STEADY THERMAL STRESS AND STRAIN RATES IN A CIRCULAR CYLINDER WITH NON- HOMOGENEOUS COMPRESSIBILITY SUBJECTED TO THERMAL LOAD

by

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Original scientific paper
DOI: 10.2298/TSCI110315080P

The non-homogeneity is assumed due to variation of modulus of compression. It has seen that in the presence of temperature, a cylinder made of non-homogeneous material k<0 (Non-homogeneity is less at internal surface than at outer surface) require high pressure to become fully plastic as is required for initial yielding and this pressure goes on increasing with the increases in temperature, showing that a cylinder made of non-homogeneous material k<0 is on the safer side of design. For homogeneous case, it has been observed that the circumferential stress has maximum value at the external surface of the cylinder made of incompressible material as compared to compressible material. For Homogeneous case, with effects of temperature reduces the stresses at the external surface of the cylinder in comparison to pressure effects only. Strain rates are found to be maximum at the internal surface of the cylinder made of compressible material and they decrease with the radius. With the introduction of temperature effect, the creep rates have higher values at the internal surface but lesser values at the external surface as compare to a cylinder subjected to pressure only.

Key words: *elastic-plastic, creep, transition, temperature, non-homogeneous, stresses, strain.*

Introduction

A thick walled circular cylinder is widely used either as pressure vessels intended for storage in industrial gases or a media transportation of high pressurized fluids. The constantly increasing industrial demand for cylindrical and spherical components has concentrated the attention of designers and scientists on this particular area of activity. The progressive, world-wide scarcity of materials, combined with their increasing cost, makes design to the elastic regime only obsolete. Thick walled cylinders of circular cross section are used commonly either as pressure vessels intended for storage in industrial gases or as media for transportation of high pressurized fluids. Thick walled cylinders under internal pressure have been analyzed by many authors [1-4] for isotropic homogeneous elastic-plastic states. Some degree of non-homogeneity is present in wide class of materials such as hot rolled copper, aluminum and magnesium alloys. Olszak and Urbanowaski [5] solved the problem of thick walled non-homogeneous cylinder subjected to internal and external pressures and showed that plastic flow may start from either surface depending on the character and intensity of the non homogeneity. However, they assumed the material to be elastically incompressible. Ghosh [6] worked on the problem involving the study of elastic- plastic stresses in a spherical

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pressure vessel of non-homogeneous material. Mukhopadhyay [7] studied the effect of non-homogeneity on yield stress in a thick walled cylindrical tube under pressure by allowing the rigidity modulus μ to obey some cosine law of its radial distance and obtained the critical pressure for yielding in terms of Bessel's function. Creep of thick-walled cylinder under internal pressure has been discussed by many authors [10-14]. Rimrott [12] analysed the above problem by considering large strain. These authors made the following assumptions:

I. The volume of the material is constant, $\dot{\varepsilon}_r + \dot{\varepsilon}_\theta + \dot{\varepsilon}_z = 0$.
II. The ratios of the principal shear strain rates to the principal shear stresses are equal, *i.e.* $\dfrac{\dot{\varepsilon}_{\theta\theta} - \dot{\varepsilon}_{rr}}{\sigma_{\theta\theta} - \sigma_{rr}} = \dfrac{\dot{\varepsilon}_{rr} - \dot{\varepsilon}_{zz}}{\sigma_{rr} - \sigma_{zz}} = \dfrac{\dot{\varepsilon}_{zz} - \dot{\varepsilon}_{\theta\theta}}{\sigma_{zz} - \sigma_{\theta\theta}}$.
III. The axial strain rate is zero, *i.e.*, $\dot{\varepsilon}_z = 0$.
IV. There is a significant stress-versus-true strain rate relationship which coincides with the true stress-versus-creep rate relationship in simple tension, e.g. Norton's Law.
V. The creep deformation is infinitesimally small.

Seth's transition theory [8] does not require any ad hoc assumptions like yield and incompressibility condition, and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. It utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deforming field and has been successfully applied to a large number of the problems in plasticity. Seth has defined the generalized principal strain measure as:

$$e_{ii} = \int_0^{\overset{A}{e}_{ii}} \left[1 - 2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ii} = \frac{1}{n}\left[1 - \left(1 - 2\overset{A}{e}_{ii}\right)^{\frac{n}{2}}\right], (i = 1,2,3) \tag{1.1}$$

where '*n*' is the measure and e_{ii} is the principal finite strain components. Taking the non-homogeneity as the compressibility of material in the cylinder as:

$$c = c_0 \left(r / b\right)^{-k} \tag{1.2}$$

where $a \le r \le b$, c_0 and *k* are real positive constants.

Governing Equations

We consider a thick-walled circular cylinder of internal radius *a* and external radius *b* respectively subjected to internal pressure *p* and steady state temperature Θ on the inner surface. The displacement components in cylindrical polar co-ordinate are given by [9]:

$$u = r(1-\beta);\ v = 0\ \ w = \mathrm{d}z \tag{2.1}$$

where β is function of $r = \sqrt{x^2 + y^2}$ only and *d* is a constant. The strain components for finite deformation are given by [9]:

$$\overset{A}{e}_{rr} = \frac{1}{2}\left[1 - \left(r\beta' + \beta\right)^2\right], \quad \overset{A}{e}_{\theta\theta} = \frac{1}{2}\left[1 - \beta^2\right], \quad \overset{A}{e}_{zz} = \frac{1}{2}\left[1 - (1-d)^2\right], \ \overset{A}{e}_{r\theta} = \overset{A}{e}_{\theta z} = \overset{A}{e}_{zr} = 0 \tag{2.2}$$

where $\beta' = d\beta / dr$ and meaning of superscripts "*A*" is Almansi. Substituting eqs. (2.2) in eq. (1.1), the generalized components of strain become:

$$e_{rr} = \frac{1}{n}\left[1 - \left(r\beta' + \beta\right)^n\right], e_{\theta\theta} = \frac{1}{n}\left[1 - \beta^n\right], e_{zz} = \frac{1}{n}\left[1 - (1-d)^n\right], e_{r\theta} = e_{\theta z} = e_{zr} = 0 \tag{2.3}$$

The stress –strain relations for thermo elastic isotropic material are given by [1]:

$$T_{ij} = \lambda \delta_{ij} I_1 + 2\mu e_{ij} - \xi \Theta \delta_{ij}, \left(i, j = 1,2,3\right) \tag{2.4}$$

where T_{ij} are the stress components, λ and μ are Lame's constants, $I_1 = e_{kk}$ is the first strain invariant, δ_{ij} is the Kronecker's delta, $\xi = \alpha(3\lambda + 2\mu)$, α being the coefficient of thermal expansion, and Θ is the temperature. Further, Θ has to satisfy:

$$\nabla^2 \Theta = 0 \tag{2.5}$$

Substituting the strain components from eq. (2.3) in eq. (2.4), the stresses are obtained as:

$$T_{rr} = \lambda I_1 + \frac{2\mu}{n}\left[1 - \left(r\beta' + \beta\right)^n\right] - \xi\Theta, T_{\theta\theta} = \lambda I_1 + \frac{2\mu}{n}\left[1 - \beta^n\right] - \xi\Theta,$$

$$T_{zz} = \lambda I_1 + \frac{2\mu}{n}\left[1 - \left(1 - d\right)^n\right] - \xi\Theta, T_{r\theta} = T_{\theta z} = T_{zr} = 0 \tag{2.6}$$

where $I_1 = \frac{1}{n}\left[3 - \left(r\beta' + \beta\right)^n - \beta^n - \left(1 - d\right)^n\right]$ and $\beta' = d\beta / dr$.

The temperature field satisfying equation (2.5) and $\Theta = \Theta_0$ at $r = a$ and $\Theta = 0$ at $r = b$, where Θ_0 is constant, is given by:

$$\Theta = \frac{\Theta_0 \log\left(r / b\right)}{\log\left(a / b\right)} \tag{2.7}$$

The equations of equilibrium are all satisfied except:

$$\frac{dT_{rr}}{dr} + \frac{T_{rr} - T_{\theta\theta}}{r} = 0. \tag{2.8}$$

Using equations (2.6) in eq. (2.8), one get a non -linear differential equation in β as:

$$nP\left(P - 1\right)^{n-1}\beta\frac{dP}{d\beta} + nP\left(P + 1\right)^n + \left(1 - c\right)nP - \left[1 - \left(P + 1\right)^n\right]c + \frac{nc\xi\overline{\Theta_0}}{2\mu\beta^n} = 0, \tag{2.9}$$

Using equation (1.2) in eqn. (2.9), one gets:

$$n\beta\left(P + 1\right)^{n-1}\frac{dP}{d\beta} = \left[\begin{array}{l} r\left(\frac{\mu'}{\mu} - \frac{c'}{c}\right)\left[\begin{array}{l}\left\{\left(3 - 2c\right) - \left(1 - c\right)\left(1 - d\right)^n\right\}\frac{1}{\beta^n} - \\ -\left(1 - c\right) - \left(P + 1\right)^n\end{array}\right] \\ +c.\left[1 - \left(P + 1\right)^n\right] + r.\frac{c}{\left[1 - \left\{2 - \left(1 - d\right)^n\right\}\frac{1}{\beta^n}\right]} \\ -nP\left[\left(1 - c\right) + \left(P + 1\right)^n - \frac{\overline{\Theta}_0 cn}{2\mu\beta^n}\left[\xi + r\xi'\log\frac{r}{b}\right]\right]\end{array}\right] \tag{2.10}$$

where c is the compressibility factor of the material in term of Lame's constant, and are given by $c = 2\mu / \lambda + 2\mu$, $r\beta' = \beta P$ and $\overline{\Theta_0} = \frac{\Theta_0}{\log(a / b)}$.

Transition points of β in equation (2.9) are $P \to \pm\infty, -1$. The boundary conditions require that:

$$T_{rr} = p \quad \text{at} \quad r = a$$
$$T_{rr} = 0 \quad \text{at} \quad r = b \tag{2.11}$$

where p is pressure applied internal surface.

The resultant force transmitted by the wall in axial direction is equal to $L = \pi a^2 p$, that is :

$$2\pi \int_a^b r T_{zz} dr = \pi a^2 p . \tag{2.12}$$

Solution through the Principal Stresses

For finding the plastic stress, the transition function is taken through the principal stress (see Seth [8, 9], Gupta [15, 16, 19 - 20], Pankaj Thakur [21 - 24]) at the transition point $P \to \pm\infty$. The transition function R is defined as:

$$R = T_{rr} - \frac{\lambda}{n} K + \alpha(3-2c)\Theta \equiv \frac{2\mu}{cn}\left[c - \beta^n\left\{(1-c)+(p+1)^n\right\}\right] - \alpha(3-2c)\left(1+\frac{2\mu}{c}\right)\Theta \tag{3.1}$$

Taking the logarithmic differentiation of equation (3.1) with respect to r and substituting the value of $dP/d\beta$ from eq. (2.9) and taking the asymptotic value $P \to \pm\infty$, one get:

$$\frac{d(\log R)}{dr} = -\frac{c}{r} \tag{3.2}$$

Integrating eq. (3.2), one get

$$R = A \exp f(r) \tag{3.3}$$

where A is a constant of integration, which can be determine by boundary condition and

$$f(r) = -\int \frac{c}{r} \mathrm{d}r \tag{3.4}$$

From equation (3.1) and eq. (3.3), we have

$$T_{rr} = A.\exp f(r) + B - \alpha(3-2c)\Theta \tag{3.5}$$

where $B = \frac{\lambda}{n} k$ and $k = \left[3 - (1-d)^n\right]$.

Using boundary condition (2.11) in equations (3.5), one get

$$T_{rr} = A.\left[\exp f(r) - \exp f(b)\right] - \alpha(3-2c)\Theta \tag{3.6}$$

Substituting of eq. (3.6) in eq. (2.8) gives

$$T_{\theta\theta} = A.\left[(1-c)\exp f(r) - \exp f(b)\right] - \alpha(3-2c)\Theta \tag{3.7}$$

$$T_{zz} = \left(\frac{1-c}{2-c}\right)(T_{rr} + T_{\theta\theta}) + \frac{c\lambda}{(1-c)}\frac{(3-2c)}{(2-c)} r e_{zz} - \frac{c\lambda\alpha}{(1-c)}\frac{(3-2c)}{(2-c)} r\Theta \tag{3.8}$$

where e_{zz} and A are obtained by using eq. (3.8) and eq. (2.12) as:

$$e_{zz} = \frac{\frac{L}{2\pi} - \left(\frac{1-c(a)}{2-c(a)}\right) Pa^2 - \int_a^b \frac{r^2 c'}{(2-c)^2} T_{rr} \mathrm{d}r + \int_a^b \frac{c.\lambda.\alpha.\Theta.r}{(1-c)(2-c)}(3-2c)\mathrm{d}r}{\lambda \int_a^b \frac{c.r.(3-2c)}{(1-c)(2-c)} \mathrm{d}r}, \quad A = \frac{\alpha.\Theta_0 \cdot \left[3 - 2c(a)\right] - P}{\exp f(a) - \exp f(b)} .$$

Equation (3.6)-(3.8) give the elastic-plastic transitional stresses for a non-homogeneous compressible cylinder under internal pressure ant temperature. Substituting equation (1.2) in equations (3.6)- (3.8),one get the transitional stresses as:

$$T_{rr} = A_1.\left[\exp\frac{c_0}{k}r^{-k} - \exp\frac{c_0}{k}b^{-k}\right] - \alpha\left(3-2c_0r^{-k}\right)\Theta \tag{3.9}$$

$$T_{\theta\theta} = T_{rr} - A_1.\left[c_0.r^{-k}\exp\frac{c_0}{k}r^{-k}\right] - \left[2k.c_0.r^{-k}.\Theta + \left(3-2c_0.r^{-k}\right)\bar{\Theta}_0\right]\alpha \tag{3.10}$$

And

$$T_{zz} = \left(\frac{1-c_0r^{-k}}{2-c_0r^{-k}}\right)\left(T_{rr}+T_{\theta\theta}\right) + A_2.e_{zz} - \alpha.A_2.\Theta \tag{3.11}$$

where

$$e_{zz} = \frac{\frac{L}{2\pi} - \left(\frac{1-c_0r^{-k}}{2-c_0r^{-k}}\right)Pa^2 - A_3\int_a^b A_4\mathrm{d}r + \alpha\int_a^b A_2\Theta\mathrm{d}r}{\int_a^b A_2\mathrm{d}r}$$

and $A_1 = \dfrac{\alpha.\Theta_0.\left[3-2c_0a^{-k}\right]-P}{\exp\frac{c_0}{k}a^{-k} - \exp\frac{c_0}{k}b^{-k}}$; $A_2 = \dfrac{\lambda.c_0.r^{1-k}.\left[3-2c_0r^{-k}\right]}{\left(1-c_0r^{-k}\right)\left(2-c_0r^{-k}\right)}$;

$$A_3 = \frac{c_0.k\left[\alpha\left(3-2c_0.r^{-k}\right)-P\right]}{\exp\frac{c_0}{k}a^{-k} - \exp\frac{c_0}{k}b^{-k}};\quad A_4 = \frac{r^{1-k}.\left[\exp\frac{c_0}{k}r^{-k} - \exp\frac{c_0}{k}b^{-k}\right]}{\left(2-c_0.r^{-k}\right)^2}.$$

From eq. (3.09)-(3.10), one gets:

$$T_{\theta\theta} - T_{rr} = A_1.\left[-c_0.r^{-k}\exp\frac{c_0}{k}r^{-k}\right] - 2k.c_0.\alpha.\Theta.r^{-k} - \alpha.\Theta_0\left(3-2c_0.r^{-k}\right) \tag{3.12}$$

It can be seen from eq. (3.12) that $\left|T_{\theta\theta} - T_{rr}\right|$ is maximum at $r = a$, therefore yielding in the cylinder will start at internal surface and in this case eq. (3.12) becomes:

$$\left|T_{\theta\theta} - T_{rr}\right|_{r=a} = \left|PA_5 - \alpha.\Theta_0.A_6\right| \equiv Y(say) \tag{3.13}$$

where Y is initial yielding stresses and

$$A_5 = \frac{c_0.a^{-k}}{1-\exp\frac{c_0}{k}\left(b^{-k}-a^{-k}\right)}$$

and $$A_6 = \frac{\left(3-2c_0.a^{-k}\right)c_0.a^{-k}}{1-\exp\frac{c_0}{k}\left(b^{-k}-a^{-k}\right)} + 2k.c_0.a^{-k}\frac{\left(3-2c_0.a^{-k}\right)}{\log\frac{b}{a}}.$$

The necessary pressure and temperature required for initial yielding is given by

$$\frac{P}{Y} = \frac{1}{A_5} + \Theta_1 \frac{A_6}{A_5} \tag{3.14}$$

where $\Theta_1 = \alpha\, \Theta_0 / Y$ .1

$$\left|T_{\theta\theta} - T_{rr}\right|_{r=b} = \left|PA_7 + 3\alpha.\Theta_0.A_8\right| \equiv Y_1\left(say\right) \tag{3.15}$$

where $A_7 = \dfrac{K}{\left(b/a\right)^k - 1}; A_8 = A_7 + \dfrac{1}{\log\dfrac{b}{a}}$.

And pressure required for full plastic state is

$$\frac{P}{Y_1} = \frac{1}{A_7} - 3\Theta_2 \frac{A_8}{A_7} \tag{3.16}$$

where Y_1 yielding stresses for fully-plastic state and $\Theta_2 = \alpha\, \Theta_0 / Y_1$.

Now stresses for full plasticity is obtained by taking $c_0 \to 0$, we have:

$$\frac{T_{rr}}{Y_1} = 3\Theta_2\left(A_9 + A_{10}\right) - \frac{P}{Y_1} A_9 \tag{3.17}$$

$$\frac{T_{\theta\theta}}{Y_1} = 3\Theta_2\left(A_{11} + A_{12}\right) - \frac{P}{Y_1} A_{11} \tag{3.18}$$

$$\frac{T_{zz}}{Y_1} = 3\Theta_2\left(A_{13} + A_{14}\right) - \frac{P}{Y_1} A_{13} \tag{3.19}$$

where $A_9 = \dfrac{\left(b/r\right)^k - 1}{\left(b/a\right)^k - 1}; A_{10} = \dfrac{\log\left(r/b\right)}{\log\left(b/a\right)}; A_{11} = \dfrac{\left(1-k\right)\left(b/r\right)^k - 1}{\left(b/a\right)^k - 1}; A_{12} = \dfrac{1 + \log\left(r/b\right)}{\log\left(b/a\right)}$.

$$A_{13} = \frac{\left(1 - \dfrac{k}{2}\right)\left(b/r\right)^k - 1}{\left(b/a\right)^k - 1}; A_{14} = \frac{0.5 + \log\left(r/b\right)}{\log\left(b/a\right)}.$$

Solution through the Principal Stress difference

It has been shown that the asymptotic solution through the principal stress difference [8, 9, 15, 16, 19-24] at the transition point $P \to -1$, gives the creep stresses. Transition function R is defined as:

$$R = T_{rr} - T_{\theta\theta} = \left(\frac{2\mu}{n}\right)\beta^n\left[1 - \left(1 - P\right)^n\right] \tag{4.1}$$

Taking the logarithmic differentiating of eq. (4.1) with respect to r and using eq. (2.10), one gets:

$$\frac{d}{dr}\left(\log R\right) = \frac{1}{r[1 - \left(P+1\right)^n]}\left\{nP\left(2-c\right) - c\left[1 - \left(P+1\right)^n\right] + \frac{nc\xi\overline{\Theta_0}}{2\mu\beta^n}\right\} \tag{4.2}$$

Taking asymptotic value of eq. (4.2) at $P \to$ -1, one gets after integration:

$$R = T_{rr} - T_{\theta\theta} = Ar^{-2n+c(n-1)} \exp f \tag{4.3}$$

where A is integration constant, determined by boundary condition and $f = \dfrac{\alpha\,\overline{\Theta}_0\left(3-2c\right)r^n}{D^n}$.

Asymptotic value of β as $P \to -1$ is D/r; D being a constant. By substituting eq. (4.3) in eq. (2.8), one gets:

$$T_{rr} = -A\int F dr + B \tag{4.4}$$

where $F = r^{-2n+c(n-1)-1} \exp f$ and B is a constant of integration, which can be determined by boundary condition. The constant A and B are obtained by using boundary condition given by eq. (2.11) in eq. (4.4) as:

$$A = \frac{-p}{\int_a^b F dr} \quad \text{and} \quad B = A\int F dr \ \text{ at } r = b\,.$$

By substituting the values of A and B into equation (4.4), one gets:

$$T_{rr} = -p\left(\int_r^b F dr \bigg/ \int_a^b F dr\right) \tag{4.5}$$

The value of $T_{\theta\theta}$ and T_{zz} are obtained from eqs. (4.3) and (2.6) respectively as:

$$T_{\theta\theta} = T_{rr} + \frac{prF}{\int_a^b F dr} \tag{4.6}$$

$$T_{zz} = \left(\frac{1-c}{2-c}\right)\left(T_{rr} + T_{\theta\theta}\right) + Ee_{zz} - E\alpha\,\Theta \tag{4.7}$$

where $E = \left[\dfrac{3-2c}{2-c}\right]2\mu$.

The term e_{zz} is obtained by using eq. (4.7) and eq. (2.12), as:

$$e_{zz} = \frac{\dfrac{ca^2 p}{(2-c)} + E\alpha\,\overline{\Theta}_0\left[a^2\log(b/a) + \dfrac{\left(a^2-b^2\right)}{2}\right]}{\left(b^2-a^2\right)} \tag{4.8}$$

Equations (4.5)-(4.7) define thermal creep stresses for a thick-walled circular cylinder under internal pressure. By introducing non-dimensional components: $R=r/b$, $R_0=a/b$, $E_1=E/p$ and $\sigma_r = \dfrac{T_{rr}}{p}$, $\sigma_\theta = \dfrac{T_{\theta\theta}}{p}$, $\sigma_z = \dfrac{T_{zz}}{p}$, eqs. (4.5)-(4.8) in non-dimensional from become:

$$\sigma_r = -\frac{\int_R^1 F_1 dr}{\int_{R_0}^1 F_1 dr} \tag{4.9}$$

$$\sigma_\theta = \sigma_r + \frac{R^{-2n+c(n-1)}\exp f_1}{\int_{R_0}^1 F_1 dr} \tag{4.10}$$

$$\sigma_z = \left(\frac{1-c}{2-c}\right)\left(\sigma_r + \sigma_\theta\right) + E_1 e_{zz} - E_1\alpha\,\Theta \tag{4.11}$$

where

$$E_1 e_{zz} = \frac{\dfrac{cR_0^2}{(2-c)} + E_1 \alpha \bar{\Theta}_0 \left[R_0^2 \log(1/R_0) + \dfrac{(R_0^2-1)}{2} \right]}{(1-R_0^2)} \tag{4.12}$$

and $F_1 = r^{-2n+c(n-1)-1} \exp f_1$ and $f_1 = \dfrac{\alpha \bar{\Theta}_0 (3-2c)(bR)^n}{D^n}$.

For incompressible material ($c \to 0$), eqs. (4.9) to (4.12) become

$$\sigma_r = -\int_R^1 F_2 dr \Big/ \int_{R_0}^1 F_2 dr \tag{4.13}$$

$$\sigma_\theta = \sigma_r + \frac{R^{-2n} \exp f_2}{\int_{R_0}^1 F_2 dr} \tag{4.14}$$

$$\sigma_z = \frac{(\sigma_r + \sigma_\theta)}{2} + E_1 e_{zz} - E_1 \alpha \Theta \tag{4.15}$$

where

$$e_{zz} = \frac{\alpha \bar{\Theta}_0 [R_0^2 \log(1/R_0) + \dfrac{(R_0^2-1)}{2}]}{(1-R_0^2)} \quad \text{and} \quad F_2 = r^{-2n-1} \exp f_2 \text{ and } f_2 = \frac{3\alpha \bar{\Theta}_0 (bR)^n}{D^n} \tag{4.16}$$

Strain Rates:

When the creep sets in, the strains should be replaced by strain rates. The stress-strain relations can be written as :

$$\dot{e}_{ij} = \frac{1+\nu}{E} T_{ij} - \frac{\nu}{E} \delta_{ij} T + \alpha \Theta \tag{4.24}$$

where $\dot{e}_{ij}$ is the strain rate tensor with respect to flow parameter *t* and $T = T_{11} + T_{22} + T_{33}$ and $\nu = 1 - c/2 - c$. By differentiating eq. (2.3) with respect to time *t*, one gets:

$$\dot{e}_{\theta\theta} = -\beta^{n-1} \dot{\beta}. \tag{4.25}$$

For SWAINGER measure (*n* = 1), from equation (4.25) it follows:

$$\dot{\varepsilon}_{\theta\theta} = \dot{\beta}. \tag{4.26}$$

The transition value of eq. (4.1) as $P \to -1$ gives

$$\beta = \left[\frac{n(3-2c)}{(2-c)} \right]^{\frac{1}{n}} (T_{rr} - T_{\theta\theta})^{\frac{1}{n}} \tag{4.27}$$

By substituting eqs. (4.25), (4.26) and (4.27) into equation (4.24), one gets:

$$\dot{\varepsilon}_{rr} = \frac{1}{E_1} \left[\frac{n(\sigma_r - \sigma_\theta)(3-2c)}{E_1(2-c)} \right]^{\frac{1}{n}-1} \left[\sigma_r - \nu(\sigma_\theta + \sigma_z) + \alpha \Theta E_1 \right],$$

$$\dot{\varepsilon}_{\theta\theta} = \frac{1}{E_1} \left[\frac{n(\sigma_r - \sigma_\theta)(3-2c)}{E_1(2-c)} \right]^{\frac{1}{n}-1} \left[\sigma_\theta - \nu(\sigma_r + \sigma_z) + \alpha \Theta E_1 \right],$$

$$\dot{\varepsilon}_{zz}=\frac{1}{E_1}\left[\frac{n\left(\sigma_r-\sigma_\theta\right)\left(3-2c\right)}{E_1\left(2-c\right)}\right]^{\frac{1}{n}-1}\left[\sigma_z-\nu\left(\sigma_r+\sigma_\theta\right)+\alpha\Theta E_1\right]. \tag{4.28}$$

where $\Theta=\Theta_0\log R/\log R_0$. From eqs. (4.28), one gets:

$$\frac{\dot{\varepsilon}_{\theta\theta}-\dot{\varepsilon}_{rr}}{\sigma_{\theta\theta}-\sigma_{rr}}=\frac{\dot{\varepsilon}_{rr}-\dot{\varepsilon}_{zz}}{\sigma_{rr}-\sigma_{zz}}=\frac{\dot{\varepsilon}_{zz}-\dot{\varepsilon}_{\theta\theta}}{\sigma_{zz}-\sigma_{\theta\theta}} \tag{4.29}$$

For incompressible material ($c\rightarrow 0$ *or* $\nu\rightarrow 1/2$) without thermal effects, the creep strain rates (4.28) using eqn. (4.23) becomes:

$$\dot{\varepsilon}_{rr}=-\left(\frac{1}{E_1}\right)^{\frac{1}{n}}\left(-n\sqrt{3}\right)^{\frac{1}{n}-1}\left(\frac{\sqrt{3}}{2}\right)^{\frac{1}{n}+1}\left(\sigma_r-\sigma_\theta\right)^{\frac{1}{n}},$$

$$\dot{\varepsilon}_{\theta\theta}=-\left(\frac{1}{E_1}\right)^{\frac{1}{n}}\left(-n\sqrt{3}\right)^{\frac{1}{n}-1}\left(\frac{\sqrt{3}}{2}\right)^{\frac{1}{n}+1}\left(\sigma_\theta-\sigma_r\right)^{\frac{1}{n}},$$

$$\dot{\varepsilon}_{zz}=0\,. \tag{4.30}$$

These constitutive equations (4.30) are the same as obtained by Odquist [17], if one put $E_1=\left(-n\sqrt{3}\right)^{n-1}=\sigma_c$ and n = 1/N. It has been shown in eqs. (4.29) and (4.30) that the assumption II and III, come out from the solution itself, not assumed [10-14, 17] prior to the solution.

Results and Discussion

Taking equation (3.13) into account curves have been drawn in the Figure 1 between pressure (*P*/*Y*) and radii ratio (*b*/*a*) for various values of temperature $\Theta_1=\alpha\Theta_0/Y$ required for initial yielding at the internal surface for k = -1, -0.5, 0.0001, 0.5, 1. For k = 0, it gives the case for cylinder made of homogeneous material. In the absence of thermal effect ($\Theta_1=0$), it is seen from Fig. 1, that for k < 0, high pressure is required for initial yielding than the homogeneous cylinder whereas reverse is the case for k > 0.

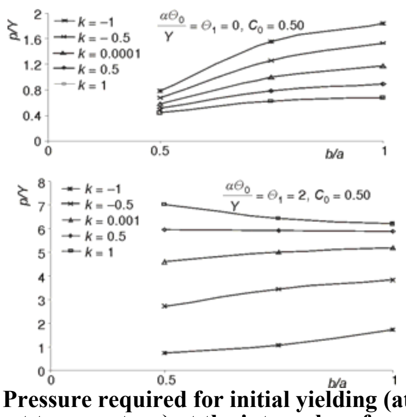

Fig. 1 Pressure required for initial yielding (at different temperature) at the internal surface of a non-homogeneous cylinder.

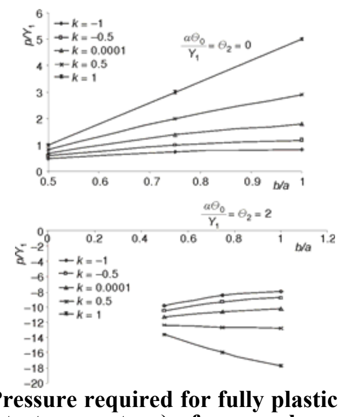

Fig. 2 Pressure required for fully plastic state (at different temperature) for non-homogeneous cylinder under internal pressure.

In the presence of thermal effects and k < 0, it can be seen from Fig.1 that the pressure required for initial yielding is less and it goes on decreasing with the increase in temperature. In the absence of thermal effects it can be seen form Fig. 1 that a high percentage decrease in the pressure is needed for the non-homogeneous cylinder to become fully plastic as is needed for its initial yielding which is shown in Fig. 2, With the introduction of thermal effect, a significant increases in pressure is needed for non-homogeneous cylinder $k < 0$ to become fully plastic, whereas opposite is the case for $k > 0$, which can be seen in Fig. 2. It means that a cylinder made of non-homogeneous material for $k < 0$ subjected to combined pressure and temperature is on the safer side of design which is also shown in Fig. 3, since the tangential stress is maximum at the outer surface for non-homogeneous material for various combinations of pressure and temperature. In the absence of temperature ($\Theta_2 = 0$), it can be seen from Fig. 3(a), that a cylinder made of non-homogeneous material ($k > 0$) is on the safer side of design is given by Gupta and Shukla [10].

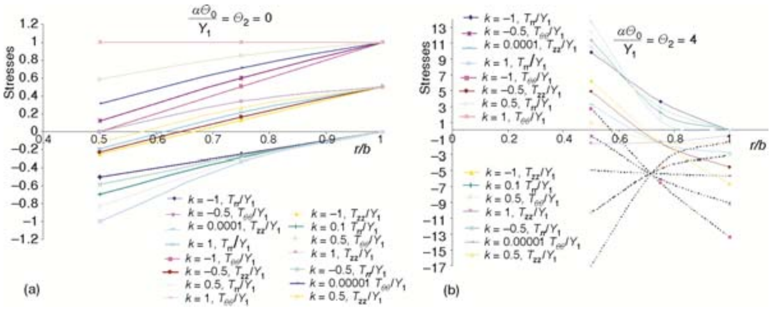

Fig. 3 Plastic stress distribution (without temperature) in non-homogeneous cylinder under internal pressure

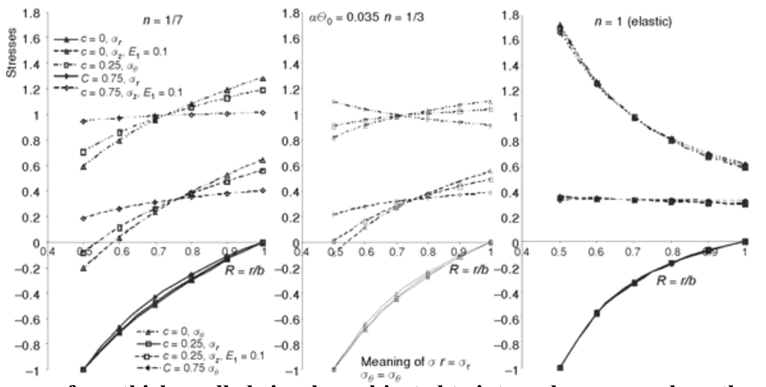

Fig. 4 Creep stresses for a thick- walled circular subjected to internal pressure along the radius R = r/b under steady state temperature.

Curves have been drawn in Fig. 4 and 5 between stresses $\sigma_r, \sigma_\theta, \sigma_z$ and radii ratio $R = r/b$ for Methyl Methacrylate material [18] with and without steady state temperature. For $n = 1/7$ (or $N = 7$), it can be seen that the circumferential stress is maximum at the external surface of a cylinder made of incompressible material as compared ti that of compressible material. For $n = 1/3$ (or $N = 3$) even though the circumferential stress has maximum value at the external surface, it has smaller values as compared to $n = 1/7$ or ($N = 7$). It has been seen the introduction of steady state temperature reduces the stresses at the outer surface. For $n = 1$, it gives elastic stress distribution.

With the effect of temperature, the value of circumferential stress is much higher than without temperature. In Fig 6 curves have been drawn for creep strain rates along the radius for $n = 1/3$ (or $N = 3$) and $E_1 = E/p = 0.1$. It has been observed that for a thick-walled cylinder made of compressible material $E_1 < 1$ the creep rates have larger values at the internal surface as compared to $E_1 \geq 1.0$ (not shown here). These values further increase at the internal surface as n decrease ($n = 1/7$) or N increases ($N = 7$) and $E_1 < 1.0$, see Fig. 7.

With the introduction of temperature effect the creep rates at the internal surface have much higher values for $n = 1/7$ as compared to $n = 1/3$. It means that a thick-walled cylinder made of compressible material subjected to both pressure and temperature have large creep rates at the internal surface for measure $n = 1/7$ (or $N = 7$) and $E_1 < 1.0$ as compared to $n = 1/3$ (or $N = 3$), $E_1 \geq 1.0$ and cylinder made of incompressible material.

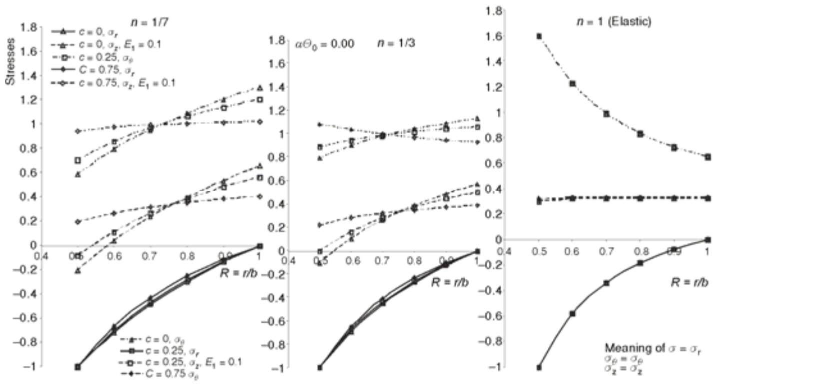

Fig. 5 Creep stresses for a thick- walled circular subjected to internal pressure along the radius R = r/b.

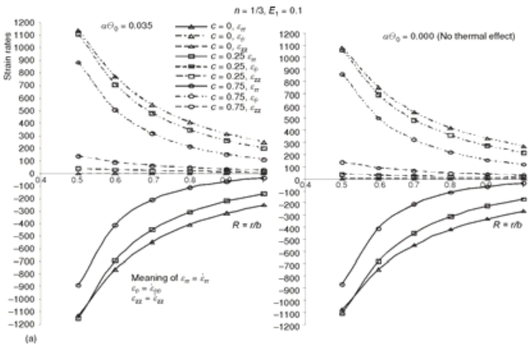

Fig. 6 Strain rate distribution for a thick-walled circular subjected to internal pressure for n = 1/3 and E_1=0.1

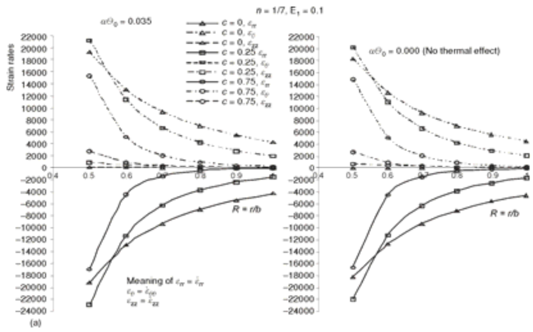

Fig. 7 Strain rate distribution for a thick- walled circular subjected to internal pressure for n = 1/7 and E_1=0.1.

Conclusion

It has seen that in the presence of temperature, a cylinder made of non-homogeneous material $k<0$ (Non-homogeneity is less at internal surface than at outer surface) require high pressure to become fully plastic as is required for initial yielding and this pressure goes on increasing with the increases in temperature, showing that a cylinder made of non-homogeneous material $k<0$ is on the safer side of design.

For homogeneous case, it has been observed that the circumferential stress has maximum value at the external surface of the cylinder made of incompressible material as compared to compressible material. With effects of temperature reduces the stresses at the external surface of the cylinder in comparison to pressure effects only. Strain rates are found to be maximum at the internal surface of the cylinder made of compressible material and they decrease with the radius. With the introduction of temperature effect, the creep rates have higher values at the internal surface but lesser values at the external surface as compare to a cylinder subjected to pressure only.

Acknowledgement

The author wishes to acknowledge his sincere thanks to Respected Prof. Simeon Oka (Editor-in- Chief Thermal Science) and Dr Vukman Bakić (Editor Thermal science) for his encouragement during the preparation of this paper.

Nomenclature

a,b - Internal and external radii, [m]
c - Compressibility factor, [-]
u,v,w - Displacement components,[m]
v - Poisson's ratio, [-]
T_{ij}, e_{ij} - stress and strain rate tensor

Greek letters

σ_r - Radial stress component, [-]
σ_θ - Circumferential stress,[-]
σ_z - Axial stress component,[-].
Θ - Temperature, [K].

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Paper received: March 15, 2011
Paper revised: May 7, 2012
Paper accepted: May 12, 2012