HEAT TRANSFER FROM A MOVING FLUID SPHERE WITH INTERNAL HEAT GENERATION

by

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In this work, we solve numerically the unsteady conduction-convection equation including heat generation inside a fluid sphere. The results of a numerical study in which the Nusselt numbers from a spherical fluid volume were computed for different ranges of Reynolds number ($0 < Re < 100$), Peclet number ($0 < Pe < 10000$) and viscosity ratio ($0 < \kappa < 10$), are presented. For a circulating drop with $Re \to 0$, steady creeping flow is assumed around and inside the sphere. In this case, the average temperatures computed from our numerical analysis are compared with those from literature and a very good agreement is found. For higher Reynolds number ($0 < Re < 100$), the Navier-Stokes equations are solved inside and outside the fluid sphere as well as the unsteady conduction-convection equation including heat generation inside the fluid sphere. It is proved that the viscosity ratio κ ($\kappa = \mu_d/\mu_c$) influences significantly the heat transfer from the sphere. The average Nusselt number decreases with increasing κ for a fixed Peclet number and a given Reynolds number. It is also observed that the average Nusselt number is increasing as Peclet number increases for a fixed Re and a fixed κ.

Key words: *fluid sphere, viscosity ratio, heat transfer, heat generation*

Introduction

The numerical solution of the Navier-Stokes equations coupled with diffusion-convection and/or the energy equation arises in many important applications in science and engineering. Some recent and useful work done for solving Navier-Stokes equations including convection and heat transfer may be found in [1-4]. Heat or mass transfer from a fluid or a rigid sphere without heat generation has been a subject of many investigations. The significant phenomena are very well explained in the books of Clift *et al.* [5] and Sadhal *et al.* [6]. Some advances on the motion and heat or mass transfer from a fluid sphere are given in the recent articles of Feng and Michaelides [7], Chhabra [8], Kishore *et al.* [9, 10], Saboni *et al.* [11, 12], Saboni and Alexandrova [13], Saboni *et al.* [14, 15], Guella *et al.* [16], Miliauskas [17], Sazhin *et al.* [18], Polyanin *et al.* [19], Faghri and Zhang [20], Sazhin [21, 22], and Sazhin *et al.* [23-25]. On

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the other hand heat or mass transfer from a fluid sphere with heat generation due to chemical or nuclear reaction has been a subject of few investigations. Kleinman and Reed [26] analyzed theoretically the transfer from a fluid sphere in the presence of isothermal chemical reaction in the continuous phase. Juncu [27] analyses the conjugate heat and mass transfer from a rigid sphere with non-isothermal first-order irreversible chemical reaction. Unsteady mass transfer from a drop with reversible second-order chemical reaction on the surface of the drop has been analysed by Juncu [28]. The mass balance equations were solved numerically for moderate Peclet numbers (Pe = 1000). Souccar and Oliver [29] studied the transfer from a droplet at high Peclet numbers with heat generation in creeping flow (Re $\to$ 0).

It should be noted that Souccar and Oliver [29] study was devoted to the creeping flow regime (Re $\to$ 0) and the results cannot be applied for a fluid sphere at higher Reynolds numbers. In addition the results relate only to fluids at extreme values of Peclet number (Pe $\to$0, Pe $\to \infty$). The results cannot be applied in the case of a fluid sphere at intermediate Peclet numbers. In this study, which is an extension of the Souccar and Oliver [29] work, we solve the unsteady conduction-convection equation including heat generation inside the fluid sphere. This paper presents a numerical study of transfer from a fluid sphere with heat generation, computed for Reynolds numbers in the range $0 \le \mathrm{Re} \le 100$, viscosity ratios between the dispersed phase and the continuous phase ($\kappa = \mu_d/\mu_c$) from 0 to 100 and Peclet numbers from 0 to 10000.

Governing equations

Consider a fluid sphere of radius a, moving slowly with uniform velocity U_∞ in another immiscible fluid of infinite extent. In the followings analysis, it will be assumed that:
- the variations in thermo physical properties of the fluid sphere are negligible,
- the heat generation is uniform throughout the fluid sphere,
- no phase change occurs during the heat transfer,
- there is no surface active agent,
- there is no oscillation and no rotation of the fluid sphere,
- the size and shape of the fluid sphere remain constant, and
- the continuous and dispersed flow fields are steady-state.

In addition the thermal resistance is assumed negligible in the continuous phase as compared to that inside the sphere. In this case, the temperature of the surface is assumed equal to that of the surrounding fluid medium ($T_s = T_\infty$). With this supposition, only the heat transfer, in the fluid sphere must be considered, so the flow around the sphere affects the heat transfer only indirectly because of the coupling between the internal flow and the external flow.

Since the flow field depends on Reynolds number, the rate of transfer will depend both on the Reynolds and on the Peclet number. Thus it is necessary to solve the Navier-Stokes equations, to obtain the velocity field and to use the latter for the solution of the conduction-convection equation. Since the flow is considered axisymmetric, the Navier-Stokes equations can be written in terms of stream function and vorticity (ψ and ω) in spherical co-ordinates r and θ [5] :

$$E^2\psi_d = \omega_d r\sin\theta \tag{1}$$

and
$$\frac{\mu_c}{\mu_d}\frac{\rho_d}{\rho_c}\frac{\mathrm{Re}}{2}\left[\frac{\partial\psi_d}{\partial r}\frac{\partial}{\partial\theta}\left(\frac{\omega_d}{r\sin\theta}\right)-\frac{\partial\psi_d}{\partial\theta}\frac{\partial}{\partial r}\left(\frac{\omega_d}{r\sin\theta}\right)\right]\sin\theta = E^2(\omega_d r\sin\theta) \tag{2}$$

where
$$E^2 = \frac{\partial^2}{\partial r^2}+\frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)$$

Outside the fluid sphere, the equations are still valid, but because of numerical reasons the radial co-ordinate r is transformed via $r = e^z$, where z is the logarithmic radial co-ordinate. The results are:

$$E^2\psi_c = \omega_c e^z \sin\theta \tag{3}$$

and

$$\frac{\mathrm{Re}}{2}\left[\frac{\partial\psi_c}{\partial z}\frac{\partial}{\partial\theta}\left(\frac{\omega_c}{e^z\sin\theta}\right)-\frac{\partial\psi_c}{\partial\theta}\frac{\partial}{\partial z}\left(\frac{\omega_c}{e^z\sin\theta}\right)\right]e^z\sin\theta = e^{2z}E^2(\omega_c e^z\sin\theta) \tag{4}$$

All variables are normalized by introducing the following dimensionless quantities: $r = r'/a$, $\omega = \omega'a/U_\infty$, $\psi' = \psi'/(U_\infty a^2)$, $\mathrm{Re} = 2aU_\infty/\nu_c$, where a is the sphere radius, Re – the Reynolds number, U_∞ – the terminal velocity, and ν – the kinematic viscosity. The primes denote the dimensional quantities and subscripts d and c refer to dispersed and continuous phase, respectively. In terms of dimensionless stream function ψ, the dimensionless radial and tangential velocities are given by:

$$u = -\frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta}, \quad \nu = \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r} \tag{5}$$

The boundary conditions to be satisfied are:

(1) far from the fluid sphere ($z = z_\infty$), undisturbed parallel flow is assumed: $\omega_c = 0$, $\psi_c = 0.5e^{2z}\sin^2\theta$,
(2) along the axis of symmetry ($\theta = 0, \pi$): $\psi_c = 0$, $\omega_c = 0$, $\psi_d = 0$, and $\omega_d = 0$, and
(3) across the interface ($z = 0$ or $r = 1$), the following relations take into account, respectively: negligible material transfer, continuity of the tangential velocity, continuity of the tangential stress (no surface tension variation):

$$\psi_c = 0, \quad \psi_d = 0, \quad \frac{\partial\psi_c}{\partial z} = \frac{\partial\psi_d}{\partial r}, \quad \frac{\mu_c}{\mu_d}\left(\frac{\partial^2\psi_d}{\partial z^2} - 3\frac{\partial\psi_c}{\partial z}\right) = \left(\frac{\partial^2\psi_d}{\partial r^2} - 2\frac{\partial\psi_d}{\partial r}\right)$$

where μ is the dynamic viscosity.

For $\mathrm{Re} \to 0$, the flows outside and inside a fluid sphere moving with a steady-state velocity were obtained analytically by Hadamard [30] and Rybczinski [31]. For such flow the non-dimensional velocity components are given by:

$$u_r = \frac{1-r^2}{2(1+\kappa)}\cos\theta \tag{6}$$

and

$$u_\theta = \frac{2r^2-1}{2(1+\kappa)}\cos\theta \tag{7}$$

At higher Reynolds numbers, no analytical solutions exist and numerical solutions must be sought. Equations (1) to (4) subjected to the boundary conditions (1) to (3) are solved simultaneously to obtain stream-function and vorticity values. Once stream function is known, the velocities can be determined from eq. (5).

Since the flow is considered axisymmetric, the unsteady convective heat transfer with heat generation in spherical co-ordinates r and θ is governed by the following dimensionless energy equation [29]:

$$\frac{\partial T}{\partial\tau} + \frac{\mathrm{Pe}}{2}\left(u_r\frac{\partial T}{\partial r} + \frac{u_\theta}{r}\frac{\partial T}{\partial\theta}\right) = \frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\left(\frac{\partial^2 T}{\partial\theta^2} + \cot\theta\frac{\partial T}{\partial\theta}\right) + 1 \tag{8}$$

where $\mathrm{Pe} = 2aU_\infty/\alpha$, α – the thermal diffusivity, τ – the dimensionless time ($\tau = \alpha t/a^2$), and T – the dimensionless temperature given by:

$$T(r,\theta,\tau) = k\,\frac{T'(r,\theta,\tau) - T'_\mathrm{s}}{\dot{q}a^2}$$

where T' is the dimensional temperature, k [Wm^{-1}K^{-1}] – the thermal conductivity, and $\dot{q}$ [Wm^{-3}] – the heat generation.

The boundary and initial conditions to be satisfied are:
(1) initial condition: $T(r, \theta, \tau = 0) = 0$,
(2) along the axis of symmetry $(\partial T/\partial\theta)|_{\theta=0} = (\partial T/\partial\theta)|_{\theta=\pi} = 0$, and
(3) across the interface: $T(\rho = 1, \theta, \tau) = 0$.

The Nusselt number is computed from the heat transfer flux from the surface of the sphere:

$$\mathrm{Nu} = -\int_0^\tau \left(\frac{\partial T}{\partial r}\right)_{r=1} \sin\theta \mathrm{d}\theta \tag{9}$$

The average temperature is computed from the equation:

$$\overline{T} = \frac{3}{2}\int_0^1\int_0^\tau T(r,\theta,\tau) r^2 \sin\theta \mathrm{d}\theta \mathrm{d}r \tag{10}$$

Method of solution

For low Reynolds numbers (creeping flow), the velocity fields are governed by eqs. (6) and (7). That is enough to incorporate these fields into the heat equation and to solve it numerically. For higher Reynolds numbers no analytical solutions exist, and numerical solutions of the Naviers-Stokes equations must be sought. The validation of the numerical procedure used here is focused on the effect of internal heat generation on the heat transfer. To validate the mathematical procedure, our numerical simulation was run at low Reynolds numbers and compared with available literature results [29]. For higher Reynolds numbers, the flow around and inside the fluid sphere is obtained by solving numerically the Navier-Stokes equations. In this case, eqs. (1) to (5), together with the above boundary conditions, are evaluated numerically. The equations of motion inside and outside the fluid sphere are solved by finite difference approximations. The elliptic stream function equations are solved iteratively, the parabolic vorticity equations are solved by means of the Alternating Direction Implicit method. Once stream function is known, the velocities are then determined from eq. (5). Detailed discussions on the accuracy of the solution procedure employed for the momentum, continuity equations, and diffusion-convection equation for the case of external resistance were made previously by Saboni and Alexandrova [13] and Saboni *et al.* [14, 15]. They used a computer code based on the same principles and investigated the effects of the viscosity ratio, the flow and contamination on the concentration profiles for a large ranges of Reynolds numbers, viscosity ratios and Peclet numbers. In these studies, the authors found a very good agreement between their numerical results and the available literature data.

To discretize the time-dependant heat equation (eq. 8), the forward-central (forward in time, central differencing in space) explicit scheme is used. Only half the physical domain was used because the line $\theta = 0$, is a line of symmetry. This assumption is justified for Reynolds less than about 200 as mentioned in the book of Panton [32]. To obtain precise results with minimal computing time, the selection of the grid was obtained by inspiration from previous studies and by making tests with different mesh sizes. The grid spacings radially and tangentially were found by decreasing the grid size by a factor two and running the simulation. Repeatability of the Nusselt number was used as a measure of the adequacy of the grid spacing. Outside the fluid sphere, a computational grid number in radial and tangential direction are 401 × 91, respectively while inside the fluid sphere, they are 61 × 91, respectively. Exploratory computations of finer grids were conducted and the resulting changes were too small compared with the increase of computation time.

First, the solution method was tested for the limiting case of a rigid sphere (or a stagnant drop, Pe = 0). In this case the dimensionless temperature is a function of r and τ only and the reduced equation has an analytical solution. This solution enabled to Souccar and Oliver [29], to get the dimensionless average temperature:

$$\overline{T}(\tau) = \sum_{n=1}^{\infty} \frac{6}{n^4 \pi^4} e^{-n^2 \pi^2 r} \tag{11}$$

Our numerical results for the dimensionless average temperature are compared in tab. 1 with the Souccar and Oliver [29] results. There is a very good agreement between our numerical results and those of Souccar and Oliver [29].

The solution method was also tested in the limiting case of a fluid sphere in the creeping flow at high Peclet number. The average temperatures computed from our numerical analysis are compared in tab. 2 with the Souccar and Oliver [29] correlation:

$$\overline{T} = \frac{8}{315}[1 - \exp(-26.8\tau)] \tag{12}$$

Very good agreement is found between our numerical results and those of Souccar and Oliver [29]. Table 2 shows that eq. (12) predicts the average temperature very well for all cases except at very low time values. This coincides with Souccar and Oliver [29] observations.

Table 1. Dimensionless average temperature as a function of time for a rigid sphere (Pe = 0)

τ	Analytical solution Souccar and Oliver [29]	Numerical results this study
0.0001	$9.10\cdot10^{-5}$	$9.50\cdot10^{-5}$
0.001	0.0009	0.0009
0.01	0.0078	0.0079
0.02	0.0142	0.0141
0.04	0.0243	0.0243
0.06	0.0322	0.0323
0.08	0.0385	0.0384
0.10	0.0436	0.0434
0.20	0.0581	0.0584
0.30	0.0634	0.0636
0.40	0.0654	0.0650

Results and discussion

Heat transfer at low Reynolds numbers (creeping flow)

Numerical results for creeping flow (Re $\rightarrow$0) and intermediate Peclet numbers (Pe_m = 0, 100, 200, 500, and 1000) are presented in figs. 1 and 2. Since the dimensionless velocities within the particle are proportional to $(1 + \kappa) - 1$ in the creeping flow regime, it was useful to introduce the modified Peclet number $Pe_m = Pe/(1 + \kappa)$. Figure 1 shows the time history of the average temperature for different Peclet numbers (Pe_m = = 0, 100, 200, 500, and 1000) in creeping flow (Re $\rightarrow$ 0).The figure shows the effect of the con-

Table 2. Dimensionless average temperature as a function of time for high Peclet number

τ	Souccar and Oliver [29] eq. 12 $Pe_m \rightarrow \infty$	This study numerical results Pe_m = 10000
0.001	0.0006	0.0009
0.01	0.0059	0.0070
0.02	0.0105	0.0116
0.04	0.0167	0.0176
0.06	0.0203	0.0210
0.08	0.0224	0.0229
0.1	0.0236	0.0241
0.2	0.0253	0.0248
0.3	0.0254	0.0248
0.4	0.0254	0.0248

duction-convection on the evolution of the average temperature inside the fluid sphere. In the absence of convection (Pe = 0), the conduction allows the sphere to reach the maximal temperature. The convection leads to the cooling of the sphere in a more or less important way depending on the value of the Peclet number. Increase in Peclet number value results in lower fluid sphere average temperature. This observation can be explained with the fact that higher Peclet number value results in higher dissipation to the fluid sphere surface. It is also evident that the increase in Peclet number faster leads to a steady state.

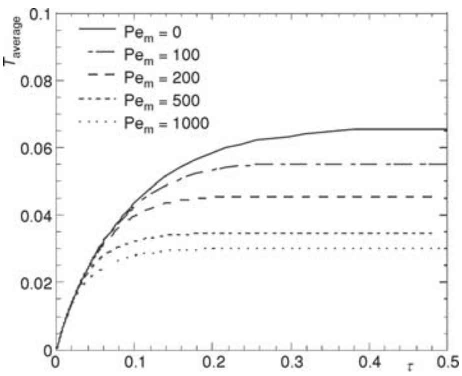

Figure 1. Average temperature *vs.* non-dimensional time for a fluid sphere in creeping flow

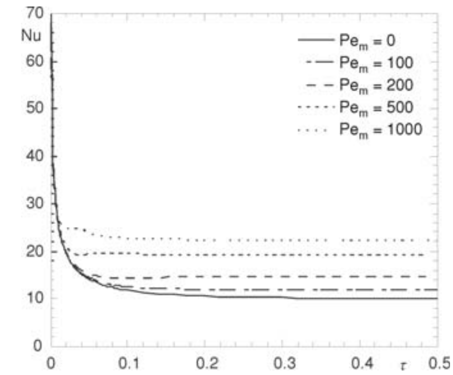

Figure 2. Average Nusselt number *vs.* non-dimensional time for a fluid sphere in creeping flow

The numerical results can be well correlated by the following equation:

$$\bar{T}(\tau) = \frac{0.5}{100 + \mathrm{Pe}_{\mathrm{m}}}[13.33(1.00 - \exp(-13.4\tau)) + 0.05\mathrm{Pe}_{\mathrm{m}}(1.00 - \exp(-40.20\tau))] \tag{13}$$

This formula gives the average temperature values which coincide with those obtained numerically for creeping flow at $0 < \mathrm{Pe_m} < 1000$ with an error not exceeding 15%.

Figure 2 shows the time history of the average Nusselt number for different Peclet numbers ($\mathrm{Pe_m}$ = 0, 10,100, 200, 500, and 1 000). From the figure it is evident that, initially, the Nusselt number is quite high and conduction dominates as a heat transfer mode. The Nusselt number decreases with time and approaches an asymptotic value depending on the predominance of the convective transfer (the asymptotic Nusselt number increases with increasing Peclet number). It is also observed that the average Nusselt number is increasing as Peclet number increases.

To present the numerical results in a more convenient form to use, we tried to correlate our numerical results by an equation which would be valid for both the low and high Peclet numbers. The equation obtained for the steady-state average Nusselt number is as follows:

$$\mathrm{Nu} = \frac{50}{\mathrm{Pe}_{\mathrm{m}} + 402}(87 + 0.57\mathrm{Pe}_{\mathrm{m}} + 0.126\sqrt[3]{\mathrm{Pe}_{\mathrm{m}}}) \tag{14}$$

The comparison shows that this correlation gives values of Nusselt number coinciding with those calculated numerically with an error not exceeding 10% for creeping flow with $0 < \mathrm{Pe_m} < 1000$.

Heat transfer at higher Reynolds numbers

Figures 3 to 4 show the steady state temperature fields within the fluid sphere for a fixed Reynolds number and different Peclet numbers and viscosity ratios κ. The numbers on the curves represent the values of the dimensionless temperature. Figure 3 presents the isotherms for Re = 100, Pe = 1000, and three values of the viscosity ratio κ = 1, 10, and 100. Examination of the isotherms plots indicates that heat transfer for low viscosity ratio is facilitated as a result of convection and conduction, while for other viscosity ratios examined, the convection does not play an important role and the transfer is essentially made by conduction. At low viscosity ratio, the convective transport has the dominant role and the isotherms become deformed under the influence of the internal circulation. This is not valid any more for high viscosity ratio and in this case the fluid sphere behaves as a rigid one.

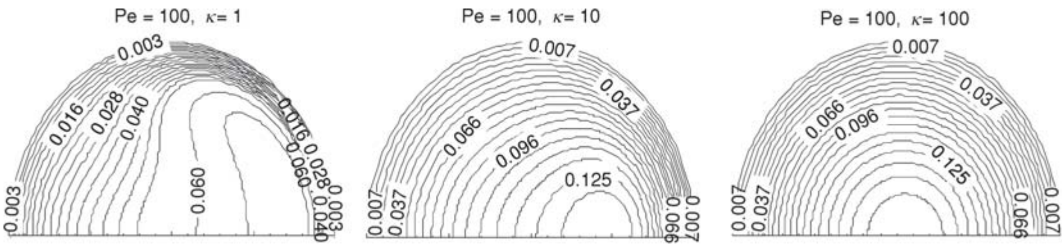

Figure 3. Steady-state temperature field within a fluid sphere for Re = 100, Pe = 100, and different viscosity ratios (κ = 1, 10, and 100)

Figure 4 presents the isotherms for Re = 100, Pe = 1000 and three values of the viscosity ratio κ = 1, 10 and 100. The same general observation as for fig. 3 can be noticed. In addition the heat transfer is more intensive, indicating that the convection is much greater due to the increased Peclet numbers. We can also note that for κ = 1, Re = 100 and Pe = 1000, the temperature contour lines tend to follow the streamlines because of the presence of a strong circulation flow inside the fluid sphere.

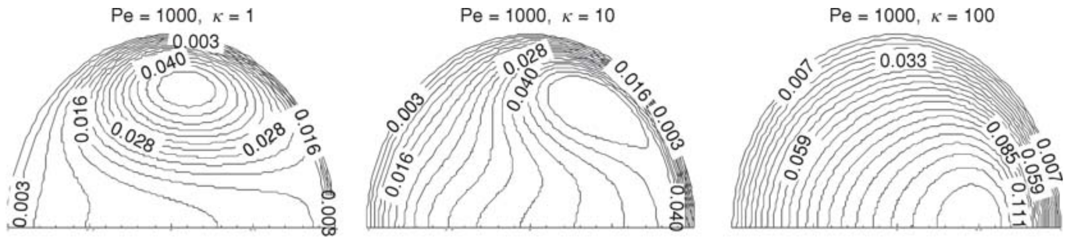

Figure 4. Steady-state temperature field within a fluid sphere for Re = 100, Pe = 1000, and different viscosity ratios (κ = 1, 10, and 100)

Figure 5 shows the time history of the average temperature for different Peclet numbers (Pe = 0, 10,100, 500, and 1000), at a fixed Reynolds number Re = 100 and viscosity ratio κ = 1. Here, also, the presence of the convection allows the cooling of the sphere in a more or less important way according to the value of the Peclet number.

Figure 6 shows the time history of the average Nusselt number for different Peclet numbers (Pe = 0, 10,100, 500, 1000, and 10000), for a fixed Reynolds number (Re = 100) and viscosity ratio ($\kappa = 1$). The figure shows that, initially, the Nusselt number is quite high and conduction dominates as a heat transfer mode. The Nusselt number decreases with time and approaches an asymptotic value depending on the predominance of the convective transfer.

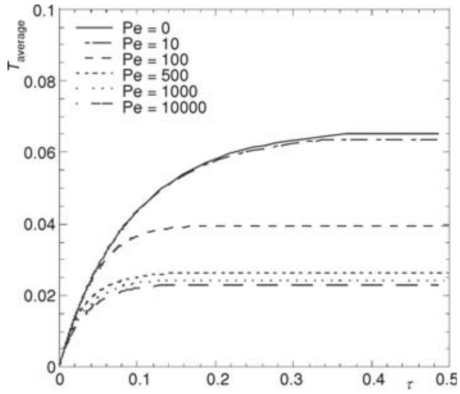

Figure 5. Average temperature as a function of non-dimensional time for Re = 100 and $\kappa = 1$

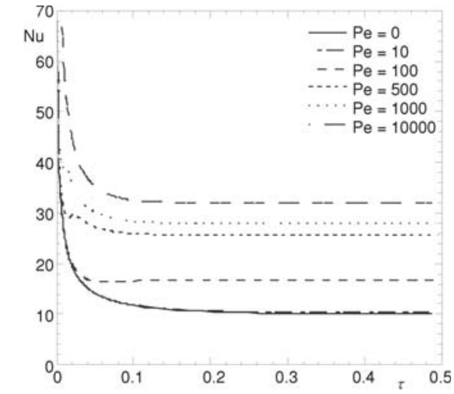

Figure 6. Average Nusselt number as a function of non-dimensional time for Re = 100 and $\kappa = 1$

Table 3 summarizes the values of average steady-state Nusselt numbers obtained from our calculations for the following range of parameters covered (Re = 10-100, $\kappa = 1$-100 and Pe = 0-10000). As expected, the average Nusselt number increases with increasing of Reynolds numbers for a fixed Peclet number and a given viscosity ratio κ. At fixed Reynolds

Table 3. Average Nusselt number at steady-state

	Pe							
Re	0	10	100	500	1000	2000	5000	10000
$\kappa = 1$								
10	10.06	10.14	13.76	22.00	24.62	26.23	27.61	28.37
50	10.06	10.23	15.69	24.62	27.07	28.67	30.20	31.05
100	10.06	10.29	16.68	25.69	28.04	29.59	31.14	31.99
$\kappa = 10$								
10	10.06	10.06	10.43	13.62	16.97	20.75	24.42	26.10
50	10.06	10.06	10.85	16.01	20.00	23.58	26.85	28.48
100	10.06	10.07	11.20	17.45	21.58	25.06	28.30	30.03
$\kappa = 100$								
10	10.06	10.06	10.16	10.24	10.48	11.20	13.96	17.46
50	10.06	10.06	10.16	10.39	10.98	12.47	16.55	20.56
100	10.06	10.06	10.17	10.53	11.40	13.38	18.09	22.16

number, the average Nusselt number decreases with the increasing of the viscosity ratio κ. It is observed that the average Nusselt number is increasing as Peclet number increases for a fixed Reynolds number and a fixed viscosity ratio κ.

Conclusions

A numerical investigation of the transfer from a moving fluid sphere with heat generation was carried out. It is assumed that the bulk of the thermal resistance resides in the fluid sphere. The Navier-Stokes equations are solved inside and outside a fluid sphere as well as the unsteady conduction-convection equation including heat generation inside the fluid sphere. The results are presented in the form of Nusselt numbers and average temperatures of the fluid sphere. For creeping flow, the comparison of our numerical results with those of other authors showed a very good agreement between them. To present the numerical results under a form easier to use we correlate our numerical results by two equations. The first formula gives the transient bulk temperature values which coincide with those obtained numerically for creeping flow with an error not exceeding 15%. A second formula gives the average Nusselt number values which coincide with those calculated numerically for creeping flow with an error not exceeding 10% for Pe_m in the range [0, 1000]. For intermediate Reynolds and Peclet numbers ($0 < Re < 100$ and $0 < Pe < 10000$), it is proved that the viscosity ratio between the dispersed and the continuous phase, influences significantly the heat transfer from the fluid sphere. During the numerical experiments were varied the Reynolds and the Peclet numbers and the viscosity ratio. The influence of these parameters on the heat transfer is elucidated.

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