

THE EFFECT OF COMPRESSIBILITY, ROTATION AND MAGNETIC FIELD ON THERMAL INSTABILITY OF WALTERS' FLUID PERMEATED WITH SUSPENDED PARTICLES IN POROUS MEDIUM

by

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The purpose of this paper is to study the effects of compressibility, rotation, magnetic field, and suspended particles on thermal stability of a layer of visco-elastic Walters' (model B') fluid in porous medium. Using linearized theory and normal mode analysis, dispersion relation has been obtained. In case of stationary convection, it is found that the rotation has stabilizing effect on the system. The magnetic field may have destabilizing effect on the system in the presence of rotation while in the absence of rotation it always has stabilizing effect. The medium permeability has destabilizing effect on the system in the absence of rotation while in the presence of rotation it may have stabilizing effect. The suspended particles and compressibility always have destabilizing effect. Due to vanishing of visco-elastic parameter, the compressible visco-elastic fluid behaves like Newtonian fluid. Graphs have also been plotted to depict the stability characteristics. The viscoelasticity, magnetic field and rotation are found to introduce oscillatory modes into the system which were non-existent in their absence.

Key words: compressibility, rotation, magnetic field, stability, visco-elastic fluid, suspended particles, porous medium, permeability

Introduction

The formulation and derivation of the basic equations of a layer of fluid heated from below in a porous medium, using the Boussinesq approximation, has been given in a treatise by Joseph [1]. A detailed account of thermal stability in a Newtonian fluid layer in the presence of magnetic field has been given by Chandrasekhar [2]. When a fluid permeates through an isotropic and homogeneous porous medium, the gross effect is represented by Darcy's law. The study of a layer of fluid heated from below in porous media is motivated both theoretically and by its practical applications in engineering.

A porous medium is a solid with holes in it, and is characterized by the manner in which the holes are imbedded, how they are interconnected and the description of their location, shape and interconnection. A macroscopic equation describing the incompressible flow of a fluid of viscosity μ through a macroscopically homogeneous and isotropic porous medium of permeability k_1 , is the well-known Darcy equation, in which the usual viscous term in the equations of fluid motion is replaced by resistance term $(-\mu/k_1)\bar{q}$, where \bar{q} is the filter velocity of the fluid. The stability of flow of a fluid through a porous medium taking into account the Darcy resistance was considered by Lapwood [3], Wooding [4] and Sunil *et al.* [5].

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The problem of thermal instability in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics.

An extensive and updated account of stability in porous media has been given by Nield and Bejan [6]. Sharma and Rana [7] have studied the thermal instability of Walters' (model B') visco-elastic fluid in the presence of a variable gravity field and rotation in porous medium and found that the rotation has a stabilizing effect as gravity increases and a destabilizing effect as gravity decreases. Sharma *et al.* [8] have considered the thermosolutal instability of Walters' (model B') rotating fluid in porous medium whereas the Rayleigh-Taylor instability of Walters' (model B') visco-elastic fluid through porous medium has been studied by Sharma *et al.* [9]. In thermal and thermosolutal convection problems, the Boussinesq approximation has been used, which is well justified in the case of incompressible fluids.

When the fluids are compressible, the equations governing the system become more complicated. Spiegel and Veronis [10] assume that the depth of the fluid layer is much smaller than the scale height as defined by them, if only motions of infinitesimal amplitude are considered and simplified the set of equations governing the flow of compressible fluids under the above assumptions. The problem of thermal instability of a compressible fluid in the presence of rotation and magnetic field has been studied by Sharma [11]. Sunil *et al.* [12] have discussed the problem of a compressible couple stress fluid permeated with suspended particles heated and soluted from below in a porous medium and found that the stable solute gradient and couple stresses have stabilizing effects whereas the suspended particles and medium permeability have destabilizing effect on the system.

In a recent study, Kumar *et al.* [13] have studied the thermal convection in a Walters' (model B') elastico-viscous dusty fluid in hydromagnetics with the effect of compressibility and rotation. Kumar *et al.* [14] has investigated the thermal instability of Walters' B' viscoelastic fluid permeated with suspended particles in hydromagnetics in porous medium. Sharma and Aggarwal [15] have studied the effect of compressibility and suspended particles on thermal stability in a Walters' (model B') visco-elastic fluid in hydromagnetics and found that compressibility and magnetic field has a stabilizing effect on the thermal stability. Rana and Kango [16] have considered the thermal instability of compressible Walters' (model B') rotating fluid in the presence of suspended particles and porous medium. Gupta and Aggarwal [17] have studied the thermal instability of compressible Walters' fluid in the presence of Hall currents and suspended particles. Kumar [18] have studied the instability streaming Walters' (model B') fluids in porous medium in hydromagnetics. Gupta *et al.* [19] have seen thermal convection of dusty compressible Rivlin-Ericksen viscoelastic fluid in the presence of Hall currents. Aggarwal and Makhija [20] have studied the combined effect of magnetic field and rotation on thermal stability of couple-stress fluid heated from below in presence of suspended particles. Motivated by the fact that knowledge regarding fluid particle mixture is not commensurate with their industrial and scientific importance of magnetic field, rotation and porous medium in many geophysical and astrophysical situations, we further extended the results reported by Gupta and Aggarwal [17] to include the effect of magnetic field, rotation and porous medium but in the absence of Hall currents for visco-elastic Walters' fluid on thermal stability.

Formulation of the problem

Consider an infinite horizontal layer of compressible, electrically conducting Walters' B' visco-elastic fluid layer of thickness d permeated with suspended particles, bounded by the planes $z = 0$ and $z = d$ in a porous medium which is heated from below so that, a uniform temperature gradient $\beta (= |dT/dz|)$ is maintained. The fluid is acted on by a uniform rotation

$\bar{\Omega}(0,0,\Omega)$, a gravity force $\bar{g}(0,0,-g)$, and a uniform magnetic field $\bar{H}(0,0,H)$. The equations of motion and continuity for a compressible Walters' B' viscoelastic fluid permeated with suspended particles in the presence of magnetic field and rotation in porous medium are:

$$\frac{1}{\varepsilon} \left[\frac{\partial \bar{q}}{\partial t} + (\bar{q} \nabla) \bar{q} \right] = -\frac{1}{\rho_0} \nabla p - \bar{g} \left(1 + \frac{\delta \rho}{\rho_0} \right) \bar{\lambda} - \frac{1}{k_1} \left(v - v' \frac{\partial}{\partial t} \right) \bar{q} + \frac{\mu_e}{4\pi\rho_0} (\nabla \times \bar{H}) \times \bar{H} + \frac{KN_0}{\rho_0\varepsilon} (\bar{q}_d - \bar{q}) + \frac{2}{\varepsilon} (\bar{q} \times \bar{\Omega}) \quad (1)$$

$$\nabla \cdot \bar{q} = 0 \quad (2)$$

where $p, \rho, T, \bar{q}(u, v, w), \bar{q}_d(\bar{x}, t), N(\bar{x}, t), v,$ and v' denote fluid pressure, density, temperature, fluid velocity, suspended particles velocity, suspended particles number density, kinematic viscosity and kinematic viscoelasticity, respectively. Here symbol ε is the medium porosity, k_1 – the medium permeability, $\bar{g}(0,0,-g)$ – acceleration due to gravity, $\bar{x} = (x, y, z), \bar{\lambda} = (0,0,1), K = 6\pi\mu\eta',$ and η' being particle radius, is the Stoke's drag coefficient. Assuming a uniform particle size, a spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term in the equation of motion (1), proportional to the velocity difference between the particles and the fluid.

Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particle on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. The effects due to pressure, gravity, Darcy's force and magnetic field on the particles are small and so are ignored. If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles, under the mentioned assumptions are:

$$mN \left\{ \frac{\partial \bar{q}_d}{\partial t} + \frac{1}{\varepsilon} (\bar{q}_d \nabla) \bar{q}_d \right\} = KN(\bar{q} - \bar{q}_d) \quad (3)$$

$$\varepsilon \frac{\partial N}{\partial t} + \nabla(N\bar{q}_d) = 0 \quad (4)$$

If $C_v, C_{pb}, T,$ and q' denote the heat capacity of fluid at constant volume, heat capacity of the particles, temperature and effective thermal conductivity of the pure fluid, respectively. Assuming that the particles and the fluid are in thermal equilibrium, the equation of heat conduction gives:

$$[\rho_0 C_v \varepsilon + \rho_s C_s (1 - \varepsilon)] \frac{\partial T}{\partial t} + \rho_0 C_v (\bar{q} \nabla) T + mN C_{pb} \left(\varepsilon \frac{\partial}{\partial t} + \bar{q}_d \nabla \right) T = q' \nabla^2 T \quad (5)$$

The Maxwell's equations yield:

$$\varepsilon \frac{\partial \bar{H}}{\partial t} = (\bar{H} \nabla) \bar{q} + \varepsilon \eta \nabla^2 \bar{H} \quad (6)$$

$$\nabla \bar{H} = 0 \quad (7)$$

where η stands for the electrical resistivity.

The equation of state for the fluid is:

$$\rho = \rho_0 [1 - \alpha(T - T_0)] \quad (8)$$

The basic motionless solution is:

$$\bar{q} = (0, 0, 0), \quad \bar{q}_d = (0, 0, 0), \quad T = T_0 - \beta z, \quad \rho = \rho_0(1 + \alpha\beta z), \quad N = N_0 = \text{constant} \quad (9)$$

where α is the coefficient of thermal expansion and the subscript zero refers to values at the reference level $z = 0$. The kinematic viscosity ν , kinematic viscoelasticity ν' , electrical resistivity η and coefficient of thermal expansion α are all assumed to be constants. Here $E = \varepsilon + (1 - \varepsilon)(\rho_s c_s / \rho_0 c_i)$ is a constant. ρ_s , c_s , and ρ_0 , c_i stand for density and specific heat of solid (porous matrix) material and fluid, respectively.

Perturbation equations

Assume small perturbations around the basic solution and let δp , $\delta \rho$, θ , and $h(h_x, h_y, h_z)$ denote, respectively, the perturbations in fluid pressure p , density ρ , temperature T , and magnetic field \bar{H} . The change in density $\delta \rho$ caused mainly by the perturbation θ in temperature is given by:

$$\delta \rho = -\alpha \rho_0 \theta \quad (10)$$

Then the linearized perturbation equations of Walters' B' viscoelastic fluid become:

$$\frac{1}{\varepsilon} \frac{\partial \bar{q}}{\partial t} = -\frac{1}{\rho_0} \nabla \delta p + g \alpha \theta \bar{\lambda} - \frac{1}{k_1} \left(\nu - \nu' \frac{\partial}{\partial t} \right) \bar{q} + \frac{\mu_e}{4\pi \rho_0} (\nabla \times \bar{h}) \times \bar{H} + \frac{KN_0}{\rho_0} (\bar{q}_d - \bar{q}) + \frac{2}{\varepsilon} (\bar{q} \times \bar{\Omega}) \quad (11)$$

$$\nabla \bar{q} = 0 \quad (12)$$

$$(E + h\varepsilon) \frac{\partial \theta}{\partial t} = \left(\beta - \frac{g}{C_p} \right) (w + h_s) + \kappa \nabla^2 \theta \quad (13)$$

$$\varepsilon \frac{\partial \bar{h}}{\partial t} = (\bar{H} \nabla) \nu + \varepsilon \eta \nabla^2 \bar{h} \quad (14)$$

$$\nabla \bar{h} = 0 \quad (15)$$

$$mN_0 \frac{\partial \bar{q}_d}{\partial t} = KN_0 (\bar{q} - \bar{q}_d) \quad (16)$$

Writing the scalar components of eq. (11), after elimination of \bar{q}_d with the help of eq. (16), and eliminating u , v , h_x , h_y , δp between them by using eqs. (10), (12), and (15), we obtain:

$$n' (\nabla^2 w) + \frac{\varepsilon}{k_1} \left(\nu - \nu' \frac{\partial}{\partial t} \right) \nabla^2 w - \varepsilon g \alpha \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - \frac{\mu_e \varepsilon H}{4\pi \rho_0} \frac{\partial}{\partial x} (\nabla^2 h_z) + 2\Omega \frac{\partial \zeta}{\partial z} = 0 \quad (17)$$

where $n' = \frac{\partial}{\partial t} 1 + \left[(mN_0 K / \rho_0) / (m \frac{\partial}{\partial t} + K) \right]$.

The z - component of eq. (14) yields:

$$\varepsilon \left(\frac{\partial}{\partial t} - \eta \nabla^2 \right) h_z = H \frac{\partial w}{\partial z} \quad (18)$$

Equation (13), on substituting for s in terms of w with the help of eq. (16) yields:

$$\left(\frac{m}{K} \frac{\partial}{\partial t} + 1 \right) \left[(E + h\varepsilon) \frac{\partial}{\partial t} - \kappa \nabla^2 \right] \theta = \left(\beta - \frac{g}{C_p} \right) \left[\frac{m}{K} \frac{\partial}{\partial t} + 1 + h \right] w \quad (19)$$

Dispersion relation

Here we analyze the disturbances into normal modes and assume that the perturbation quantities are of the form:

$$[w, \theta, \xi, \zeta, h_z] = [W(z), \Theta(z), Z(z), X(z), K(z)] \exp(ik_x x + ik_y y + nt) \tag{20}$$

where k_x, k_y are wave numbers along x- and y-directions, respectively, $k = [(k_x^2 + k_y^2)^{-1/2}]$ is the resultant wave number of the disturbances, and n is the growth rate. Using eq. (20), eqs. (17)-(19) in non-dimensional form become:

$$(D^2 - a^2) \left[\frac{\sigma}{\varepsilon} \left(1 + \frac{f}{1 + p_1 \sigma \tau} \right) - \frac{(1 - F \sigma)}{P_l} (D^2 - a^2) \right] \cdot W - \frac{\mu_e H d}{4\pi \rho_0 \nu} (D^2 - a^2) DK + \frac{g \alpha d^2}{\nu} a^2 \Theta + \frac{2 \Omega d^3}{\nu \varepsilon} DZ = 0 \tag{21}$$

$$\left[\frac{1 - F \sigma}{P_l} (D^2 - a^2) - \sigma \left(1 + \frac{f}{1 + p_1 \sigma \tau} \right) \right] Z = \frac{\mu_e H d}{4\pi \rho_0 \nu} DX + \frac{2 \Omega d}{\nu \varepsilon} DW \tag{22}$$

$$(D^2 - a^2 - p_2 \sigma) K = - \left(\frac{H d}{\varepsilon \eta} \right) DW \tag{23}$$

$$(D^2 - a^2 - p_2 \sigma) X = - \left(\frac{H d}{\varepsilon \eta} \right) DZ \tag{24}$$

$$(1 + p_1 \sigma \tau) (D^2 - a^2 - E p_1 \sigma) \Theta = - \frac{\beta d^2}{\kappa} \left(\frac{G - 1}{G} \right) (H_1 + p_1 \sigma \tau) W \tag{25}$$

where $a = kd$, $\sigma = nd^2/\nu$, $p_1 = (\nu/\kappa)$ is the Prandtl number, $p_1 = (\nu/\eta)$ – the magnetic Prandtl number, $F = (\nu/d^2)$ – the dimensionless kinematic viscoelasticity, $G = (C_p \beta/g)$ – the dimensionless compressibility, $h = (m N_0 C_p / \rho_0 C_v)$, $f = (m N_0 / \rho_0)$, $\tau = (m \kappa / k d^2)$. We have expressed the co-ordinators x, y , and z in new units of length d , time t in the new unit of length (d^2/κ) , and let $H_1 = 1 + h_1$, $\tau_1 = (\tau \nu / d^2)$, $x^* = (x/d)$, $y^* = (y/d)$, $z^* = (z/d)$, and $D = (d/dz^*)$. Stars (*) have been omitted hereafter, for convenience.

Eliminating Θ, K , and Z between eqs. (21)-(25), we obtain:

$$(D^2 - a^2) \left[\frac{\sigma}{\varepsilon} \left(1 + \frac{f}{1 + p_1 \sigma \tau} \right) - \frac{1 - F \sigma}{P_l} (D^2 - a^2) \right] W + \frac{Q(D^2 - a^2)}{(D^2 - a^2 - p_2 \sigma)} D^2 W - \text{Ra}^2 \left(\frac{G - 1}{G} \right) \left[\frac{H_1 + p_1 \sigma \tau}{(1 + p_1 \sigma \tau)(D^2 - a^2 - E p_1 \sigma)} \right] W + {}_{+T_A} \left[\frac{1 - F \sigma}{P_l} (D^2 - a^2) - \sigma \left(1 + \frac{f}{1 + p_1 \sigma \tau} \right) + Q \frac{D^2}{D^2 - a^2 - p_2 \sigma} \right]^{-1} D^2 W = 0 \tag{26}$$

Here $Q = (\mu_e H^2 d^2 / 4\pi \rho_0 \eta)$ is the Chandrasekhar number and $R = (g \alpha \beta d^4 / \nu \kappa)$ is the Rayleigh number.

Consider the case in which both the boundaries are free, the medium adjoining the fluid is perfectly conducting and temperatures at the boundaries are kept fixed. The appropriate boundary conditions, for which eqs. (21)-(25) must be solved, are:

$$W = D^2W = 0, \quad DZ = 0, \quad \Theta = 0, \quad \text{at } z = 0, 1; \quad DX = 0, \quad K = 0 \quad (27)$$

on a perfectly conducting boundary.

Using the boundary conditions, it can be shown that all even order derivatives of W must vanish for $z = 0$ and the proper solution of W characterizing the lowest mode is:

$$W = W_0 \sin \pi z \quad (28)$$

where W_0 is a constant.

Substituting the proper solution $W = W_0 \sin \pi z$ in eq. (26), we obtain the dispersion relation:

$$\begin{aligned} (1+x) \left[\frac{i\sigma_1}{\varepsilon} \left(1 + \frac{f}{1+ip_1\sigma\tau} \right) + \frac{1-i\pi^2 F\sigma_1}{P} \right] W - Q_1 \frac{1+x}{1+x+ip_2\sigma_1} W - \\ - R_1 x \frac{G-1}{G} \left(\frac{H_1 + i\pi^2 p_1\sigma_1\tau}{(1+i\pi^2 p_1\sigma_1\tau)(1+x+iEp_1\sigma_1)} \right) W + \\ + T_{A_1} \left[\frac{1-i\pi^2 F\sigma_1}{P} - \frac{i\sigma_1}{\varepsilon} \left(1 + \frac{f}{1+i\pi^2 p_1\sigma_1\tau} \right) + \frac{Q_1}{1+x+i\pi^2 p_2\sigma_1} \right]^{-1} W = 0 \end{aligned} \quad (29)$$

where $Q_1 = Q/\pi^2$, $R_1 = R/\pi^4$, $T_{A_1} = (2\Omega d^2 / \nu \pi^2)^2$, $i\sigma_1 = \sigma/\pi^2$, $x = a^2/\pi^2$, $P = \pi^2 P_1$.

Equation (29) is the required dispersion relation including the effects of compressibility, rotation, magnetic field, suspended particles, and medium permeability. This dispersion relation is identical with that derived by Gupta and Aggarwal [17] in the absence of magnetic field, rotation, and porous medium but in the presence of Hall currents. In the absence of magnetic field, this dispersion relation is identical with that derived by Rana and Kango [16]. Also in the absence of rotation and porous medium, eq. (29) is same as derived by Sharma and Aggarwal [15] whereas in the absence of porous medium, eq. (29) reduces to the one derived by Kumar *et al.* [13].

The stationary convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (29) reduces to:

$$R_1 = \frac{G}{G-1} \frac{1+x}{xH_1} \left[Q_1 + \frac{(1+x)^2}{P} + T_{A_1} \frac{P(1+x)}{(1+x)^2 + PQ_1} \right] \quad (30)$$

which expresses the modified Rayleigh number R_1 as a function of dimensionless wave number x and the parameters G , H_1 , Q_1 , P , and T_{A_1} . We thus find that for stationary convection the viscoelastic parameter F vanishes with σ_1 and the visco-elastic Walters' B' fluid behaves like an ordinary Newtonian fluid. In the absence of magnetic field, rotation, and porous medium eq. (30) reduces to:

$$R_1 = \frac{G}{G-1} \frac{1+x}{xH_1} \left[(1+x)^2 + \frac{Q_1[(1+x)^2 + Q_1]}{(1+x)(1+x+M) + Q_1} \right]$$

which is identical with the expression for R_1 derived by Gupta and Aggarwal [17] wherein thermal instability of a compressible Walters' fluid in the presence of Hall currents and suspended particles has been considered.

In the absence of magnetic field, the eq. (30) reduces to:

$$R_1 = \frac{G}{G-1} \frac{1+x}{xH_1} \left[\frac{(1+x)^2}{P} + T_{A_1} \frac{P}{1+x} \right]$$

which is identical with the expression for R_1 derived by Rana and Kango [16] wherein thermal instability of a compressible Walters' B' rotating fluid in porous medium is studied in the presence of suspended particles. In the absence of rotation and porous medium, eq. (30) reduces to:

$$R_1 = \frac{G}{G-1} \frac{1+x}{xH_1} [Q_1 + (1+x)^2]$$

which is same as given by Sharma and Aggarwal [15]. In the absence of porous medium eq. (30) reduces to:

$$R_1 = \frac{G}{G-1} \frac{1+x}{xH_1} \left\{ Q_1 + (1+x)^2 + T_{A_1} \left[\frac{(1+x)}{(1+x)^2 + Q_1} \right] \right\}$$

which is identical to the one derived by Kumar *et al.* [13].

For stationary convection, eq. (30) implies that the compressible Walters' visco-elastic fluid (model B') behaves like an ordinary Newtonian fluid.

To study the effects of suspended particles, magnetic field, rotation, medium permeability, and compressibility, we examine the nature of dR_1/dH_1 , dR_1/dQ_1 , dR_1/dT_{A_1} , dR_1/dP , and dR_1/dG , analytically.

Firstly, to investigate the effect of suspended particles, we examine the nature of dR_1/dH_1 . Equation (30) yields:

$$\frac{dR_1}{dH_1} = - \frac{G}{G-1} \frac{1+x}{xH_1^2} \cdot \left[Q_1 + \frac{(1+x)^2}{P} + T_{A_1} \frac{P(1+x)}{(1+x)^2 + PQ_1} \right] \quad (31)$$

The negative sign implies that, for a stationary convection, the suspended particles have destabilizing effect on the system. This is in agreement with the results of fig. 1, where, Rayleigh number R_1 is plotted against suspended particles parameter, H_1 for different values of wave numbers x ($= 2, 4, 6, 8, 10$), and H_1 ($= 100, 200, \dots, 600$). In this figure the value of H_1 increases with the decrease in Rayleigh number implying that the suspended particles have destabilizing effect on the system.

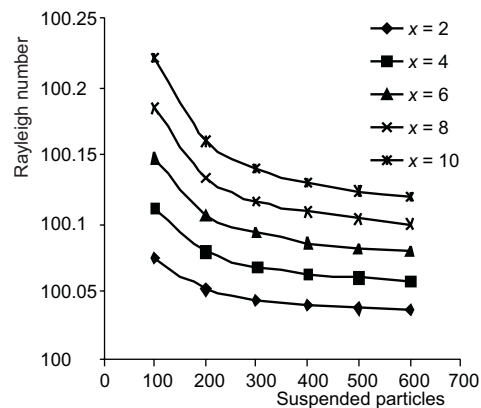


Figure 1. Variation of R_1 with H_1 for fixed $Q_1 = 100$, $T_{A_1} = 1$, $G = 100$, $P = 1$ for different values of x ($= 2, 4, 6, 8, 10$), and H_1 ($= 100, 200, \dots, 600$)

For analyzing the effect of magnetic field, we examine the nature of dR_1/dQ_1 . Equation (30) yields:

$$\frac{dR_1}{dQ_1} = \frac{G}{G-1} \frac{1+x}{xH_1} \left[1 - T_{A_1} \frac{P^2(1+x)}{[(1+x)^2 + PQ_1]^2} \right] \quad (32)$$

which shows that magnetic field has stabilizing effect in the absence of rotation. In the presence of rotation, magnetic field will have destabilizing effect if:

$$T_{A_1} > \frac{(1+x + PQ_1)^2}{P^2(1+x)} \quad \text{and stabilizing effect if: } T_{A_1} < \frac{(1+x + PQ_1)^2}{P^2(1+x)}$$

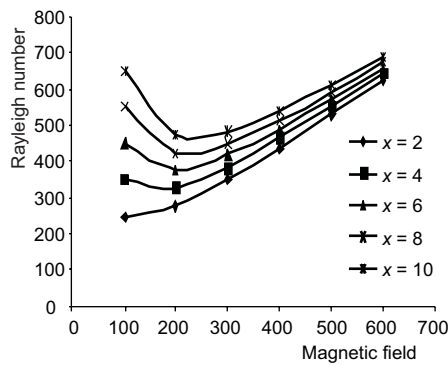


Figure 2. Variation of R_1 with Q_1 for fixed $H_1 = 1000, T_{A_1} = 5000, G = 2500, P = 2000$ for different values of $x (= 2, 4, 6, 8, 10)$, and $Q_1 (= 100, 200, \dots, 600)$

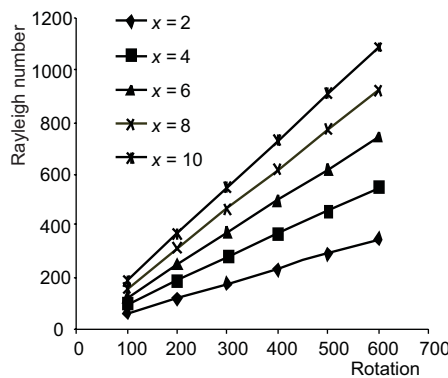


Figure 3. Variation of R_1 with T_{A_1} for fixed $Q_1 = 5, H_1 = 10, G = 20, P = 10$ for different values of $x (= 2, 4, 6, 8, 10)$, and $T_{A_1} (= 100, 200, \dots, 600)$

These analytical results are in agreement with fig. 2 numerically, for the permissible range of values of various parameters, where Rayleigh number R_1 is plotted against magnetic field parameter Q_1 for different values of wave numbers $x (= 2, 4, 6, 8, 10)$, and $Q_1 (= 100, 200, \dots, 600)$.

To study the effect of rotation, we examine the nature of dR_1/dT_A . From eq. (30), we obtain:

$$\frac{dR_1}{dT_{A_1}} = \frac{G}{G-1} \frac{1+x}{xH_1} \frac{P(1+x)}{(1+x)^2 + PQ_1} \quad (33)$$

which reflects the stabilizing effect of rotation parameter on the system. Also in fig. 3, Rayleigh number is plotted with respect to rotation parameter T_A for different values of $x (= 2, 4, 6, 8, 10)$, and $T_A (= 100, 200, \dots, 600)$ where T_A increases with the increase in Rayleigh number which implies the stabilizing effect on the system.

For analyzing the effect of permeability, we examine the nature of dR_1/dP . Equation (30) yields:

$$\frac{dR_1}{dP} = -\frac{1}{P^2} \frac{G}{G-1} \frac{(1+x)^2}{xH_1} \cdot \left[1 - T_{A_1} \frac{P^2(1+x)}{[(1+x)^2 + PQ_1]^2} \right] \quad (34)$$

It is clear from eq. (34) that, for stationary convection, the medium permeability has destabilizing effect on the system in the absence of rotation. In the presence of rotation, it has stabilizing effect if $T_{A_1} > [(1+x + PQ_1)^2]/[P^2(1+x)]$, and stabilizing effect if $T_{A_1} < [(1+x + PQ_1)^2]/[P^2(1+x)]$ which is in agreement with fig. (4) where Rayleigh number R_1 is plotted

against permeability P for different values of x ($= 2, 4, 6, 8, 10, 12, 14$), and P ($= 20, 40, \dots, 200$).

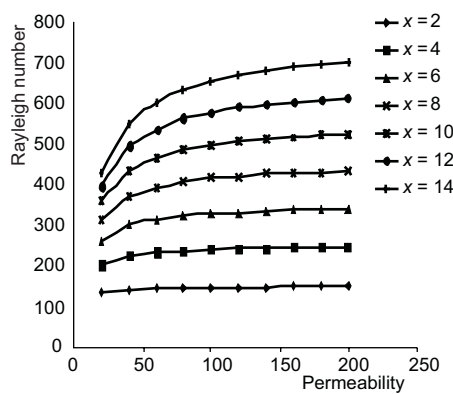


Figure 4. Variation of R_1 with P for fixed $Q_1 = 1$, $T_{A_1} = 50$, $G = 50$, $H_1 = 1$ for different values of x ($= 2, 4, 6, 8, 10, 12, 14$), and P ($= 20, 40, \dots, 200$)

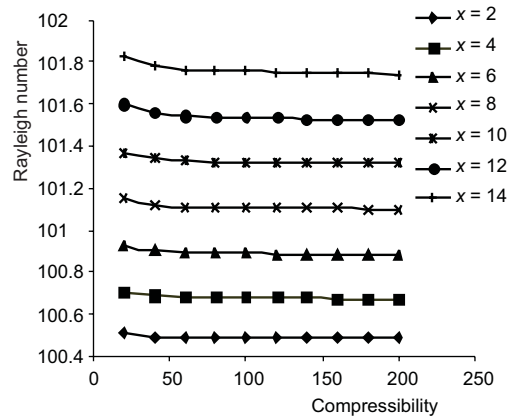


Figure 5. Variation of R_1 with G for fixed $Q_1 = 200$, $T_{A_1} = 2$, $P = 1$, $H_1 = 10$ for different values of x ($= 2, 4, 6, 8, 10$), and P ($= 20, 40, \dots, 200$)

The effect of compressibility can be studied by examining the nature of dR_1/dG , eq. (30) yields;

$$\frac{dR_1}{dG} = -\frac{1}{(G-1)^2} \frac{1+x}{xH_1} \left[Q_1 + \frac{(1+x)^2}{P} + T_{A_1} \frac{P(1+x)}{(1+x)^2 + PQ_1} \right] \quad (35)$$

It is further evident from eq. (35) that compressibility always has destabilizing effect on the system. Figure (5) confirms the above result numerically for the permissible range of values of various parameters.

Stability of system and oscillatory modes

Multiplying eq. (21) by W^* , the complex conjugate of W , integrating over the range of z and using eqs. (22)-(25) together with the boundary conditions (27), we obtain:

$$\begin{aligned} & \frac{\sigma}{\varepsilon} \left(1 + \frac{f}{1+p_1\sigma\tau} \right) I_1 + \frac{1-F\sigma}{P_l} I_2 - \frac{\mu_e \varepsilon \eta}{4\pi\rho_0\nu} (I_3 + p_2\sigma^* I_4) - \\ & - \frac{g\alpha\kappa a^2}{\beta\nu} \frac{G}{G-1} \frac{1+p_1\sigma^*\tau}{H_1 + p_1\sigma^*\tau} (I_5 + Ep_1\sigma^* I_6) + \\ & + d^2 \left[\frac{1-F\sigma^*}{P_l} I_7 + \sigma^* \left(1 + \frac{f}{1+p_1\sigma^*\tau} \right) I_8 + \frac{\mu_e \varepsilon \eta d^2}{4\pi\rho_0\nu} (I_9 + p_2\sigma^* I_{10}) \right] = 0 \end{aligned} \quad (36)$$

where

$$I_1 = \int_0^1 (|DW|^2 + a^2 |W|^2) dz, \quad I_2 = \int_0^1 (|D^2W|^2 + 2a^2 |DW|^2 + a^4 |W|^2) dz,$$

$$\begin{aligned}
I_3 &= \int_0^1 \left(|D^2 K|^2 + 2a^2 |DK|^2 + a^4 |K|^2 \right) dz, & I_4 &= \int_0^1 \left(|DK|^2 + a^2 |K|^2 \right) dz, \\
I_5 &= \int_0^1 \left(|D\Theta|^2 + a^2 |\Theta|^2 \right) dz, & I_6 &= \int_0^1 \left(|\Theta|^2 \right) dz, & I_7 &= \int_0^1 \left(|DZ|^2 + a^2 |Z|^2 \right) dz, \\
I_8 &= \int_0^1 \left(|Z|^2 \right) dz, & I_9 &= \int_0^1 \left(|DX|^2 + a^2 |X|^2 \right) dz, & I_{10} &= \int_0^1 \left(|DX|^2 \right) dz
\end{aligned} \quad (37)$$

and σ^* is the complex conjugate of σ . The integrals I_1, I_2, \dots, I_{10} are all positive definite.

Putting $\sigma = i\sigma_i$, and equating imaginary parts of eq. (37), we obtain:

$$i\sigma_i \left\{ \begin{aligned} & \frac{1}{\varepsilon} \left(1 + \frac{f}{1 + p_1^2 \sigma_i^2 \tau} \right) I_1 - \frac{F}{P_1} I_2 - \frac{\mu_e \varepsilon \eta}{4\pi \rho_0 \nu} p_2 I_4 + \\ & + \frac{g\alpha\kappa a^2}{\beta\nu} \frac{G}{G-1} \left[\frac{\tau(H_1 - 1)I_5 + (H_1 + p_1^2 \sigma_i^2 \tau^2)H_1 p_1 I_6}{H_1 + p_1^2 \sigma_i^2 \tau^2} \right] + \\ & + d^2 \left[\frac{F}{P_1} I_7 - \left(1 + \frac{f}{1 + p_1^2 \sigma_i^2 \tau} \right) I_8 \right] - \frac{\mu_e \varepsilon \eta d^2}{4\pi \rho_0 \nu} p_2 I_{10} \end{aligned} \right\} = 0 \quad (38)$$

Equation (38) yields that $\sigma_i = 0$ or $\sigma_i \neq 0$, which means that modes may be non-oscillatory or oscillatory. In the absence of viscoelasticity, magnetic field, and rotation, eq. (38) reduces to:

$$i\sigma_i \left\{ \frac{1}{\varepsilon} \left(1 + \frac{f}{1 + p_1^2 \sigma_i^2 \tau} \right) I_1 + \frac{g\alpha\kappa a^2}{\beta\nu} \frac{G}{G-1} \left[\frac{\tau(H_1 - 1)I_5 + (H_1 + p_1^2 \sigma_i^2 \tau^2)H_1 p_1 I_6}{H_1 + p_1^2 \sigma_i^2 \tau^2} \right] \right\} = 0 \quad (39)$$

and the quantity inside the brackets is positive definite. Thus $\sigma_i = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is valid. The viscoelasticity, magnetic field and rotation introduce oscillatory modes (as σ_i may not be zero) into the systems which were non-existent in their absence. This result is in agreement with the result derived by Kumar *et al.* [14] wherein the effect of suspended particles and magnetic field is studied on thermal instability of Walters' fluid in the presence of porous medium.

Conclusions

In case of stationary convection, we have investigated the effects of compressibility, rotation and magnetic field on thermal stability of visco-elastic fluid permeated with suspended particles in porous medium. The principal conclusions from the analysis are as follows:

- The compressible Walters' (model B') visco-elastic fluid behaves like an ordinary Newtonian fluid due to vanishing of the viscoelastic parameter.
- Compressibility has always destabilizing effect on the system which is evident from eq. (35), and is supported by fig. 5.

- The suspended particles have destabilizing effect on the system which is evident from eq. (31), and it is cleared from fig. 1.
- From eq. (33), it is clear that, rotation has stabilizing effect on the system which is in agreement with fig. 2.
- It is evident from eq. (32) that, magnetic field has stabilizing effect on the system in the absence of rotation whereas in the presence of rotation, it has destabilizing effect if $T_{A_1} > [(1 + x + PQ_1)^2]/[P^2(1 + x)]$, and stabilizing effect if $T_{A_1} < [(1 + x + PQ_1)^2]/[P^2(1 + x)]$ which is supported by fig. 3.
- It is observed from eq. (34) that the medium permeability has destabilizing effect in the absence of rotation whereas in the presence of rotation it has stabilizing effect if $T_{A_1} > [(1 + x + PQ_1)^2]/[P^2(1 + x)]$, and destabilizing effect if $T_{A_1} < [(1 + x + PQ_1)^2]/[P^2(1 + x)]$, which is in agreement with fig. 4.
- The principle of exchange of stabilities is found to hold true in the absence of viscoelasticity, magnetic field and rotation which means that oscillatory modes are introduced due to the presence of viscoelasticity, magnetic field and rotation which were non-existent in their absence.

Nomenclature

C_{pt} – heat capacity of particles [$\text{Jkg}^{-1}\text{K}^{-1}$]
 C_s – heat capacity of the solid (porous matrix) material, [$\text{Jkg}^{-1}\text{K}^{-1}$]
 C_v – heat capacity of fluid at constant volume, [$\text{Jkg}^{-1}\text{K}^{-1}$]
 d – depth of layer, [m]
 F – dimensionless kinematic viscoelasticity, [–]
 \bar{g} – acceleration due to gravity, [ms^{-2}]
 \vec{H} – magnetic field vector, [G]
 K – Stokes' drag coefficient, [kgs^{-1}]
 k – wave number, [m^{-1}]
 k_x, k_y – components of wave number k along x-axis, and y-axis, [m^{-1}]
 k_1 – medium permeability, [m^2]
 m – mass of single particle, [kg]
 N – suspended particle number density, [m^{-3}]
 n – growth rate, [s^{-1}]
 p – fluid pressure, [Pa]
 p_1 – Prandtl number, [–]
 p_2 – magnetic Prandtl number, [–]
 Q – dimensionless Chandrasekhar number, [–]

q' – effective thermal conductivity of pure fluid, [$\text{Wm}^{-1}\text{K}^{-1}$]
 \bar{q} – filter velocity, [ms^{-1}]
 \bar{q}_d – suspended particle velocity, [ms^{-1}]
 R_1 – dimensionless Rayleigh number, [–]
 T – temperature, [K]
 t – time, [s]
 x – dimensionless wave number, [–]

Greek symbols

α – coefficient of thermal expansion, [K^{-1}]
 β – uniform temperature gradient, [Km^{-1}]
 ε – medium porosity, [–]
 η – electrical resistivity, [m^2s^{-1}]
 η' – particle radius, [m]
 θ – perturbation in temperature, [K]
 κ – thermal diffusivity, [m^2s^{-1}]
 μ – dynamic viscosity [$\text{kgm}^{-1}\text{s}^{-1}$]
 μ_e – magnetic permeability, [H m^{-1}]
 ν – kinematic viscosity, [m^2s^{-1}]
 ν' – kinematic viscoelasticity, [m^2s^{-1}]
 ρ – density, [kg m^{-3}]

References

- [1] Joseph, D. D., *Stability of Fluid Motions*, vol. II, Springer-Verlag, Berlin, 1976
- [2] Chandrasekhar, S., *Hydrodynamic and Hydromagnetics Stability*, Dover Publication, New York, USA, 1981
- [3] Lapwood, E. R., Convection of a Fluid in a Porous Medium, *Proc. Camb. Phil. Soc.*, 44 (1948), 4, pp. 508-521
- [4] Wooding, R. A. Rayleigh Instability of a Thermal Boundary Layer in Flow through a Porous Medium, *J. Fluid Mech.*, 9 (1960), 2, pp. 183-192
- [5] Sunil, Sharma, A., et al., Effect of Magnetic Field Dependent Viscosity on Thermal Convection in a Ferromagnetic Fluid, *Chemical Engineering Communications*, 195 (2008), 5, pp. 571-583
- [6] Nield, D. A., Bejan, A., *Stability in Porous Medium*, Springer-Verlag, New York, 1999

- [7] Sharma, V., Rana, G. C., Thermal Instability of a Walters' B' Elastic-Viscous Fluid in the Presence of a Variable Gravity Field and Rotation in a Porous Medium, *J. Non Equilib. Thermodyn.*, 26 (2001), pp. 31-40
- [8] Sharma, R. C., Sunil Chand, S., Thermosolutal Instability of Walters' Rotating Fluid (model B') in Porous Medium, *Arch. Mech.*, 51 (1999), 2, pp. 181-191
- [9] Sharma R. C., et al., Rayleigh Taylor Instability of Walters' B' Visco-Elastic Fluid through Porous Medium, *Int. J. Appl. Mech. Engg.*, 7 (2002), pp. 433-44
- [10] Spiegel, E. A., Veronis, G., On the Boussinesq Approximation for a Compressible Fluid, *Astrophys. J.*, 131 (1960), pp. 442-444
- [11] Sharma, R. C., Thermal Instability in Compressible Fluid in the Presence of Rotation and Magnetic Field, *J. Math. Anal. Appl.*, 60 (1997), 1, pp. 227-235
- [12] Sunil, D., et al., Compressible Couple-Stress Fluid Permeated with Suspended Particles Heated and Soluted from below in Porous Medium, *Indian J. of Pure Appl. Phys.*, 41 (2003), 8, pp. 612-620
- [13] Kumar, V., et al., Thermal Convection in a Walters' (model B') Elastic-Viscous Dusty Fluid in Hydromagnetics with the Effect of Compressibility and Rotation, *Int. J. Appl. Mech. Eng.*, 15 (2010), 1, pp. 51-62
- [14] Kumar, P., et al., Thermal Instability of Walters' B' Viscoelastic Fluid Permeated with Suspended Particles in Hydromagnetics in Porous Medium, *Thermal Science*, 18 (2004), 1, pp. 51-61
- [15] Sharma, R. C., Aggarwal A. K., Effect of Compressibility and Suspended Particles on Thermal Stability in a Walters' B' Visco-Elastic Fluid in Hydromagnetics, *Int. J. Appl. Mech. Eng.*, 11 (2006), 2, pp. 391-399
- [16] Rana, G. C., Kango, S. K., Thermal Instability of Compressible Walters' (model B') Rotating Fluid Permeated with Suspended Dust Particles in Porous Medium, *Advances in Appl. Sci. Research.*, 2 (2011), 3, pp. 586-597
- [17] Gupta, U., Aggarwal, P., Thermal Instability of Compressible Walters' B' Fluid in the Presence of Hall Currents and Suspended Particles, *Thermal Science*, 15 (2011), 2, pp. 487-500
- [18] Kumar, S., The Instability of Streaming Walters' (model B') Fluids in Porous Medium in Hydromagnetics, *Int. J. of Appl. Math and Mech.*, 7 (2011), 11, pp. 33-45
- [19] Gupta, U., et al., Thermal Convection of Dusty Compressible Rivlin Ericksen Viscoelastic Fluid with Hall Currents, *Thermal Science*, 16 (2012), 1, pp. 177-191
- [20] Aggarwal, A. K., Makhija, S., Combined Effect of Magnetic Field and Rotation on Thermal Stability of Couple-Stress Fluid Heated from Below in Presence of Suspended Particles, *International Journal of Applied Mechanics and Engineering*, 16 (2011), 4, pp. 931-942