HALL EFFECT ON THERMAL STABILITY OF FERROMAGNETIC FLUID IN POROUS MEDIUM IN THE PRESENCE OF HORIZONTAL MAGNETIC FIELD

by

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This paper deals with the theoretical investigation of the effect of Hall currents on the thermal stability of a ferromagnetic fluid heated from below in porous medium. For a fluid layer between two free boundaries, an exact solution is obtained using a linearized stability theory and normal mode analysis. A dispersion relation governing the effects of medium permeability, a uniform horizontal magnetic field, magnetization and Hall currents is derived. For the case of stationary convection, it is found that the magnetic field and magnetization have a stabilizing effect on the system, as such their effect is to postpone the onset of thermal instability whereas Hall currents are found to hasten the onset of thermal instability. The medium permeability hastens the onset of convection under certain conditions. The principle of exchange of stabilities is not valid for the problem under consideration whereas in the absence of Hall currents (hence magnetic field), it is valid under certain conditions.

Key words: *Hall currents, thermal stability, ferromagnetic fluid, porous medium*

Introduction

Ferrohydrodynamics deals with the mechanics of fluid motions influenced by strong forces of magnetic polarization. Ferrohydrodynamics concerns usually non-conducting liquids with magnetic properties and constitutes an entire field of physics close to magnetohydrodynamics but still different. The polarization force and the body couple are the two main features that distinguish ferromagnetic fluid from ordinary fluid. Ferromagnetic fluids are electrically non-conducting colloidal suspensions of solid ferromagnetic particles in a non-electrically conducting carrier fluid like water, kerosene, hydrocarbon, *etc.* These fluids behave as a homogeneous continuum and exhibit a variety of interesting phenomena. Ferromagnetic fluids are not found in nature but are artificially synthesized.

Soon after the method of formation of ferromagnetic fluids in the early or mid 1960s, the importance of ferrohydrodynamics was realized. Due to the wide ranges of application of ferromagnetic fluid to instrumentation, lubrication, printing, vacuum technology, vibration damping, metal recovery, acoustics and medicine, its commercial usage includes vacuum feedthroughs for semiconductor manufacturing and related uses [1], pressure seals for compressors and blowers [2]. They are also used in liquid cooled loudspeakers that involve small bulk quantities of the ferromagnetic fluid to conduct heat away from the speaker coils [3]. This innovation increases the amplifying power of the coil, and hence it leads the loudspeakers to produce high fidelity sound. In order to bring the drugs to a target site in a human

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body, a magnetic field can pilot the path of a drop of the ferromagnetic fluid in the human body [4]. The novel zero leakage rotating shaft seals are used in computer disk drives [5].

Experimental and theoretical physicists and engineers gave significant contributions to ferrohydrodynamics and its applications [6]. During the last half century, research on magnetic liquids has been very productive in many fields. Strong efforts have been undertaken to synthesize stable suspensions of magnetic particles with different performances in magnetism, fluid mechanics or physical chemistry.

An authoritative introduction to this fascinating subject has been discussed in detail in the celebrated monograph by Rosensweig [7]. This monograph reviews several applications of heat transfer through ferromagnetic fluids. One such phenomenon is enhanced convective cooling having a temperature dependent magnetic moment due to magnetization of the fluid. This magnetization, in general, is a function of magnetic field, temperature and density of the fluid. The variation of anyone of these causes a change of body force. This leads to convection in ferromagnetic fluids in the presence of magnetic field gradient. This mechanism is known as ferroconvection, which is similar to Bėnard convection [8]. In our analysis, we assume that the magnetization is aligned with the magnetic field. Convective instability of a ferromagnetic fluid for a fluid layer heated from below in the presence of uniform vertical magnetic field has been considered by Finlayson [9]. He explained the concept of thermo-mechanical interaction in ferromagnetic fluids. Thermoconvective stability of ferromagnetic fluids without considering buoyancy effects has been investigated by Lalas and Carmi [10] whereas Shliomis [11] analyzed the linearized relation for magnetized perturbed quantities at the limit of instability.

The Bėnard convection in ferromagnetic fluids has been considered by many authors [12-17]. The medium has been considered to be non-porous in all the above studies. There has been a lot of interest, in recent years, in the study of the breakdown of the stability of a fluid layer subjected to a vertical temperature gradient in a porous medium and the possibility of the convective flow. The stability of flow of a fluid through a porous medium taking into account the Darcy resistance was considered by Lapwood [18], Wooding [19] and Sunil *et al.* [20]. A porous medium is a solid with holes in it, and is characterized by the manner in which the holes are imbedded, how they are interconnected, and the description of their location, shape and interconnection. However, the flow of a fluid through a homogeneous and isotropic porous medium is governed by Darcy's law. A macroscopic equation describing incompressible flow of a fluid of viscosity μ, through a macroscopically homogeneous and isotropic porous medium of permeability k_1, is well-known Darcy's equation, in which the usual viscous term in the equations of fluid motion is replaced by the resistance term $(-\mu/k_1)\vec{q}$, where $\vec{q}$ is the filter velocity of the fluid. The thermoconvective instability in a ferromagnetic fluid saturating a porous medium of very large permeability subjected to a vertical magnetic field has been studied using the Brinkman model by Vaidyanathan *et al.* [21], and indicated that only stationary convection can exist.

In the presence of strong electric field, the electric conductivity is affected by the magnetic field. Consequently, the conductivity parallel to the electric field is reduced. Hence, the current is reduced in the direction normal to both electric and magnetic field. This phenomenon in the literature is known as Hall effect. The Hall current is likely to be important in flows of laboratory plasmas as well as in many geophysical and astrophysical situations. The effect of Hall current on thermal instability has also been studied by several authors [22-28].

In the present problem, we have studied the effect of Hall current on thermal stability of ferromagnetic fluid heated from below in porous medium in the presence of horizontal magnetic field. Here, we have extended the results reported by Kumar *et al.* [29] to include the effect of Hall currents for ferromagnetic fluids in porous medium.

Mathematical formulation of the problem

Consider an infinite, incompressible, electrically non-conducting thin ferromagnetic fluid, bounded by the planes $z = 0$ and $z = d$ as shown in fig. 1. This layer is heated from below so that the uniform temperature gradient $\beta(=|\mathrm{d}T/\mathrm{d}z|)$ is maintained. A uniform horizontal magnetic field $\vec{\mathrm{H}}(H,0,0)$ and gravity force $\vec{\mathrm{g}}(0,0,-\mathrm{g})$ pervade the system. This fluid layer is flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k_1.

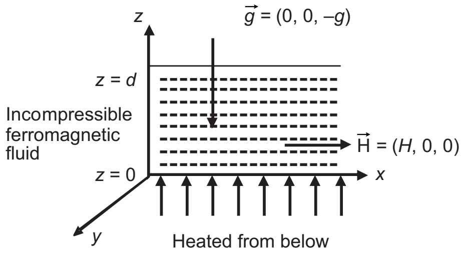

Figure 1. Geometrical configuration

Ferromagnetic fluids respond so rapidly to a magnetic torque that we can assume the following conditions to hold:

$$\vec{\mathrm{M}} \times \vec{\mathrm{H}} = 0 \tag{1}$$

where $\vec{\mathrm{M}}$ is the magnetization, and $\vec{\mathrm{H}}$ – the magnetic field intensity. In addition to the above equation, the ferromagnetic fluids also satisfy Maxwell's equations. Assuming the fluid is electrically non-conducting and that the displacement current is negligible, Maxwell's equations become:

$$\nabla \cdot \vec{\mathrm{B}} = 0, \quad \nabla \times \vec{\mathrm{H}} = 0 \tag{2}$$

In Chu formulation of electrodynamics [30], the magnetic field, magnetization, and magnetic induction are related by:

$$\vec{\mathrm{B}} = \mu_0(\vec{\mathrm{H}} + \vec{\mathrm{M}}) \tag{3}$$

We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field and temperature so that:

$$\vec{\mathrm{M}} = \frac{\vec{\mathrm{H}}}{H} M(\vec{\mathrm{H}}, \vec{\mathrm{T}}) \tag{4}$$

Let p, ρ, T, α, g, η, μ_e, N, e, and $\vec{\mathrm{q}}(u,v,w)$ denote the fluid pressure, density, temperature, thermal coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability, electron number density, charge of an electron, and fluid (filter) velocity, respectively. The equations expressing the conservation of momentum, mass, temperature, and equation of state of ferromagnetic fluids through porous medium are:

$$\frac{1}{\varepsilon}\left[\frac{\partial \vec{\mathrm{q}}}{\partial t} + \frac{1}{\varepsilon}(\vec{\mathrm{q}} \cdot \nabla)\vec{\mathrm{q}}\right] = -\frac{1}{\rho_0}\nabla p + \vec{\mathrm{g}}\left(1 + \frac{\delta\rho}{\rho_0}\right) + M\nabla\vec{\mathrm{H}} - \frac{1}{k_1}\nu\vec{\mathrm{q}} + \frac{\mu_e}{4\pi\rho_0}(\nabla \times \vec{\mathrm{H}}) \times \vec{\mathrm{H}} \tag{5}$$

$$\nabla \cdot \vec{\mathrm{q}} = 0 \tag{6}$$

$$E\frac{\partial T}{\partial t} + (\vec{\mathrm{q}} \cdot \nabla)T = \kappa\nabla^2 T \tag{7}$$

$$\rho = \rho_0\left[1 - \alpha(T - T_0)\right] \tag{8}$$

where the suffix zero refers to the values at the reference level $z = 0$ and in writing eq. (5) use has been made of the Boussinesq approximation which states that the density variations are ignored in all terms in the equation except the external force term. The magnetic permeability μ_e, the kinematic viscosity ν and the thermal diffusivity κ are all assumed to be constants. Here $E = \varepsilon + (1-\varepsilon)(\rho_s c_s/\rho_0 c_i)$ is a constant. Density and specific heat of solid (porous matrix) material and fluid are ρ_s, c_s, and ρ_0, c_i. ∇H is the magnetic field gradient:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \; H = |\vec{\mathrm{H}}|, \, B = |\vec{\mathrm{B}}| \quad \text{and} \quad M = |\vec{\mathrm{M}}|$$

The Maxwell's equation in the presence of Hall currents yield:

$$\varepsilon \frac{\partial \vec{\mathrm{H}}}{\partial t} = \nabla \times (\vec{\mathrm{q}} \times \vec{\mathrm{H}}) + \varepsilon\eta\nabla^2 \vec{\mathrm{H}} - \frac{\varepsilon}{4\pi N'e} \nabla \times \left[(\nabla \times \vec{\mathrm{H}}) \times \vec{\mathrm{H}} \right] \tag{9}$$

$$\nabla \cdot \vec{\mathrm{H}} = 0 \tag{10}$$

The equation of state specifying M by two thermodynamic variables only (H and T), is necessary to complete the system. In the present case, we consider magnetization to be independent of the magnetic field intensity so that $M = M(T)$ only. As a first approximation, we assume that:

$$M = M_0 \left[1 - \gamma (T - T_0) \right] \tag{11}$$

where M_0 is the magnetization at $T = T_0$ with T_0 being the reference temperature, and:

$$\gamma = \frac{1}{M_0} \left(\frac{\partial M}{\partial T} \right)_H$$

The basic state is assumed quiescent and is given by:

$$\vec{\mathrm{q}} = (0,0,0), \quad p = p(z), \; T = T(z) = T_0 - \beta z, \; \rho = \rho(z) = \rho_0 (1 + \alpha\beta z), \; \vec{\mathrm{M}} = \vec{\mathrm{M}}(z) \tag{12}$$

The perturbation equation

Assume small perturbations around the basic state, and let $\delta\rho$, δp, δM, θ, $\vec{\mathrm{h}}\,(h_x, h_y, h_z)$ and $\vec{\mathrm{q}}\,(u, v, w)$ denote the perturbations in density, pressure p, magnetization M, temperature T, magnetic field $\vec{\mathrm{H}}(H,0,0)$, and filter velocity $\vec{\mathrm{q}}$ (zero initially), respectively. The change in magnetization δM and density $\delta\rho$ caused by the perturbations θ and γ in temperature and concentration, is given by:

$$\delta M = -\gamma M_0 \theta \tag{13}$$

$$\delta\rho = -\rho_0 \alpha\theta \tag{14}$$

Then the linearized perturbation equations for ferromagnetic fluids under Boussinesq approximation are:

$$\frac{1}{\varepsilon} \frac{\partial \vec{\mathrm{q}}}{\partial t} = -\frac{1}{\rho_0} \nabla \delta p - \vec{\mathrm{g}}\alpha\theta - \frac{\gamma M_0 \nabla \vec{\mathrm{H}}}{\rho_0} \theta - \frac{1}{k_1} \nu \vec{\mathrm{q}} + \frac{\mu_e}{4\pi\rho_0} (\nabla \times \vec{\mathrm{h}}) \times \vec{\mathrm{H}} \tag{15}$$

$$\nabla \cdot \vec{\mathrm{q}} = 0 \tag{16}$$

$$E\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta \tag{17}$$

$$\varepsilon\frac{\partial\vec{\mathrm{h}}}{\partial t} = (\vec{\mathrm{H}}\cdot\nabla)\vec{\mathrm{q}} + \varepsilon\eta\nabla^2\vec{\mathrm{h}} - \frac{\varepsilon}{4\pi N'e}\nabla\times\left[(\nabla\times\vec{\mathrm{h}})\times\vec{\mathrm{H}}\right] \tag{18}$$

$$\nabla\vec{\mathrm{h}} = 0 \tag{19}$$

Writing the scalar components of eq. (15) and eliminating u, v, h_x, and δp between them by using eqs. (16) and (19), we obtain:

$$\left(\frac{1}{\varepsilon}\frac{\partial}{\partial t} + \frac{1}{k_1}\nu\right)\nabla^2 w = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\alpha\, g - \frac{\gamma M_0\nabla\cdot H}{\rho_0}\right)\theta + \frac{\mu_e H}{4\pi\rho_0}\nabla^2\frac{\partial h_z}{\partial x} \tag{20}$$

Again, from eq. (16), we get:

$$\left(\frac{1}{\varepsilon}\frac{\partial}{\partial t} + \frac{1}{k_1}\nu\right)\zeta = \frac{\mu_e H}{4\pi\rho_0}\frac{\partial\xi}{\partial x} \tag{21}$$

where $\zeta = (\partial v/\partial x) - (\partial u/\partial y)$ is the z-component of vorticity, $\xi = (\partial h_y/\partial x) - (\partial h_x/\partial y)$ is the z-component of current density.

From eq. (18) on using eq. (19), we obtain:

$$\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)\xi = H\frac{\partial\zeta}{\partial x} + \frac{H}{4\pi N'\mathrm{e}}\frac{\partial}{\partial x}(\nabla^2 h_z) \tag{22}$$

z -component of eq. (18) is:

$$\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)h_z = H\frac{\partial w}{\partial x} - \frac{H}{4\pi N'\mathrm{e}}\frac{\partial\xi}{\partial x} \tag{23}$$

From eq. (17), we obtain:

$$\left(E\frac{\partial}{\partial t} - \kappa\nabla^2\right)\theta = \beta w \tag{24}$$

Normal mode analysis

Analyzing the disturbances into normal modes, we assume that perturbation quantities are of the form:

$$\left[w, \theta, \xi, \zeta, h_z, \gamma\right] = \left[W(z), \Theta(z), X(z), Z(z), K(z), \Gamma(z)\right]\exp(ik_x + ik_y + nt) \tag{25}$$

where k_x and k_y are wave numbers along x- and y-directions, respectively, $k[= (k_x^2 + k_y^2)^{-1/2}]$ is the resultant wave number of the disturbance and n is the growth rate (in general, a complex constant). For functions with this dependence on x, y and t, $\partial^2/\partial x^2 + \partial^2/\partial y^2 = -k^2$ and $\nabla^2 = \partial^2/\partial z^2 - k^2$.

Using eq. (25), eqs. (20)-(24) in non-dimensional form become:

$$\left(\frac{\sigma}{\varepsilon} + \frac{1}{P_1}\right)(D^2 - a^2)W = -\frac{\alpha a^2 d^2}{\nu}\left(g - \frac{\gamma M_0\nabla H}{\rho_0\alpha}\right)\Theta + \frac{ik_x\mu_e H d^2}{4\pi\rho_0\nu}(D^2 - a^2)K \tag{26}$$

$$\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right)Z=\frac{ik_x\mu_e Hd^2}{4\pi\rho_0\nu}X \tag{27}$$

$$(D^2-a^2-p_2\sigma)K=-\frac{ik_xHd^2}{\varepsilon\eta}W+\frac{ik_xHd^2}{4\pi N'e\eta}X \tag{28}$$

$$(D^2-a^2-p_2\sigma)X=-\frac{ik_xHd^2}{\varepsilon\eta}Z-\frac{ik_xH}{4\pi N'e\eta}(D^2-a^2)K \tag{29}$$

$$(D^2-a^2-Ep_1\sigma)\Theta=-\frac{\beta d^2}{\kappa}W \tag{30}$$

where we have expressed the co-ordinators x, y, and z in new units of length d, time t in the new unit of length d^2/κ, and let $a = kd$, $\sigma = nd^2/\nu$, $x^* = x/d$, $y^* = y/d$, $z^* = z/d$, and $D = d/dz^*$. $p_1 = \nu/\kappa$ is the Prandtl number, $p_2 = \nu/\eta$ is the magnetic Prandtl number, $p'_1 = \nu/\kappa'$ and $P_1 = k_1/d^2$ is the dimensionless medium permeability. Stars (*) have been omitted hereafter, for convenience.

Exact solution for free boundaries

Here we consider the case where both boundaries are free as well as being perfect conductors of heat, while the adjoining medium is perfectly conducting. The appropriate boundary conditions for the problem are [8]:

$$W=D^2W=X=DZ=0,\quad \Theta=0\quad \text{at}\quad z=0\quad \text{and}\quad 1$$

$$DX=0,\ K=0\ \text{on a perfectly conducting boundary} \tag{31}$$

Eliminating Θ, K, X, and Z from eqs. (26)-(30), we obtain:

$$a^2R_fW=\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right](D^2-a^2)(D^2-a^2-E\,p_1\sigma)W-$$

$$-\frac{Qd^2k_x^2}{\varepsilon}\left\{(D^2-a^2p_2\sigma)\left[(D^2-a^2p_2\sigma)\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right)-\frac{Qd^2k_x^2}{\varepsilon}\right]\right\}(D^2-a^2)(D^2-a^2-E\,p_1\sigma)\cdot$$

$$\cdot\left\{(D^2-a^2p_2\sigma)\left[(D^2-a^2p_2\sigma)\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right)-\frac{Qd^2k_x^2}{\varepsilon}\right]-Md^2k_x^2\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right)(D^2-a^2)\right\}^{-1}W \tag{32}$$

where $R_f=[g-(\gamma M_0\nabla H)/(\rho_0\alpha)](\alpha\beta d^4)/(\nu\kappa)$ is the Rayleigh number for ferromagnetic fluids, $Q=(\mu_eH^2d^2)/(4\pi\rho_0\nu\eta)$ – the Chandrasekhar number, and $M=[H/(4\pi N'e\eta)]^2$ – the Hall parameter.

If $g>(\gamma M_0\nabla H)/(\rho_0\alpha)$ then $R_f > R$, which implies that convection starts in the ferromagnetic fluids at a lower thermal Rayleigh number and if $g<(\gamma M_0\nabla H)/(\rho_0\alpha)$ then $R_f > R$, which implies that the convection start in the ferromagnetic fluids at a higher thermal Rayleigh number.

Using the boundary conditions (31) we can show that all the even order derivatives of W must vanish for $z = 0$ and 1 and hence the proper solution W characterizing the lowest mode is:

$$W=W_0\sin\pi z$$

where W_0 is constant. Substituting for W in eq. (32), we obtain the dispersion relation:

$$R_1 = \frac{1+x}{x}(1+x+iE\,p_1\sigma_1)\left(\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}\right)+$$
$$+Q_1\cos^2\theta(1+x)(1+x+iE\,p_1\sigma).\left[(1+x+ip_2\sigma_1)\left(\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}\right)+Q_1x\cos^2\theta\right]\cdot$$
$$.\left\{(1+x+ip_2\sigma_1)\left[(1+x+ip_2\sigma_1)\left(\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}\right)+Q_1x\cos^2\theta\right]+Mx\cos^2\theta\left(\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}\right)(1+x)\right\}^{-1} \tag{33}$$

where $x=\dfrac{a^2}{\pi^2}$, $R_1=\dfrac{R_f}{\pi^4}$, $i\sigma_1=\dfrac{\sigma}{\pi^2}$, $Q_1=\dfrac{Q}{\varepsilon\pi^2}$, $k_x=k\cos\theta$, and $P=\pi^2P_1$.

Equation (33) is the required dispersion relation including the effects of magnetic field, Hall currents, and medium permeability on a layer of ferromagnetic fluid heated from below in porous medium in the presence of a uniform horizontal magnetic field and Hall currents. In the absence of Hall current ($M = 0$), the dispersion relation (33) reduces to the one derived by Kumar *et al.* [29].

The case of stationary convection

When instability sets in stationary convection, the marginal state will be characterized by $\sigma_1 = 0$, the dispersion relation (33) reduces to:

$$R_1=\frac{1+x}{x}\frac{\left(\dfrac{1+x}{P}+Q_1x\cos^2\theta\right)^2+\dfrac{Mx\cos^2\theta(1+x)}{P^2}}{\dfrac{1+x+Mx\cos^2\theta}{P}+Q_1x\cos^2\theta} \tag{34}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x, medium permeability parameter P and Hall current parameter M. In the absence of Hall currents, the above expression for Rayleigh number R_1 reduces to:

$$R_1=\frac{1+x}{x}\left(\frac{1+x}{P}+Q_1x\cos^2\theta\right)$$

which is identical with the expression for R_1 derived by Kumar *et al.* [29] in the absence of suspended particles.

In order to investigate the effects of magnetic field, medium permeability and Hall current, we examine the behavior of $\mathrm{d}R_1/\mathrm{d}Q_1$, $\mathrm{d}R_1/\mathrm{d}P$, and $\mathrm{d}R_1/\mathrm{d}M$ analytically.

Equation (34) yields:

$$\frac{\mathrm{d}R_1}{\mathrm{d}Q_1}=(1+x)\cos^2\theta\frac{\dfrac{1+x}{P}+Q_1x\cos^2\theta+\dfrac{Q_1M(x\cos^2\theta)^2}{P\left(\dfrac{1+x+Mx\cos^2\theta}{P}+Q_1x\cos^2\theta\right)}}{\left(\dfrac{1+x+Mx\cos^2\theta}{P}+Q_1x\cos^2\theta\right)^2} \tag{35}$$

$$\frac{\mathrm{d}R_1}{\mathrm{d}M} = -Q_1 x \cos^4\theta(1+x)\frac{\dfrac{1+x}{P} + Q_1 x\cos^2\theta}{P\left(\dfrac{1+x+Mx\cos^2\theta}{P} + Q_1 x\cos^2\theta\right)^2} \tag{36}$$

$$\frac{\mathrm{d}R_1}{\mathrm{d}P} = -\frac{(1+x)}{xP^2}\frac{\left[\begin{array}{c}\dfrac{1+x}{P^2}(1+x+Mx\cos^2\theta)^2 + \dfrac{2Q_1 x\cos^2\theta(1+x)(1+x+Mx\cos^2\theta)}{P} + \\ + Q_1^2(x\cos^2\theta)^2(1+x-Mx\cos^2\theta)\end{array}\right]}{\left(\dfrac{1+x+Mx\cos^2\theta}{P} + Q_1 x\cos^2\theta\right)^2} \tag{37}$$

It is clear from eq. (35) that for stationary convection the magnetic field has a stabilizing effect. Equation (36) shows that Hall currents have a destabilizing effect. These results are in agreement with those of Sharma *et al*. [25] in which the Hall effect on thermal stability of Rivlin-Ericksen fluid have been investigated. From eq. (37) it is clear that the medium permeability have destabilizing effect for all wave numbers $(1 + x) > Mx\cos^2\theta$.

Replacing R_1 by R_f/π^4 in eq. (34), we get the following result:

$$R_1 = \frac{1+x}{x}\frac{\left(\dfrac{1+x}{P} + Q_1 x\cos^2\theta\right)^2 + \dfrac{Mx\cos^2\theta(1+x)}{P^2}}{\dfrac{1+x+Mx\cos^2\theta}{P} + Q_1 x\cos^2\theta}\frac{\pi^4}{1-\dfrac{\gamma M_0\nabla H}{\rho_0\alpha g}} \tag{38}$$

To see the effect of magnetization, we examine the behavior of $\mathrm{d}R_1/\mathrm{d}M_0$ analytically. Equation (38) yields:

$$\frac{\mathrm{d}R_1}{\mathrm{d}M_0} = \frac{1+x}{x}\frac{\left(\dfrac{1+x}{P} + Q_1 x\cos^2\theta\right)^2 + \dfrac{Mx\cos^2\theta(1+x)}{P^2}}{\left(\dfrac{1+x+Mx\cos^2\theta}{P} + Q_1 x\cos^2\theta\right)}\frac{\pi^4\gamma\nabla H}{\rho_0\alpha g\left(1-\dfrac{\gamma M_0\nabla H}{\rho_0\alpha g}\right)^2} \tag{39}$$

Equation (39) shows that magnetization has stabilizing effect on the system.

Graphs have been plotted between the modified Rayleigh number R_1 and magnetic field parameter Q_1, Hall current parameter M, medium permeability parameter P, and magnetization M_0 for various values of wave number x(= 2, 4, 6, 9, 10). It is evident from fig. (2) that magnetic field has a stabilizing effect whereas fig. (3) depict that the Hall currents have a destabilizing effect on the thermal convection. In fig. (4), R_1 is plotted against P for $M \ll 1$ and it is found that the medium permeability always hastens the onset of convection for all wave numbers as the Rayleigh number decreases with an increase in medium permeability parameter. In fig. (5), R_1 is plotted against x for $M > 1$ and it depicts that the medium permeability hastens the onset of convection for small wave numbers (near x =1) as the Rayleigh number decreases with an increase in medium permeability parameter and postpones the onset of convection for higher wave numbers as the Rayleigh number increases with an increase in medium permeability parameter. From fig. (6), it is evident that magnetization has stabilizing effect on the system.

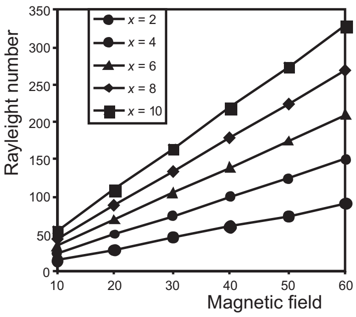

Figure 2. Variation of R_1 with Q_1 for fixed P = 50, θ = 45°, M = 10, Q_1 (= 10, 20,…, 60)

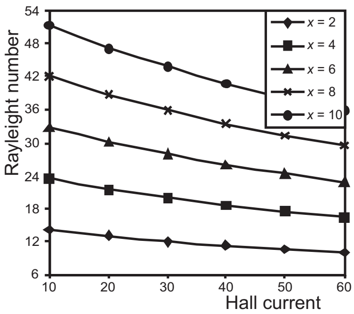

Figure 3. Variation of R_1 and M for a fixed P = 10, θ = 45°, Q_1 = 10, M (= 10, 20,…, 60)

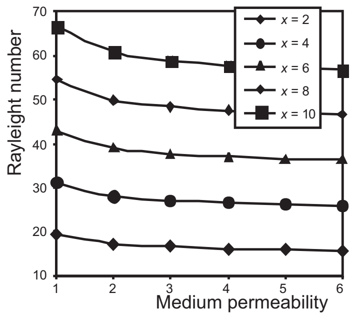

Figure 4. Variation of R_1 and P for fixed M = 0.1, Q_1 = 10, θ = 45°, and P (= 1, 2,…, 6)

The case of oscillatory modes

Here, we consider the possibility of oscillatory modes, if any, on the thermal stability due to the presence of magnetic field, Hall currents and medium permeability.

Multiplying eq. (26) by W^*, the complex conjugate of W, integrating over the range of values of z and making use of eqs. (27)-(30) together with the boundary conditions (31), we obtain:

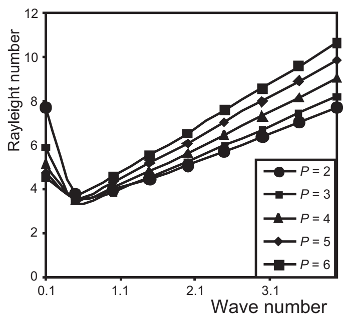

Figure 5. Variation of R_1 and x for a fixed M = 100, Q_1 = 10, θ = 45°, for x (= 0.1, 0.5, 1,…, 4)

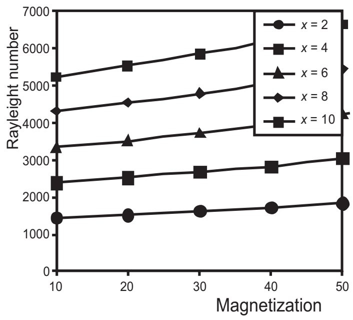

Figure 6. Variation of R_1 and M_0 for a fixed M = 10, Q_1 = 10, θ = 45°, α = 10, γ = 0.5, ρ_0 = 10, ∇H = 10 for different values of x (= 2, 4, 6, 8, 10) and M_0 (=10, 20,…, 50)

$$\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right)I_1-\frac{\alpha\kappa a^2}{\beta\nu}\left(g-\frac{\gamma M_0\nabla H}{\rho_0\alpha}\right)(I_2+E\,p_1\sigma^* I_3)+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}(I_4+p_2\sigma^* I_5)+$$
$$+d^2\cdot\left[\left(\frac{\sigma^*}{\varepsilon}+\frac{1}{P_1}\right)I_6+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}(I_7+p_2\sigma^* I_8)\right]=0 \tag{40}$$

where

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)\mathrm{d}z,\quad I_2=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)\mathrm{d}z,\quad I_3=\int_0^1\left(|\Theta|^2\right)\mathrm{d}z,$$
$$I_4=\int_0^1\left(\left|D^2K\right|^2+2a^2|DK|^2+a^4|K|^2\right)\mathrm{d}z,\quad I_5=\int_0^1\left(|DK|^2+a^2|K|^2\right)\mathrm{d}z,\quad I_6=\int_0^1|Z|^2\,\mathrm{d}z$$
$$I_7=\int_0^1\left(|DX|^2+a^2|X|^2\right)\mathrm{d}z,\quad I_8=\int_0^1|X|^2\,\mathrm{d}z \tag{41}$$

The integrals I_1-I_8 are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$ ($\sigma^* = \sigma_r + i\sigma_i$) ($\sigma_r$, σ_i are real) in eq. (40) and equating real and imaginary parts, we obtain:

$$\sigma_r\left[\frac{I_1}{\varepsilon}-\frac{\alpha a^2\kappa}{\nu\beta}\left(g-\frac{\gamma M_0\nabla H}{\rho_0\alpha}\right)E_1p_1I_3+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}p_2I_5+\frac{d^2}{\varepsilon}I_6+\frac{\mu_e\varepsilon\eta d^2}{4\pi\rho_0\nu}p_2I_8\right]=$$

$$=-\left[\frac{I_1}{P_1}-\frac{\alpha a^2\kappa}{\nu\beta}\left(g-\frac{\gamma M_0\nabla H}{\rho_0\alpha}\right)I_2+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}I_4+\frac{d^2}{P_1}I_6+\frac{\mu_e\varepsilon\eta d^2}{4\pi\rho_0\nu}p_2I_7\right] \tag{42}$$

$$\sigma_i\left[\frac{I_1}{\varepsilon}+\frac{\alpha a^2\kappa}{\nu\beta}\left(g-\frac{\gamma M_0\nabla H}{\rho_0\alpha}\right)E_1p_1I_3-\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}p_2I_5-\frac{d^2}{\varepsilon}I_6+\frac{\mu_e\varepsilon\eta d^2}{4\pi\rho_0\nu}p_2I_8\right]=0 \tag{43}$$

It is evident from eq. (42) that σ_r is either positive or negative. The system is, therefore, either stable or unstable. It is clear from eq. (43) that σ_i may be either zero or non-zero, which means that the modes may be non-oscillatory or oscillatory and the principle of exchange of stabilities is not satisfied for the problem.

In the absence of magnetic field (hence hall currents), eq. (43) reduces to:

$$\sigma_i\left[\frac{I_1}{\varepsilon}+\frac{\alpha a^2\kappa}{\nu\beta}\left(g-\frac{\gamma M_0\nabla H}{\rho_0\alpha}\right)E_1p_1I_3+\frac{\mu_e\varepsilon\eta d^2}{4\pi\rho_0\nu}p_2I_8\right]=0 \tag{44}$$

If $g > (\gamma M_0\nabla H)/(\rho_0\alpha)$, then the terms in the bracket are positive definite, which implies that $\sigma_i = 0$. Therefore, oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied if $g > (\gamma M_0\nabla H)/(\rho_0\alpha)$.

Conclusions

In the present paper, the combined effect of medium permeability, horizontal magnetic field, Hall currents, and magnetization has been considered on the thermal stability of a ferromagnetic fluid. The effect of various parameters such as magnetic field, Hall currents, magnetization, and medium permeability has been investigated analytically as well as numerically. The main results from the analysis of the paper are as follows.

- In order to investigate the effects of magnetic field, Hall currents, magnetization and medium permeability, we examine the behavior of dR_1/dQ_1, dR_1/dM, dR_1/dM_0, and dR_1/dP analytically.
- It is found that Hall currents have a destabilizing effect whereas magnetic field and magnetization have a stabilizing effect on the system. Figures 2, 3, and 6 support the analytic results graphically. The reasons for stabilizing effect of magnetic field and destabilizing effect of Hall currents are accounted by Chandrasekhar [8]. These are valid for second-order fluids as well.
- For $M \ll 1$, the medium permeability always hastens the onset of convection for all wave numbers as the Rayleigh number decreases with an increase in medium permeability parameter whereas for $M > 1$, the medium permeability hastens the onset of convection for small wave numbers as the Rayleigh number decreases with an increase in medium permeability parameter and postpones the onset of convection for higher wave numbers as the Rayleigh number increases with an increase in medium permeability parameter.

- The principle of exchange of stabilities is not valid for the problem under consideration whereas in the absence of Hall currents (and hence magnetic field), it is valid under certain conditions.

Nomenclature

C_s – heat capacity of the solid (porous matrix) material, [$\mathrm{Jkg^{-1}K^{-1}}$]
C_v – heat capacity of fluid at constant volume, [$\mathrm{Jkg^{-1}K^{-1}}$]
d – depth of layer, [m]
e – charge of an electron, [C]
$\vec{g}$ – acceleration due to gravity, [$\mathrm{ms^{-2}}$]
$\vec{H}$ – magnetic field vector, [G]
$\vec{h}$ – perturbation in magnetic field $\vec{H}$, [G]
K – Stokes' drag coefficient, [$\mathrm{kgs^{-1}}$]
k – wave number, [$\mathrm{m^{-1}}$]
k_x, k_y – components of wave number k along x-axis and y-axis, [$\mathrm{m^{-1}}$]
k_1 – medium permeability, [$\mathrm{m^2}$]
M – dimensionless Hall current parameter, [–]
N' – electron number density, [$\mathrm{m^{-3}}$]
n – growth rate, [$\mathrm{s^{-1}}$]
P_1 – dimensionless medium permeability, [–]
p – fluid pressure, [Pa]
p_1 – Prandtl number, [–]
p_2 – magnetic Prandtl number, [–]
Q – dimensionless Chandrasekhar number, [–]
q' – effective thermal conductivity of pure fluid, [$\mathrm{Wm^{-1}K^{-1}}$]
$\vec{q}$ – filter velocity, [$\mathrm{ms^{-1}}$]
R_1 – dimensionless Rayleigh number, [–]
T – temperature, [K]
t – time, [s]
x – dimensionless wave number, [–]

Greek symbols

α – coefficient of thermal expansion, [$\mathrm{K^{-1}}$]
β – uniform temperature gradient, [$\mathrm{K\,m^{-1}}$]
ε – medium porosity, [–]
η – electrical resistivity, [$\mathrm{m^2s^{-1}}$]
θ – perturbation in temperature, [K]
κ – thermal diffusivity, [$\mathrm{m^2s^{-1}}$]
μ – dynamic viscosity [$\mathrm{kgm^{-1}s^{-1}}$]
μ_e – magnetic permeability, [$\mathrm{H\,m^{-1}}$]
ν – kinematic viscosity, [$\mathrm{m^2s^{-1}}$]
ν' – kinematic viscoelasticity, [$\mathrm{m^2s^{-1}}$]
ρ – density [$\mathrm{kgm^{-3}}$]

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