Temperature has a close relationship with heat circulation and water flow in the environment, and thus it plays an important role in human's daily life [1, 2] and it is important to forecast extreme temperature accurately [3]. Since many factors attribute a lot to the temperature change, the accurate prediction of extreme temperature is faced with a high degree of scientific uncertainty, which traditional deterministic mathematical model cannot solve perfectly. Numerical simulation method can solve the problem better [3-5].

In recent years, artificial neural network (ANN) algorithms are widely used to deal with forecasting meteorological objects [6]. Based on the GA and particle swarm algorithm, Yang designed the back propagation (BP) neural networks to establish the multi-factor time series forecasting model [7]. At the same time, set pair analysis (SPA) model which is easy to operate and gives good prediction results is also popular in meteorological forecast field [4, 5]. Jin applied the set pair analysis method to the set pair analysis based on similarity forecast model of water resources change (SPA-SF), and the application results showed that the statistic and physical concepts of SPA-SF were distinct and its precision was high [8].

However, SPA model does not provide a unified standard of quantifying the set element symbols, while the prediction results are probably correlative with it. Using the similarity of time series as rank and using the rank set pair analysis [6-9] to transform the rank of set's elements in the R-SPA model, the results are proved to be better. Nonetheless, three aspects [8] given by the connection degree [9] in the R-SPA model have the same weight. Therefore the coefficients cannot reflect all the information about the difference between sets, which may cause non-negligible deviation of predicted value.

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To overcome the defects of the models mentioned above, refined rank set pair analysis (RRSPA) model is proposed in the paper. First, the three-element connection number is extended to multivariate connection number, and linear function is chosen for rank transformation function *f*, which transforms the elements based on the rank and helps avoid choosing the quantization standard. The numerical size and proportion of *f*'s value is corresponding to that of elements in the original set, describing the change trend of elements in the set nicely. Then linear function is also applied to the uncertain coefficient function, which gives a precise description of weight. At last, comparing RRSPA model with traditional R-SPA model, BP model, and LR model, we get the conclusion.

Refined rank set pair analysis model

The procedure of the establishment of this RRSPA model is as follows.

Step 1. Data processing

Considering time series $x_1, x_2, \ldots, x_n$, we assume that x_t has dependence with the front *T* history numbers, namely $x_{t-1}, x_{t-2}, \ldots, x_{t-T}$. Move the time series, and we can get history sets $A_1, A_2, \ldots, A_{n-T}$, and the current set *B*. Every set has the size of *T*. History set $A_k (k = 1, 2, \ldots, n-T)$ is corresponding to a subsequent value x_{T+i}.

Usually, because of weak dependence of the time series, the size of history set and current set is an integer from 4-6 [6].

Step 2. Determination of refined rank transformation function

The traditional rank transformation of a set [6] is ordering the elements of the set from 1 to *T* according to the rank they belong to. But there exists a defect in this transformation, as it does not make a refined treatment on the size and multiple relationships of the elements in a set. To overcome this defect of the traditional rank transformation, refined rank transformation function *f* is proposed.

Similar to the traditional rank transformation, the refined rank transformation function must confirm that the minimum of a set turns out to be 1, maximum is *T*, and the rest is transformed according to the proportion.

Let $f(a_i)$ denotes the transformation result of the ith element a_i in A_k, and $f(b_i)$ denotes the transformation result of the ith element b_i in *B*. Since the linear function is transformed according to numerical proportion, linear function is chosen for *f*:

$$f(a_i) = \frac{a_i - \min(A_k)}{\max(A_k) - \min(A_k)}(T-1)+1, \quad f(b_i) = \frac{b_i - \min(B)}{\max(B) - \min(B)}(T-1)+1 \qquad (1)$$

After a simple verification, the refined rank transformation function does guarantee that the transformation result of minimum in a set is 1, while maximum is *T*.

The definition of *f* avoids the selection of quantification standard. At the same time, the co-domain of *f* covers all the real number in the interval [1, *T*], which reflects proportion and value of elements in the original set. Therefore *f* describes the change trend of elements in the set perfectly.

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Step 3. Determination of difference function

For set pairs (A_k, B), $k = 1, 2, \ldots n-T$, function $d(a_i, b_i)$ should describe the differences between $a_i \in A_k$ and $b_i \in B$, and thus the absolute value of the difference is a good choice. The $d(a_i, b_i)$ is:

$$d(a_i, b_i) = |f(a_i) - f(b_i)|, \quad (a_i, b_i) \in (A_k, B) \tag{2}$$

It is easy to be observed that the co-domain of $d(a_i, b_i)$ is $[0, T-1]$.

If $d = 0$, we consider that the pair (a_i, b_i) is identical. If $d = T-1$, we consider that the pair (a_i, b_i) is contrary. If $d \neq 0, T-1$, we consider that the pair (a_i, b_i) is discrepant.

In order to incorporate the traditional SPA theory, the definition of difference uncertain coefficient function $c(d)$ must meet the three following conditions:

(1) When $d = 0$, which indicates pair (a_i, b_i) is identical, $c(d) = 1$.
(2) When $d = T-1$, which indicates pair (a_i, b_i) is contrary, $c(d) = -1$.
(3) When $d \neq 0, T-1$, which indicates pair (a_i, b_i) is discrepant, $c(d) \in (-1, 1)$.

Step 4. Determination of difference uncertain coefficient function
and connection degree based on SPA

In order to confirm that the definition of difference uncertain coefficient function $c(d)$ meets the three conditions based on SPA theory, the linear function is also a good choice:

$$c(d) = -\frac{2}{T-1}d + 1 \tag{3}$$

Therefore, the connection degree of (A_k, B) can be calculated:

$$U_{A-B} = \frac{1}{N} \sum_{(a_i, b_i) \in (A,B)} c\left[d(a_i, b_i)\right] \tag{4}$$

Step 5. Determination of similar history sets and prediction

According to the connection degree maximum principle, some A_k similar to B are chosen from all the history sets. The way to choose history set A_k is given:

$$K = \{k \mid k = \arg\max\{U_{A_j - B} \mid 1 \leq j \leq n - T\}\} \tag{5}$$

Let m denote the size of K, and K can be expressed:

$$K = \{k_1, k_2 \cdots k_m\} \tag{6}$$

Thus the prediction value x'_{n+1} of x_{n+1} is expressed:

$$x'_{n+1} = \frac{1}{m} \sum_{j=1}^{m} w_j x_{T+k_j} \tag{7}$$

where w_i denotes the ratio of mean of elements in B to mean of elements in A_{k_j}.

Some real extreme temperature time series are chosen to examine the performance of RRSPA model, and is compared with traditional R-SPA model, LR model and BP model.

Applications in extreme temperature

Case 1

Study area and data description

The RRSPA model is applied to predict the highest temperature time series observed at Taiyuan station N37° 51', E112° 33') in China, in July 1951-2010.

We obtain the time series $x_1, x_2,\ldots, x_n$ ($n = 45$). Considering the weak dependence in average temperature series, the value of T, which should not be too large or too small, is taken as 6 in the paper. According to this, the history sets are $A_1, A_2,\ldots, A_{n-6}$.

Predicting the highest temperature in a year, such as the year 2000, we take the front T years (1994-1999) as set B. When predicting the highest temperature of July in the next year 2001, we assume the data in 1995-2000 is known.

The prediction results and analysis

Prediction results of the highest temperature in Taiyuan in July 2000-2010 and the corresponding relative error (RE) are presented in tab. 1.

Table 1. Predictions of the highest temperature in Taiyuan and relative errors

Year	Measured value	LR model		R-SPA model		BP model		RRSPA model	
		T [0.1/°C]	RE [%]	T [0.1/°C]	RE [%]	T [0.1/°C]	RE [%]	T [0.1/°C]	RE [%]
2000	338	351.35	3.95	361.40	6.92	346.45	2.50	342.08	1.21
2001	365	346.69	5.02	342.20	6.25	350.80	3.89	347.06	4.91
2002	359	348.27	2.99	356.22	0.78	346.91	3.37	344.80	3.95
2003	334	346.95	3.88	320.88	3.93	341.56	2.26	351.72	5.31
2004	334	346.46	3.73	347.94	4.17	359.56	7.65	343.90	2.96
2005	359	341.39	4.91	358.27	0.20	338.87	5.61	350.75	2.30
2006	350	354.31	1.23	351.90	0.54	358.15	2.33	357.12	2.03
2007	358	344.88	3.67	330.54	7.67	359.12	0.31	358.55	0.15
2008	353	340.20	3.63	317.32	10.11	333.59	5.50	317.32	10.11
2009	356	345.00	3.09	342.88	3.69	342.23	3.87	352.70	0.93
2010	394	351.18	10.87	354.38	10.05	353.90	10.18	363.43	7.76

To verify the prediction effect of RRSPA model, we take traditional R-SPA model, LR model, and BP model to make comparisons. To be the same, the temperature data in 1951-1994 is used as trained sets in BP model [6] and the data of 2000-2010 are tested sets, with 500 trained times and the study rate 0.3.

Compared with the results of BP model, R-SPA model, and RRSPA model, RRSPA model has the smallest deviation with the measured value in those 11 years in general. The number of relative errors below 3% in three models is, respectively, 3, 4, and 7. According to those points, RRSPA model has a better effect.

The precision of prediction is evaluated by three measurement indices in this paper, namely the mean relative error (MRE), mean absolute error (MAE), and root mean square error (RMSE). The definitions of the three indices can be given as:

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$$\text{MRE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - x_i}{x_i}\right|, \quad \text{MAE} = \frac{1}{n}\sum_{i=1}^{n}\left|y_i - x_i\right|, \quad \text{and} \quad \text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - x_i)^2} \qquad (8)$$

The results are calculated in tab. 2 (Taiyuan).

Table 2 indicates that the MRE of LR, traditional R-SPA model and BP model is, respectively, 4.27%, 4.94%, and 4.31%, while RRSPA model MRE is 3.78%, whose precision is relatively increased by 11.48%, 23.48%, and 12.30%. For the MAE and RMAE, RRSPA model also gives the smallest: as for MAE, RRSPA model precision has relatively increased by 11.9%, 23.29%, and 12.45%, respectively, compared with that of LR, and R-SPA, BP model; as for RMAE, RRSPA model precision has relatively increased by 4.05%, 20.58%, and 6.69%, respectively, compared with that of LR, R-SPA, and BP model. They all indicate that RRSPA model is a better choice for prediction.

Table 2. The error analysis in three models in Taiyuan

Models	LR model	R-SPA model	BP model	RRSPA model
MRE (%)	4.27	4.94	4.31	3.78
MAE	15.41	17.69	15.50	13.57
RMAE	18.02	21.77	18.53	17.29

Case 2

We also apply the same method to predict the highest temperature time series observed at Shijiazhuang station (N38°03', E114°26') in China in July 2000-2010, known data in 1956-2010. The corresponding measurement indices MRE, MAE, and RMAE of Shijiazhuang are calculated in tab. 3.

Table 3 indicates that the MRE of LR, traditional R-SPA model and BP, respectively, is 6.27%, 6.67%, and 6.11%, while that of RRSPA model is 5.73%, whose precision is also relatively increased. For the MAE and RMAE, the RRSPA model also gives the smallest: as for MAE, RRSPA model precision has relatively increased by 4.30%, 10.59%, and 4.20%, respectively, compared with that of LR, R-SPA, and BP model; as for RMAE, RRSPA model precision has relatively increased by 3.42%,

Table 3. The error analysis in three models in Shijiazhuang

Models	LR model	R-SPA model	BP model	RRSPA model
MRE (%)	6.27	6.67	6.11	5.73
MAE	24.8901	26.64	22.86	23.82
RMAE	27.8581	30.62	28.65	28.81

5.91%, and 0.56%, respectively, compared with that of LR, R-SPA, and BP model. Measured indices MAE and RMAE of RRSPA model are both the smallest, which indicates that RRSPA model is a better choice for prediction.

In sum, two cases both indicate that RRSPA model has the best estimation in prediction of high temperature, among LR, R-SPA, and BP neural network models.

Conclusions and prospect

A refined R-SPA model is proposed in this paper. The annual highest temperature time series in July of Taiyuan and Shijiazhuang are studied by using the new model. The main conclusions are:

- In the set pair analysis method, we establish a new formula to calculate the connection degree, which makes definition of the connection degree more explicit and simplify the

calculation. We apply it to RRSPA model. In addition, refined rank transformation function is proposed in this paper. The function makes a refined treatment on the size and multiple relationships of the elements in a set.

- In the view of error analysis, compared with traditional R-SPA model, RRSPA model gives smaller MRE, MAE, and RMSE. Taking Taiyuan as an example, as for MRE, RRSPA model precision has relatively increased by 11.48%, 23.48%, and 12.30%, respectively, compared with that of LR, R-SPA, and BP model; as for MAE, RRSPA model precision has relatively increased by11.9%, 23.29%, and 12.45%, respectively, compared with that of LR, R-SPA, and BP model; as for RMAE, RRSPA model precision has relatively increased by 4.05%, 20.58%, and 6.69%, respectively compared with that of LR, R-SPA, and BP model.

- Since linear function is chosen for both difference uncertain coefficient function $c(d)$ and refined rank transformation function f without putting more information about the prediction background in the RRSPA model, it may give a better result if some optimization algorithms are applied to the RRSPA model.

Acknowledgments

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