THE CHARACTERISTIC LENGTH ON NATURAL CONVECTION FROM A HORIZONTAL HEATED PLATE FACING DOWNWARDS

by

Bulent KOZANOGLU [*] ***and Francisco RUBIO***

Mechanical Engineering Department, University of the Americas-Puebla, Puebla, Mexico

Original scientific paper
DOI: 10.2298/TSCI110127087K

Natural convection from a downward facing horizontal heated plate was analyzed. An expression for the thickness of the thermal boundary layer was obtained in terms of Rayleigh number. Assuming this thickness as the characteristic length of the problem, the data published by other authors were modified and an equation for Nusselt number is presented. It was observed that this equation correlates the data more precisely than the commonly known equations in the literature that employ the ratio of the area to the perimeter or the shorter side of the plate as the characteristic length. It is concluded that taking the thermal boundary layer as the characteristic length of phenomenon is a proper approach and correlates all the data closely.

Key words: *natural convection, thermal boundary layer, characteristic length, horizontal plate, downward facing surface*

Introduction

Natural convection heat transfer adjacent to horizontal plates has been of considerable interest to scientists and engineers because of its application to cooling of electronic equipment, space heating, power production, solidification process, solar collectors, energy storage devices, as well as natural phenomena. The physical phenomenon encountered in natural convection along a horizontal heated plate facing downwards is the same as the one observed along a horizontal cooled plate facing upwards. In this type of convection, the fluid layer just underneath the heated plate moves towards the edges of the surface under the influence of a horizontal pressure gradient, while the movement is towards the center in case of heated plate facing upwards.

Saunders *et al.* [1], Weise [2], and Fishenden and Saunders [3] carried out the pioneer experimental studies on this phenomenon. Singh and Birkebak [4] presented an analysis by assuming a finite thickness of the boundary layer at the edges of the plate and observed that the boundary layer thickness as well as the Nusselt number depends on Rayleigh number. In their experimental study Aihara *et al.* [5] observed that the velocity field near the downward facing horizontal heated plate is rather unstable, however, in the boundary layer, particularly in the vicinity of the free edges, a relatively stable flow is established. They also concluded that the similarity solutions cannot be obtained for this problem, since the boundary layer does not satisfy the fundamental requirement of similarity that the boundary layer should grow towards the downstream. However, Fujii *et al.* [6] studied the same problem theoretically by an approximate integral treatment, for a case of uniform heat flux, and reported that local Nusselt number is proportional to one-sixth power of modified Grashof number, whereas average Nusselt number

*Corresponding author; e-mail: bulentu.kozanoglu@udlap.mx

is proportional to one-fifth power of Grashof number. Goldstein *et al.* [7] carried out experiments using naphthalene sublimation technique. These authors took the attention to the fact that the most of the available experimental data lie well above the predictions of the boundary layer theory, while experimentally and theoretically determined transfer coefficients display different dependences on the Rayleigh number. Edge and aspect ratio effects on natural convection from isothermally heated horizontal plates facing downwards were studied by Hatfield and Edwards [8]. These authors proposed a correlation taking into consideration edge effects and adiabatic extensions. Using interferometric techniques, Gryzagoridis [9] analyzed natural convection from an isothermal surface and observed the growth of the boundary layer and enhancement of the local heat transfer coefficient with increasing temperature difference between the plate and the ambient. Schulenberg [10, 11] analytically studied natural convection heat transfer below downward facing horizontal surfaces applying the method of matched asymptotic expansions. This researcher obtained local similarity solutions for low Prandtl numbers [10], application to liquid metals, and high Prandtl numbers [11]. The local Nusselt numbers predicted through this analytical procedure were found to be closer to the experimental values than those previously evaluated by integral methods. Onur and Aktas [12] experimentally observed that plate spacing has a strong influence over the natural heat convection between two parallel plates where the hot plate is isothermal and facing downwards. Kwak and Song [13] experimentally and numerically studied the variation of the Nusselt number around rectangular grooved fins attached to downward facing horizontal plates in laminar natural convection. Dubovsky *et al.* [14] compared their experimental results with numerical simulations in a ventilation system heated by a downward facing, constant temperature plate. A two dimensional numerical simulation for laminar natural convection flow below a downward facing heated surface was presented by Friedrich and Angirasa [15]. These researchers observed that boundary layer assumptions cannot be applied at the edge of this geometry. Radziemska and Lewandowski [16] observed a great stability of the flow below a downward facing horizontal round plate. Their experimental flow visualizations and numerical calculations demonstrated that the natural convection boundary layer has its maximum and minimum thicknesses at the center and at the edges of the plate, respectively. Dayan *et al.* [17] conducted a combined theoretical, experimental and numerical study to analyze natural convection below a horizontal rectangular hot fin array and determined the minimum fin height that provides the necessary heat transfer rate. Later on, Mittelman *et al.* [18] extended this study to inclined hot fin arrays and concluded that the heat transfer rate is substantially enhanced beyond a certain angle of inclination.

The studies cited above do not have a common agreement over the definition of the characteristic length of the phenomenon. Some of them consider the shorter side of the plate as the characteristic length [8, 19], while the others take it as one-half of the longer side [1, 9, 20] or one-half of the shorter side [5]. In this type of convection, the most commonly accepted characteristic length is the one suggested by Goldstein *et al.* [7]. These researchers proposed a characteristic length of $L = A/P$, where A and$P$ are the area and the perimeter of the surface, respectively. However, limitations with the range of their experimental data did not allow them to present a correlation. Later on, applying an electrochemical technique, Lloyd and Moran [21] studied natural convection mass transfer adjacent to horizontal surfaces of circular, square, rectangular as well as right triangular planforms, and, employing the same characteristic length suggested by Goldstein *et al.* [7], proposed an equation for Nusselt number with a 1/4 power dependence on Rayleigh number.

In a recent study, taking the thickness of the thermal boundary layer as the characteristic length of the problem, Kozanoglu and Lopez [22] studied natural convection heat transfer above horizontal surfaces and concluded that the concept of the thermal boundary layer thickness as the characteristic length might provide the opportunity to express data points of different regimes at one single equation.

In the present study, the natural convection from a horizontal heated plate facing downwards has been studied, compiling the experimental data available in the technical literature and reconsidering them employing the thickness of the thermal boundary layer as the characteristic length.

Analysis, results, and discussion

A very extensive review of the published work on natural convection heat transfer below a horizontal heated plate facing downwards was carried out. The experimental data presented by different authors were compiled [1, 5, 8, 9, 16, 19, 20]. A list of geometries, experimental conditions as well as the characteristic lengths employed by these authors is presented in tab. 1. It should be noted that this phenomenon has been studied by many other researchers and a good number of experimental works are available in the technical literature. However, the above listed publications are the only ones including all the experimental details and geometries required for a reconsideration and modification of the data.

Figure 1 shows the original data by these authors, altogether 154 data points, considering the characteristic length suggested by each one of them. As seen, this group of data is little bit scattered and can be correlated to provide the expression:

$$\mathrm{Nu} = 0.312\ \mathrm{Ra}^{0.252} \quad (1)$$

with a correlation coefficient of only 0.896, in the range of Rayleigh numbers from $1.5 \cdot 10^2$ to $9.0 \cdot 10^{10}$.

Fuji and Imura [19] proposed a relation with one-fifth power of Rayleigh number:

$$\mathrm{Nu} = 0.58\ \mathrm{Ra}^{0.2} \quad (2)$$

This power of Rayleigh number was also obtained in theoretical studies based on the boundary layer approximation. Wagner [23] and Singh *et al.* [24] solving integral equations, and Clifton and Chapman [25] introducing the minimum mechanical energy principle expressed Nusselt number through this type of relation. In eq. (2) the characteristic length was taken as the shorter side of the plate. The data points presented in fig. 1 were modified according to the characteristic length defined by Fuji and Imura [19], and plotted in fig. 2 together with eq. (2). This equation correlates these data points with a very low coefficient of 0.788.

Figure 3 displays these same data points compiled earlier, now modified according to

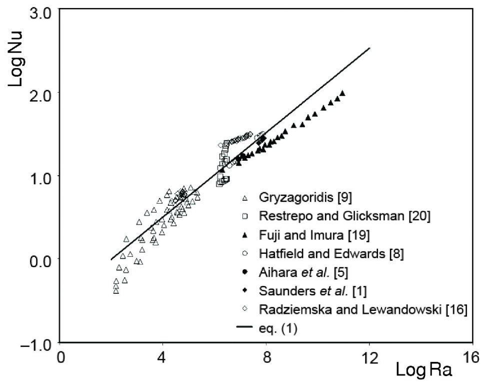

Figure 1. Average Nusselt number *vs.* Rayleigh number, considering the originalcharacteristic lengths proposed by each one of the authors

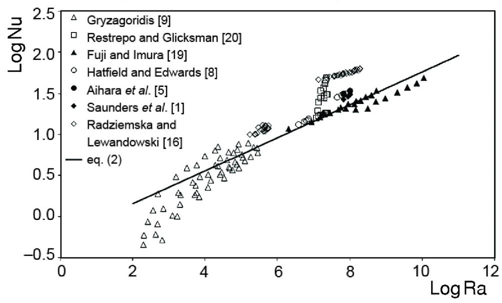

Figure 2. Average Nusselt number *vs.* Rayleigh number, considering the characteristic length proposed by Fuji and Imura [19]

Table 1. List of geometries, experimental conditions, and characteristic lengths by various authors considered in this work

Author	Characteristic length, *L*	Medium	Temperature[K]	Geometry
Saunders *et al.* [1]	Total side length	Air	–	Rectangular plates
Aihara *et al.* [5]	Half of the plate width	Air	321-343	Rectangular plates
Hatfield and Edwards [8]	Short side of the plate	Air	–	Rectangular and square plates
Gryzagoridis [9]	Half of the length of the plate	Air	294-306	Rectangular plates
Radziemska and Lewandowski [16]	Radius of the plate	Waterand air	298-308	Circular plates
Fujii and Imura [19]	Short side of the plate	Water	313	Rectangular plates
Restrepo and Glicksman [20]	Half of the length of the plate	Air	295-361	Square plates

the characteristic length suggested by Goldstein *et al.* [7]. This characteristic length, as mentioned before, is defined as the ratio of the surface area to the perimeter of it and receives a widespread acceptance in the field. Figure 3 also presents eq. (3) that is recommended by Lloyd and Moran [21]:

$$\mathrm{Nu} = 0.27\ \mathrm{Ra}^{0.25} \tag{3}$$

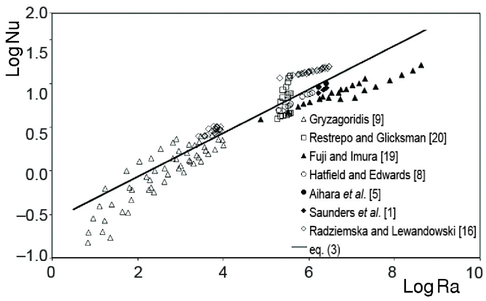

Figure 3. Average Nusselt number *vs.* Rayleigh number, considering the characteristic length suggested by Goldstein *et al.* [7]

employing the characteristic length proposed by Goldstein *et al.* [7]. In this case, a relatively low coefficient of correlation of 0.866 is obtained between eq. (3) and the data points presented in fig. 3. It should be noted that Goldstein *et al.* [7] as well as Lloyd and Moran [21] carried out mass transfer experiments on natural convection adjacent to horizontal surfaces, and reported that this phenomenon is analogous to the natural convection heat transfer phenomenon adjacent to a horizontal plate with an uniform surface temperature. Based on this analogy, both papers interpreted their Sherwood number results as results on Nusselt number.

Finally, the characteristic length proposed by Kozanoglu and Lopez [22], the thickness of the thermal boundary layer, was applied to the same group of data. The first step of this analysis is obtaining an expression for the thickness of the thermal boundary layer. The experimental data presented in fig. 4 [5, 20], were correlated, with a coefficient of 0.851, to result the equation:

$$\delta = 0.00082\mathrm{Ra}_{\delta}^{0.31} \tag{4}$$

The dependency of the thickness of the thermal boundary layer upon Rayleigh number has been reported by various researchers [4, 5, 20, 22, 25]. Restrepo and Glicksman [20] informed that the thermal boundary layer thickness decreases with increasing Rayleigh number. These researchers defined the Rayleigh number taking the characteristic length as one-half of the plate length. Modifying their original data according to the characteristic length concept introduced in this study, this relationship is inversed and the thickness of the thermal boundary layer increases with the increasing boundary layer Rayleigh number, Ra_δ, as seen in fig. 4. It should be noticed that the Rayleigh numbers in fig. 4 and eq. (4) are Ra_δ, based on δ, the thermal boundary layer thickness.

Then, employing eq. (4), the group of data compiled and analyzed earlier was modified on the basis of the characteristic length proposed in this study. This new set of data is correlated with a very high coefficient of 0.994 to give eq. (5):

$$Nu_\delta = 0.116 Ra_\delta^{0.32} \tag{5}$$

for a very wide range of Rayleigh numbers, as observed in fig. 5. As seen in this figure, eq. (5) represents almost all the data precisely. Furthermore, this method eliminates the need to develop distinct expressions in laminar and turbulent regions, since the boundary layer thickness already contains the information over the regimes.

Thus, it can be concluded that eq. (5) provides a better representation of the experimental data than the commonly known eqs. (2) and (3), byFuji and Imura [19] and Lloyd and Moran [21], respectively. Consequently, the thickness of the thermal boundary layer seems to be physically a more relevant characteristic length than the ones proposed in the technical literature.

Figure 6 presents a comparison of eqs. (1), (2), (3), and (5). As observed in this figure, all the equations provide values of the Nusselt number in the same order of magnitude and the slopes of the lines are very similar to each other. It is common in the technical literature to assume relations with 1/4 and 1/3 power dependence of Rayleigh number, in the laminar and

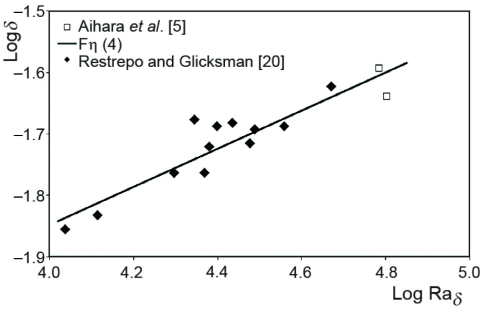

Figure 4. Thickness of the thermal boundary layer as a function of Rayleigh number

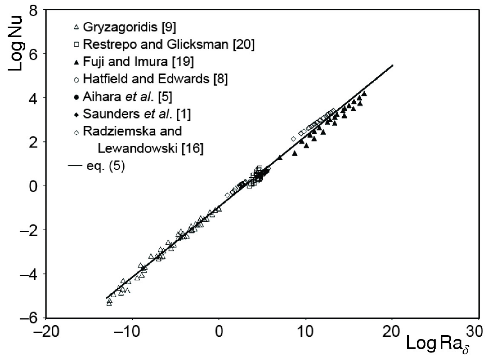

Figure 5. Average Nusselt number *vs.* Rayleigh number, employing the thickness of the thermal boundary layer as the characteristic length

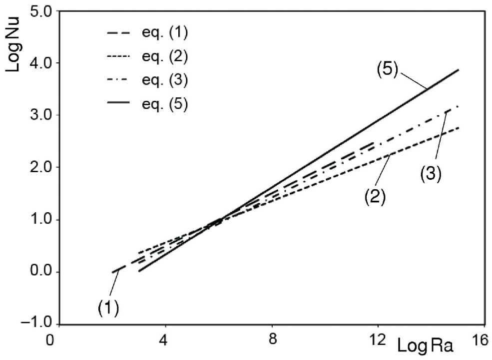

Figure 6. Comparison of eqs. (1), (2), (3), and (5)

turbulent regimes, respectively. Some researchers, such as Lloyd and Moran [21], forced their experimental data to obtain equations with these powers. In this work, such a procedurewasnotapplied since through a power dependence of 1/3 the influence of the characteristic length is eliminated.

Conclusions

Employing the data published by Aihara *et al.* [5] and Restrepo and Glicksman [20], eq. (4) was obtained to express the thermal boundary layer thickness as a function of Rayleigh number. Utilizing eq. (4), 154 data points, presented by various authors for different shapes of horizontal plates and distinct experimental conditions, were modified according to the characteristic length proposed in this work. This characteristic length seems to bring all the experimental data into a common correlation with a very high coefficient, as seen in fig. 5. This correlation coefficient was much higher than the coefficients obtained in case of applying the characteristic lengths proposed by Fuji and Imura [19] and Lloyd and Moran [21] to the same group of data.

Thus, it is concluded that treating the thickness of the thermal boundary layer as the characteristic length of the problem is a physically sounding proposal and yields better results than the commonly recognized definitions of characteristic lengths in the technical literature for this phenomenon. This conclusion also supports the results presented by Kozanoglu and Lopez [22] who applied the same concept of characteristic length to the problem of natural convection over a horizontal heated plate, but facing upwards. This approach also has the advantage of expressing data points of different regimes through one single equation.

Nomenclature

A – area of the surface, [m^2]
g – gravitational acceleration, [ms^{-2}]
h – convection coefficient, [$W\,m^{-2}K^{-1}$]
k – thermal conductivity, [$Wm^{-1}K^{-1}$]
L – characteristic length, [m]
P – perimeter of the plate, [m]
T – temperature, [K]

Greek symbols

α – thermal diffusivity, [m^2s^{-1}]
β – thermal expansion coefficient, [K^{-1}]
δ – thickness of the thermal boundary layer, [m]
ν – kinematic viscosity, [m^2s^{-1}]

Non-dimensional groups

Nu – Nusselt number (= hL/k), [–]
Nu_δ – Nusselt number based on δ(= $h\delta/k$), [–]
Ra – Rayleigh number (= $g\beta\Delta T L^3/\nu\alpha$), [–]
Ra_δ – Rayleigh number based on δ(= $g\beta\Delta T\delta^3/\nu\alpha$), [–]

References

[1] Saunders, O. A., *et al.*, Some Measurements of Convection by an Optical Method, *Engineering, 139* (1935), pp. 483-485
[2] Weise, R., Heat Transfer by Free Convectionon Square Plates (in German), *Forsch. Geb. Ing.-Wes., 6* (1935), 6, pp. 281-292
[3] Fishenden, M., Saunders, O. A., *An Introduction to Heat Transfer*, Oxford University Press, London, 1961
[4] Singh, S. N., Birkebak, R. C., Laminar Free Convection from a Horizontal Infinite Strip Facing Downwards, *Z. Angew. Math. Phys.*, *20* (1969), 4, pp. 454-461
[5] Aihara, T., *et al*., Free Convection along the Downward-Facing Surface of a Heated Horizontal Plate, *Int. J. Heat Mass Tran.*, *15* (1972), 12, pp. 2535-2549
[6] Fujii, T., *et al*., A Theoretical Study of Natural Convection Heat Transfer from Downward-Facing Horizontal Surfaces with Uniform Heat Flux, *Int. J. Heat Mass Tran.*, *16* (1973), 3, pp. 611-627
[7] Goldstein, R. J., *et al*., Natural Convection Mass Transfer Adjacent to Horizontal Plates, *Int. J. Heat Mass Tran.*, *16* (1973), 5, pp. 1025-1035

[8] Hatfield, D. W., Edwards, D. K., Edge and Aspect Ratio Effects on Natural Convection from the Horizontal Heated Plate Facing Downwards, *Int. J. Heat Mass Tran., 24* (1981), 6, pp. 1019-1024
[9] Gryzagoridis, J., Natural Convection from an Isothermal Downward Facing Horizontal Plate, *Int. Comm. Heat Mass Transfer*, *11* (1984), 2, pp. 183-190
[10] Schulenberg, T., Natural Convection Heat Transfer to Liquid Metals below Downward Facing Horizontal Surfaces, *Int. J. Heat Mass Tran.*, *27* (1984), 3, pp. 433-441
[11] Schulenberg, T., Natural Convection Heat Transfer below Downward Facing Horizontal Surfaces, *Int. J. Heat Mass Tran.*, *28* (1985), 2, pp. 467-477
[12] Onur, N., Aktas, M. K., An Experimental Study on the Effect of Opposing Wall on Natural Convection along an Inclined Hot Plate Facing Downward, *Int. Comm. Heat Mass Transfer*, *25* (1998), 3, pp. 389-397
[13] Kwak, C. E., Song, T. H., Natural Convection around Horizontal Downward-Facing Plate with Rectangular Grooves: Experiments and Numerical Simulations, *Int. J. Heat Mass Tran.*, *43* (2000), 5, pp. 825-838
[14] Dubovsky, V., *et al.*, Natural Convection Inside Ventilated Enclosure Heated by Downward-Facing Plate: Experiments and Numerical Simulations, *Int. J. Heat Mass Tran., 44* (2001), 16, pp. 3155-3168
[15] Friedrich, M. K., Angirasa, D., The Interaction between Stable Thermal Stratification and Convection under a Heated Horizontal Surface Facing Downward, *Int. J. Non-linear Mech.*, *36* (2001), 5, pp. 719-729
[16] Radziemska, E., Lewandowski, W. M., Heat Transfer by Natural Convection from an Isothermal Downward Facing Round Plate in Unlimited Space, *Applied Energy*, *68* (2001), 4, pp. 347-366
[17] Dayan, A., *et al.*, Laminar Free Convection Underneath a Downward Facing Hot Fin Array, *Int. J. Heat Mass Tran.*, *47* (2004), 12-13, pp. 2849-2860
[18] Mittelman, G., *et al.*, Laminar Free Convection Underneath a Downward Facing Inclined Hot Fin Array, *Int. J. Heat Mass Tran.*, *50* (2007), 13-14, pp. 2582-2589
[19] Fujii, T., Imura, H., Natural Convection Heat Transfer from a Plate with Arbitrary Inclination, *Int. J. Heat Mass Tran.*, *15* (1972), 4, pp. 755-767
[20] Restrepo, F., Glicksman, L. R., The Effect of Edge Conditions on Natural Convection from a Horizontal Plate, *Int. J. Heat Mass Tran.*, *17* (1974), 1, pp. 135-142
[21] Lloyd, J. R., Moran, W. R., Natural Convection Adjacent to Horizontal Surfaces of Various Planforms, *J. Heat Tran.*, *96* (1974), 4, pp. 443-451
[22] Kozanoglu, B., Lopez, J., Thermal Boundary Layer and the Characteristic Length on Natural Convection over a Horizontal Plate, *Heat Mass Tran.*, *43* (2007), 4, pp. 333-339
[23] Wagner, C., Discussion on Integral Methods in Natural Convection Flow, *Trans. Am. Soc. Mech. Engrs., 23E* (1956), pp.320-321
[24] Singh, S. N., *et al*., Laminar Free Convection Heat Transfer from Downward-Facing Horizontal Surfaces of Finite Dimensions, *Prog. Heat Mass Transfer, 2* (1969), pp. 87-98
[25] Clifton, J. V., Chapman, A. J., Natural-Convection on a Finite-Size Horizontal Plate, *Int. J. Heat Mass Transfer*, *12* (1969), 12, pp. 1573-1584

Paper submitted: January 27, 2011
Paper revised: June 15, 2011
Paper accepted: June 15, 2011