A METHOD FOR CALCULATION OF FORCES ACTING ON AIR COOLED GAS TURBINE BLADES BASED ON THE AERODYNAMIC THEORY

by

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The paper presents the mathematical model and the procedure for calculation of the resultant force acting on the air cooled gas turbine blade(s) based on the aerodynamic theory and computation of the circulation around the blade profile. In the conducted analysis was examined the influence of the cooling air mass flow expressed through the cooling air flow parameter λ_c*, as well as, the values of the inlet and outlet angles* β_1 *and* β_2*, on the magnitude of the tangential and axial forces. The procedure and analysis were exemplified by the calculation of the tangential and axial forces magnitudes.*

Key words: *gas turbines, blade cooling, blade forces*

Introduction

Air cooled gas turbine blades have smaller full metal cross-section area, compared to non-cooled blades, due to cavities within the blade bodies where the specific cooling configurations are placed. On the other hand, the blade profile cross-section area is generally determined by the flow, as well as by the energy transformation requirements. In principle profile with a smaller cross-section area generate smaller energy losses and thus enables higher efficiency. At the same time, the smaller cross-section area puts larger strains and stresses on the blade body material. In the case of cooled blades, the smaller profile cross-section area leaves smaller space for the cooling configurations, or conditions an additional increase of stresses on the metal body, or even both. In such circumstances, accurate determination of the total stress on the blade body expressed as the sum of the mechanical and thermal stresses on the designing phase becomes extremely important. The accurate calculation of the forces acting on the blade is highly important for the determination of the mechanical stresses.

In principle there are two main types of cooling configurations in the gas turbine blades. The first one is convective cooling, where the cooling air passes through the blade body, and leaves it at the outlet profile edge fig. 1(a). The second one is film cooling where the cooling air is, after partly passing through the body of the blades, derived at the gas side of the blades through one or several arrays of wholes, forming the film of the cooling air that separates the blade body from hot gas stream, fig. 1(b). In the second case, the part of the gas quantity can also be derived through the outlet profile edge. The effusion cooling, where the holes are spread all over the blade body, is practically the special case of the film cooling. Other known cooling configurations are also used mainly as complementary to these two.

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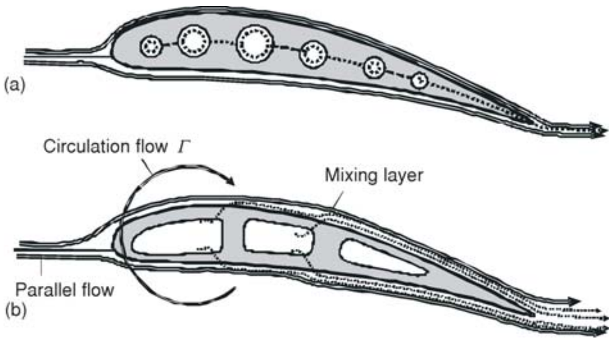

Figure 1. The main cooling configurations and the composition of the parallel and circulation flow

It is well known that more efficient cooling of the thermal of high loaded components enable higher turbine inlet temperatures and, thus, higher cycle efficiency at the same reliability level. On the other hand, cooling influences cascade aerodynamic features, generate additional losses in energy transformation process, and effect turbine capacity. Aerodynamics of gas turbine cascades, both linear, and annular, was investigated mainly due to develop the knowledge about pressure loading and pressure distribution around the blade, what is highly important for heat transfer calculations, as well as for future understanding of structural damping. The investigations were performed by computational simulations using different computational fluid dynamics (CFD) codes, as well as by experimental research using appropriate test facilities. A wide overview of the investigations is presented in [1]. Additional losses in a turbine stage and, therefore, the appropriate entropy creation are associated with film cooling. The entropy creation occurs in the coolant and mainstream mixing process, as is analytically described in [2]. The effect of film cooling on turbine capacity is also analyzed in [3].

The use of the aerodynamic theory for the analysis and determination of forces acting on non-cooled cascade, or blade is wide spread for a long time [4, 5]. The cooling air flow carries the disturbance into the gas flow around the blade and/or behind the blade, and, thus, influences the resultant force acting on the cascade. The application of aerodynamic theory for the analysis and determination of forces acting on air cooled cascade or air cooled blades, in the way how it is treated in this paper, is not found in the literature.

The procedure for calculation of the resultant force increase based on the canal turbine theory using the impulse equation is presented in [6]. On the other hand, the gas turbine blade profiles are usually shaped more like the airplane wing profile, than like the impulse bucket profile, leaving space for the application of the aerodynamic theory for the calculation of the forces acting on the blades. This paper presents the procedure for the calculation of the resultant force acting on the air cooled gas turbine blades based on the aerodynamic theory and the computation of the circulation around the blade profile.

Mathematical model and analysis

When the spacing to the blade chord length ratio is $t/L > 4$, the fluid density change through the cascade, as well as, the flow deviation are in principle small, and the aerodynamic theory can be applied [7]. The cascade that fulfills these conditions is sketched in fig. 2(a).

In the course of the procedure development two groups of assumptions have to be adopted. In the first one there are those conditioned by application of the well known aerodynamic theory on the non-cooled cascade [4], as follows. The blade cascade is the array of single blades. Theoretically, the mutual influence of the neighboring blades is neglected, practically the condition $t/L > 4$, as proposed in [7], is considered. The flow is plain (the influence of both profile ends is neglected). The flow is potential. The flow is subsonic and, therefore, the density change of the fluid passing through the cascade can be neglected [4, 6].

In the second group are the assumptions necessary to uniform physical model of air cooled gas turbine blades and the mathematical one. For the purpose of research of aerodynamic losses in a gas turbine stage with film cooling in [8] are adopted following assumptions. The cooling air is mixing with main gas stream at the constant pressure in the mixing layer. The densities in the mixing layer and in the undisturbed flow are the same and equal to the average density in the cross-section. As a consequence, the mixing process is considered as the isothermal one.

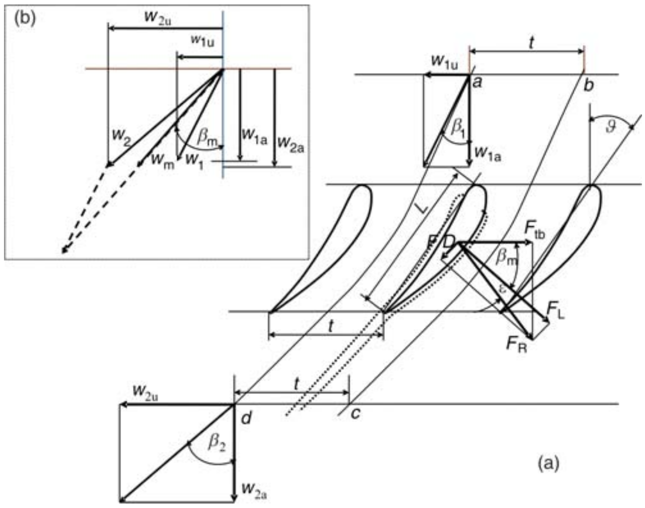

Figure 2. Schematic presentation of the aerodynamic forces acting on an air cooled blade profile

In addition to the above assumptions, for the purpose of the model development we consider the thickness of the mixing layer greater than the boundary layer and the velocities in the mixing layer and in the undisturbed flow as the same and equal to the average velocity in the cross-section.

The assumptions, mentioned above, are acceptable for the spacing to the blade chord length ratio $t/L > 4$. However, additional corrections should be introduced for the spacing to the blade chord length ratio $1 < t/L < 4$.

The flow over the blade can be considered as being composed of the parallel flow and of the circulation flow, as schematically presented in fig. 1(a). The circulation is defined as the contour integral of the velocity around the closed curve, and can be computed according to the equation $\Gamma = \oint (\vec{w}, d\vec{s})$.

The gas flow acts on the blade with the resultant aerodynamic force that can be decomposed on two orthogonal aerodynamic forces. The lift, and drag forces are:

$$F_L = \rho c_L \frac{w^2}{2} LH \quad \text{and} \quad F_D = \rho c_D \frac{w^2}{2} LH \tag{1}$$

The subscripts D and L refer to the drag and the lift, respectively. The lift is also determined by the equation:

$$F_L = \rho_\infty w_\infty H\Gamma \tag{2}$$

The blade length is denoted with H.

The fluid flow passing through the cascade changes its direction, since the angles β_1 and β_2* (of inlet and outlet velocities w_1 and w_2,) are different. Therefore, the velocity w_∞ should be determined adequately, to fulfill the condition that lift lays orthogonally on it. Based on the

* In the paper is accepted the designation for angles and profile parameters according to NASA designation, presented in [7]

literature, for example [4], we suppose that this condition will be fulfilled if we define w_∞ as the vector half sum of the velocities w_1 and w_2, according to the equation $\vec{w}_\infty = \vec{w}_m = 0.5(\vec{w}_1 + \vec{w}_2)$.

Above equation is graphically presented in fig. 2(b).

The important consequence of the blade air cooling is a certain quantity of air that is exhausted outside the blade and mixed with the main gas stream. The quantity of air depends on the needs for cooling caused by adopted gas temperatures, effectiveness of the applied cooling configuration, as well as, the cooled blade surface A_c. In the following analysis will be used the cooling air flow parameter λ_c defined [9] as the cooling air flow divided by the product of the hot gas flow and the cooled surface, according to the equation:

$$\lambda_c = \frac{\dot{M}_{ca}}{\dot{M}_{hs} A_c} \tag{3}$$

The aerodynamic forces: the lift and the drag will be correlated with the tangential and the axial forces, as well as, with velocity diagram, usually used in the canal theory.

From the continuity equation for the inlet and outlet cross-sections and from the mass balance equation for the outlet cross-section, we have:

$$\dot{M}_1 = A_1 \rho_1 w_{1a} \tag{4}$$

$$\dot{M}_2 = \dot{M}_1 (1 + \lambda_c A_c) = A_1 \rho_1 w_{1a} (1 + \lambda_c A_c) = A_2 \rho_2 w_{2a} \tag{5}$$

From the eqs. (4) and (5) follows $w_{2a} \approx w_{1a}(1 + \lambda_c A_c)$

The subscript c refers to the air cooling.

The following equation came out of the velocity diagram in fig. 2(b):

$$w_{2u} = w_{2a} \operatorname{tg} \beta_2 = w_{1a}(1 + \lambda_c A_c) \operatorname{tg} \beta_2 = w_{1u}(1 + \lambda_c A_c) \frac{\operatorname{tg} \beta_2}{\operatorname{tg} \beta_1} \tag{6}$$

Finally, we obtain the expression for the circulation by integration along the closed loop a-b-c-d, fig. 2(b), in the form:

$$\Gamma = t(w_{2u} - w_{1u}) = t w_{1u} \left[(1 + \lambda_c A_c) \frac{\operatorname{tg} \beta_2}{\operatorname{tg} \beta_1} - 1 \right] \tag{7}$$

The resultant force acting on the blade F_R equals the vector sum of the lift and drag forces F_L and F_D, respectively. Projecting the resultant force on the tangential direction leads, with small obvious simplifications, to the expression:

$$F_{tBc} = F_R \cos(\beta_m + \varepsilon) = F_L \cos \beta_m = \rho t H w_{1u} \left[(1 + \lambda_c A_c) \frac{\operatorname{tg} \beta_2}{\operatorname{tg} \beta_1} - 1 \right] w_m \cos \beta_m \tag{8}$$

The term $w_m \cos\beta_m$ is determined by the equation:

$$w_m \cos \beta_m = \frac{\dot{M}_1}{\rho t H z} \left(1 + \frac{\lambda_c A_c}{2} \right) \tag{9}$$

After obvious transformations the final expression for the total tangential force acting on all blades in the annular cascade with z blades is obtained in the form:

$$F_{tc} = z F_{tBc} = \dot{M}_1 w_1 \sin \beta_1 \left[1 + \frac{\lambda_c A_c}{2} \right] \left[(1 + \lambda_c A_c) \frac{\operatorname{tg} \beta_2}{\operatorname{tg} \beta_1} - 1 \right] \tag{10}$$

The inlet and outlet velocities w_1 and w_2 are considered to lie in the same quadrant, as schematically presented in fig. 2(a). In the case when these velocities lie in different quadrants, sign – should be replaced with sign +.

The total force acting on the annular cascade in axial direction is composed of the resultant aerodynamic force projected in axial direction and of the force produced by the cascade inlet to outlet pressure difference, as described by the equation:

$$F_{\mathrm{ac}} = zF_{\mathrm{aBc}} + p_1 A_{1\mathrm{f}}\left(1 - \frac{p_2}{p_1}\right) \tag{11}$$

We have denoted with $F_{\mathrm{aB}}^{\mathrm{c}}$ the projection in the axial direction of the resultant aerodynamic force acting on the single blade F_{R}, while p_1 and p_2^{c} are the static pressures at the inlet and at the outlet of the cascade. The cross-sections surface is denoted with $A_{1\mathrm{f}}$.

The projection on the axial direction of the resultant aerodynamic force acting on the single blade $F_{\mathrm{aB}}^{\mathrm{c}}$, analogous to the tangential force, is defined by the equation:

$$F_{\mathrm{aBc}} = \frac{M_1}{2z} w_1 \sin\beta_1 \operatorname{tg}\beta_1 \left\{\left[(1+\lambda_{\mathrm{c}}A_{\mathrm{c}})\frac{\operatorname{tg}\beta_2}{\operatorname{tg}\beta_1}\right]^2 - 1\right\} \tag{12}$$

The cascade outlet flow is greater in the case of air cooling than in the case of non-cooling and therefore the outlet to the inlet pressure ratio p_2^{c}/p_1 should be lower than the pressure ratio p_2/p_1 in the case of non-cooling.

For deriving the expression of the pressure ratio we will put the differential equation of the first law of thermodynamics for the unadiabatic (or diabatic) and irreversible flow in the form:

$$-\mathrm{d}h = -c_{\mathrm{p}}\mathrm{d}T = -v\mathrm{d}p + \mathrm{d}q_{\mathrm{ir}} - \mathrm{d}q_{\mathrm{c}} \tag{13}$$

– the elementary quantity of the heat, which is transferred in cooling process, can be considered, according to [5], as proportional to the elementary technical work $\mathrm{d}q_{\mathrm{c}} = \zeta_{\mathrm{c}}v\mathrm{d}p$,
– the elementary heat quantity produced by the process irreversibility can be expressed according to [5] as a part of the elementary technical work in the form $\mathrm{d}q_{\mathrm{ir}} = (1 - \eta_{\mathrm{p}})v\mathrm{d}p$.

The polytropic efficiency defined as: $\eta_{\mathrm{p}} = v\mathrm{d}p/\mathrm{d}h_{\mathrm{rev}}$ is denoted with η_{p} [5]. Upon substituting expressions for $\mathrm{d}q_c$ and $\mathrm{d}q_{\mathrm{ir}}$ into eq. (13) and after integration we obtain the relation between the pressure ratio and the temperature ratio in the form:

$$\frac{T_{2\mathrm{c}}}{T_1} = \left(\frac{p_{2\mathrm{c}}}{p_1}\right)^{(\eta_{\mathrm{p}}+\zeta_{\mathrm{c}})\frac{\kappa-1}{\kappa}} \tag{14}$$

Finally, from the everything mentioned, as well as, from the energy equation of the diabatic irreversible flow can be derived the expression for calculation of the required outlet to inlet pressure ratio in air cooled cascade in the form:

$$\frac{p_{2\mathrm{c}}}{p_2} = \left(1 - \left\{\frac{w_1^2}{2}\left[(1+\lambda_c A_c)^2\frac{\cos^2\beta_1}{\cos^2\beta_2} - 1\right] + q_c\right\}\frac{\kappa-1}{\kappa}\frac{1}{\mathrm{R}T_1}\right)^{\frac{\kappa}{\kappa-1}\frac{1}{\eta_p+\zeta_c}} \tag{15}$$

The exponent of the reversible expansion is denoted with κ.

Results and discussion

A set of calculations is performed on the basis of the explained mathematical model. The relative increases of the tangential and the axial forces are defined by the equations:

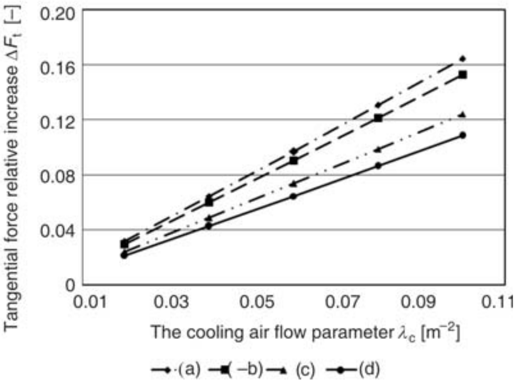

Figure 3. Relative increase of tangential force (a) β_1/β_2 = 10/40; (b) β_1/β_2 = 5/35; (c) β_1/β_2 = –5/25; (d) β_1/β_2 = –10/20

$$\Delta F_t = \frac{F_{tc} - F_t}{F_t} \quad \text{and} \quad \Delta F_a = \frac{F_{ac} - F_a}{F_a} \tag{16}$$

The relative increase of the tangential force as function of the cooling air flow parameter λ_c, with angles β_1 and β_2 as known parameters presented in fig. 3.

The relative increase of the tangential force within considered interval of the cooling air flow parameter λ_c, is not great, but it is important. Also, there is certain influence of the flow deviation in the cascade described by the angle difference $\beta_1 - \beta_2$. Additional calculations show that the change of the angle β_2, by constant value of the angle β_1, has no influence on the tangential force relative increase. We conclude that practically all the relative increase of the tangential force due to the angle difference $\beta_1 - \beta_2$ presented in fig. 3, can be attributed to the change of the angle β_1.

In fig. 4 is presented the relative increase of the axial force as function of the cooling air flow parameter λ_c, with angles β_1 and β_2 as known parameters. The relative increase of the axial force within considered interval of the cooling air flow parameter λ_c, is greater than the relative increase of the tangential force. However, the increase is mostly caused by the outlet pressure decrease. In principle there is possibility to increase the exit flow area by increasing the exit length of the blades. However, such change would cause the change in the cooled surface of the blades and due to that, additional increase of the cooling air flow expressed though the additional increase of the cooling air flow parameter λ_c.

Moreover, the influence of the flow deviation in the cascade described by the angle difference $\beta_1 - \beta_2$, on the relative increase of the axial force is stronger than the influence on the relative increase of the tangential force.

In the case of the axial force, there is significant influence of the angle β_2 value, by constant value of the angle β_1, on the relative force increase, as presented in fig. 5. However, this influence covers a part of the overall relative increase of the axial force. The other part of the increase can be attributed to the change of the angle β_1.

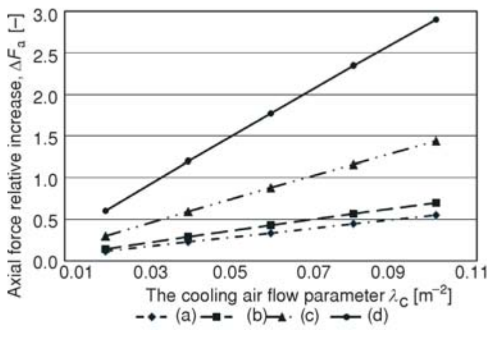

Figure 4. Relative increase of axial force (a) β_1/β_2 = 10/40; (b) β_1/β_2 = 5/35; (c) β_1/β_2 = –5/25; (d) β_1/β_2 = –10/20

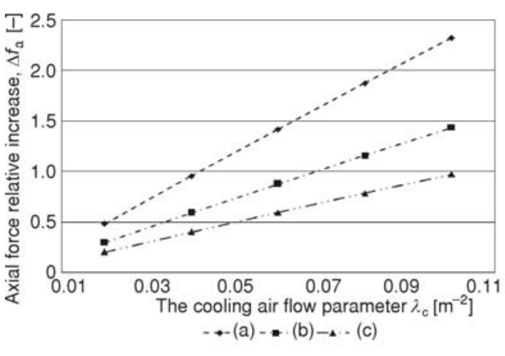

Figure 5. Relative increase of axial force (a) β_1/β_2 = –5/20; (b) β_1/β_2 = –5/25; (c) β_1/β_2 = –5/30

In the calculations, performed for the figs. 3 and 4, angles β_1 and β_2, as well as the value of cooling air flow parameter λ_c, varied, while all other parameters from eq. (12) and (15) are kept constant. The angles β_1/β_2 have values: 10/40, 5/35, –5/25, and –10/20 degrees (difference $\beta_2 - \beta_1$ is constant). In fig. 5 vary only the angle β_2 and λ_c, while all other parameters are kept constant. The angles β_1/β_2 have values: –5/20, –5/25 and –5/30 degrees. In NASA designation system the angles β_1 and β_2 between positive direction of the velocities vectors w_1 and w_2 and positive direction of the axial axis determine the direction of the vectors w_1 and w_2, in the co-ordinate system.

The mathematical model presented in the paper is developed for the case of NASA designation system. The numerical values of the forces, as well as, the relative increases of the forces are the same regardless to the selected designation system.

Conclusions

The paper presents the mathematical model based on the aerodynamic theory and the procedure for the calculation of the forces acting on the gas turbine cascade with air cooled blades. The procedure is strictly valid for the spacing to the blade chord length ratio $t/L > 4$. Additional corrections should be introduced for the spacing to the blade chord length ratio $1 < t/L < 4$. The results obtained from the conducted analysis confirm the assumption that the resultant force acting on the air cooled gas turbine cascade is greater than those acting on non-cooled one, under the same stage flow conditions. The increase of the force is caused by the cooling air flow described by the cooling air flow parameter λ_c.

The resultant force is decomposed into the tangential and axial component due to easier comparison with results based on the canal theory and application of the impulse equation, presented in the literature. The force caused by the cascade inlet to outlet pressure difference is a part of the axial component.

The specific conclusions regarding the magnitude of the forces can be summarized as follows.

- The magnitude of the axial component is stronger affected by the cooling air flow (expressed through the cooling air flow parameter λ_c), than magnitude of the tangential one.
- The magnitude of the relative increase of the tangential force is, to the certain extent, influenced by the angle difference $\beta_1 - \beta_2$, but practically not by the angle β_2 alone.
- The magnitude of the relative increase of the axial force is influenced by the angle difference $\beta_1 - \beta_2$ to the greater extent than the magnitude of the tangential one. The influence of the angle β_2 significantly participates in this increase.
- The magnitudes of both the tangential and the axial forces are influenced by the value of the angle β_1.

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Nomenclature

A – area, [m^2]
A_c – cooled area, [m^2]
A_f – equivalent cascade cross-section area, [m^2]
c_D – drag coefficient
c_L – lift coefficient
F – force, [N]
H – blade length, [m]
L – chord length, [m]

M – mass flow, [kgs^{-1}]
p – pressure, [Nm^{-2}]
q – heat per mass unit, [Jkg^{-1}]
T – absolute temperature, [K]
t – blade spacing, [m]
v – specific volume, [m^{3}kg^{-1}]
w – velocity, [ms^{-1}]
z – number of blades

Greek symbols

β – angle between positive direction of the velocity vector $\vec{w}$ and positive direction of the axial axis (indicated on fig. 2)
Γ – circulation
λ_χ – cooling air flow parameter
ρ – density, [kgm^{-3}]

Subscripts

a – axial direction
B – refers to blade
c – refers to cooling
ca – cooling air
D – draft
hs – hot gases
L – lift
u – refers to tangential direction
1 – inlet into the cascade
2 – outlet out of the cascade
∞ – refers to indefinite

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