STEADY THERMAL STRESS AND STRAIN RATES IN A ROTATING CIRCULAR CYLINDER UNDER STEADY STATE TEMPERATURE

by

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Thermal stress and strain rates in a thick walled rotating cylinder under steady state temperature have been derived by using Seth's transition theory. For elastic-plastic stage, it is seen that with the increase of temperature, the cylinder having smaller radii ratios requires lesser angular velocity to become fully plastic as compared to cylinder having higher radii ratios The circumferential stress becomes larger and larger with the increase in temperature. With increase of thickness ratio stresses must decrease. For the creep stage, it is seen that circumferential stresses for incompressible materials maximum at the internal surface as compared to compressible material, which increase with the increase in temperature and measure n.

Key words: *Elastic-plastic stress, Creep, Rotating cylinder, Compressibility*.

Introduction

For an ideally plastic material without strain-hardening, the stress distribution in solid rotating cylinder has been described by Nadai [1]. The additional of a central hole and the consideration of rigid-plastic material with linear strain hardening have been discussed by Devis and Connelly [2] using small strain theory. Rimrott [3] considered the hollow cylinder in the fully plastic state, using large strain theory and Mises yield condition. Hodge Jr. and Balaban [7] studied the elastic-plastic problem of a rotating cylinder and compare results obtained with finite and infinitesimal strain. In analyzing the problem, these authors used some simplifying assumptions. First, the deformations assumed to be small enough to make infinitesimal strain theory applicable. Second, simplifications were made regarding the constitutive equations of the material like incompressibility of the material and yield condition. Incompressibility of material is one of the most important assumptions simplifying the problem. In fact, in most of the case, it is not possible to find a solution in closed from without this assumption. The problem of creep in thick-walled rotating cylinder without thermal effects has been discussed by Rimrott and Luke [8] for large strains and Dev [9] for plain strain. In analyzing the problems these authors assumed incompressibility of the material, yield condition and power relationship between stress and strain. Incompressibility of the material in creep problems is an assumption which simplifies the problem. In fact it is not possible to find a solution in closed form with this assumption. Seth's transition theory

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does not require any of these assumptions. This theory gives the same results or rather more general results, without making un-necessary assumptions. By applying the concepts of Seth's transition theory, Gupta [11] analyzed the above problems with no thermal effects. Seth's transition theory utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deforming field and has been successfully applied to a large number of the problems in plasticity and creep. Seth [10] has defined the generalized principal strain measure as:

$$e_{ii} = \int_0^{\overset{A}{e}_{ii}} \left[1 - 2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ii} = \frac{1}{n}\left[1 - \left(1 - 2\overset{A}{e}_{ii}\right)^{\frac{n}{2}}\right], (i = 1,2,3) \quad (1.1)$$

where *'n'* is the measure and $\overset{A}{e}_{ii}$ are the Almansi finite strain components. In Cartesian framework we can rapidly write down the generalized measure in terms of any other measure. In terms of $\overset{A}{e}_{ii}$, the generalized principal strain components $\overset{M}{e}_{ii}$ are

$$\overset{M}{e}_{ii} = \left[\frac{1}{n}\left\{1 - \left(1 - 2\overset{A}{e}_{ii}\right)^{\frac{n}{2}}\right\}\right]^m \quad (1.2)$$

For uniaxial case it is given by:

$$e = \left[\frac{1}{n}\left\{1 - \left(\frac{l_0}{i}\right)^n\right\}\right]^m \quad (1.3)$$

where m is the irreversibility index and l_0 and l are the initial and strained lengths respectively. In this research article, we analyses the steady thermal stresses and strain in a rotating thick walled cylinder under steady state temperature.

Governing equations

Consider a thick walled rotating cylinder of internal and external radii a and b respectively, rotating about its axis with an angular speed ω of gradually increasing speed about its axis and subjected to a steady state temperature Θ on the internal surface at $r = a$. The components of displacements in cylindrical co-ordinates are given by Seth [10]:

$$u = r(1 - \beta); v = 0; w = dz \quad (2.1)$$

where β is position function, depending on $r = \sqrt{x^2 + y^2}$ only and d is a constant.

The generalized components of strain are given by Seth [8]:

$$e_{rr} = \frac{1}{n^m}\left[1 - \left(\beta + r\beta'\right)^n\right]^m, e_{\phi\phi} = \frac{1}{n^m}\left[1 - \beta^n\right]^m, e_{zz} = \frac{1}{n^m}\left[1 - (1-d)^n\right]^m,$$

$$e_{r\phi} = e_{\phi z} = e_{zr} = 0 \quad (2.2)$$

where $\beta' = d\beta/dr$. The thermo-elastic stress-strain relations for isotropic material are given by Parkus [12] and Fung [13]:

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij} - \xi\Theta\delta_{ij}, (i, j = 1, 2, 3) \quad (2.3)$$

where T_{ij} are the stress components, λ and μ are Lame's constants, $I_1=e_{kk}$ is the first strain invariant, δ_{ij} is the Kronecker's delta, $\xi=\alpha(3\lambda+2\mu), \alpha$ being the coefficient of thermal expansion, and Θ is the temperature. Further, Θ has to satisfy:

$$\nabla^2\Theta = 0 \tag{2.4}$$

Substituting eqn. (2.2) in eqn. (2.3), one gets:

$$T_{rr} = \frac{(\lambda+2\mu)}{n^m}\left[1-(r\beta'+\beta)^n\right]^m + \frac{\lambda}{n^m}\left[1-\beta^n\right]^m + \lambda k - \xi\Theta,$$

$$T_{\phi\phi} = T_{zz} = \frac{\lambda}{n^m}\left[1-(r\beta'+\beta)^n\right]^m + \frac{(\lambda+2\mu)}{n^m}\left[1-\beta^n\right]^m + \lambda k - \xi\Theta,$$

$$T_{zz} = \frac{\lambda}{n^m}\left[1-(r\beta'+\beta)^n\right]^m + \frac{\lambda}{n^m}\left[1-\beta^n\right]^m + (\lambda+2\mu)k - \xi\Theta$$

$$T_{r\phi} = T_{\phi z} = T_{zr} = 0 \tag{2.5}$$

where $\beta' = d\beta/dr$ and $k = \frac{1}{n^m}\left[1-(1-d)^n\right]^m$. Equations of motion are all satisfied, except:

$$\frac{d}{dr}(T_{rr}) + \frac{(T_{rr}-T_{\phi\phi})}{r} + \rho r\omega^2 = 0 \tag{2.6}$$

where ρ is the density of the material.

The temperature satisfying Laplace eqn. (2.4) with boundary condition: $\Theta=\Theta_0$ at $r = a$, $\Theta = 0$ at $r=b$, where Θ_0 is constant, given by:

$$\Theta = \frac{\Theta_0 \log r/b}{\log a/b} \tag{2.7}$$

Using eqs. (2.5) in eq. (2.6), one get a non- linear differential equation in β as:

$$P(P+1)^{n-1} n\frac{dP}{d\beta}\left[1-\beta^n(P+1)^n\right]^{m-1} + P(P+1)^n\left[1-\beta^n(P+1)^n\right]^{m-1}$$
$$+(1-c)P\left(1-\beta^n\right)^{m-1} + \frac{n^m c\xi\bar{\Theta}_0}{2\mu\beta^n mn} - \frac{c}{\beta^n mn}\left[\left\{1-\beta^n(P+1)^n\right\}^m - \left(1-\beta^n\right)^m\right] - \frac{c\rho\omega^2 r^2 n^m}{2\mu mn\beta^n} = 0 \tag{2.8}$$

where c is the compressibility factor of the material in term of Lame's constant, and are given by $c = 2\mu/\lambda+2\mu$, $r\beta' = \beta P, C = 2\mu/\lambda+2\mu$ and $\bar{\Theta}_0 = (\Theta_0)/(\log a/b)$.

For m = 1, which holds good for secondary stage of creep, the equation (2.8) reduces to:

$$nP(P+1)^{n-1}\beta\frac{dP}{d\beta} + nP(P+1)^n + (1-c)nP - \left[1-(P+1)^n\right]c + \frac{nc\xi\bar{\Theta}_0}{2\mu\beta^n} - \frac{nc\rho\omega^2 r^2}{2\mu\beta^n} = 0 \tag{2.9}$$

Transition points of β in eqn. (2.9) are $P \to -1$ and $P \to \pm\infty$. The boundary conditions are:

(i) $T_{rr} = 0$ *at* $r = a$ (2.10)

(ii) The resultant force normal to the plane z = constant must vanish: $\int_a^b rT_{zz}dr = 0$ (2.11)

Solution Though the Principal Stress

It has been shown [10-11, 14-15, 18-22] that the asymptotic solution through the principal stress leads from elastic to plastic state at the transition point $P \to \pm\infty$, we define the transition function R as:

$$R = 1 - \frac{n}{3\lambda+2\mu}\left[T_{rr} + c\xi\Theta + \frac{\rho\omega^2 r^2}{2}\right] = \left[(1-c)\beta^n + \beta^n (P+1)^n + \frac{n\xi\Theta(1-c)}{(\lambda+2\mu)} - \frac{n\rho\omega^2 r^2}{2(\lambda+2\mu)}\right] \quad (3.1)$$

Taking the logarithmic differentiation of equation (3.1) with respect to r and using equation (2.9), one gets:

$$\frac{d}{dr}(\log R) = \frac{n\beta^n}{r}\frac{\left[\frac{c}{n}\left\{1-(P+1)^n\right\} - \frac{c^2\xi\bar{\Theta}_0}{2\mu\beta^n}\right]}{\left[(1-c)\beta^n + \beta^n (P+1)^n + \frac{n\xi\Theta(1-c)}{(\lambda+2\mu)} - \frac{n\rho\omega^2 r^2}{2(\lambda+2\mu)}\right]} \quad (3.2)$$

Taking the asymptotic value of equation (3.2) as $P \to \pm\infty$, one gets: $\frac{d}{dr}(\log R) = -\frac{c}{r}$ (3.3)

Integrating of eq. (3.3), one gets: $R = Ar^{-c}$ (3.4)

where A is a constant of integration, which can be determined by the boundary condition. From equation (3.1) and (3.4), one gets:

$$T_{rr} = \frac{(3\lambda+2\mu)}{n}\left(1 - Ar^{-c}\right) - c\xi\Theta - \frac{\rho\omega^2 r^2}{2} \quad (3.5)$$

The value of E in the transition range is given by [11]:

$$Y = \frac{E}{n} = \frac{(3-2c)}{(2-c)}.\frac{2\mu}{n} \quad (3.6)$$

where Y is the yield stress in tension. Using equation (3.6) in (3.5), one gets

$$T_{rr} = \frac{(2-c)}{c}Y\left(1 - Ar^{-c}\right) - c\xi\Theta - \frac{\rho\omega^2 r^2}{2} \quad (3.7)$$

Substituting equations (3.7) and (2.6) gives:

$$T_{\phi\phi} - T_{rr} = (2-c)YAr^{-c} - c\xi\bar{\Theta}_0 \quad (3.8)$$

$$\Rightarrow \quad T_{\phi\phi} = \frac{(2-c)}{c}Y\left(1 - Ar^{-c}\right) + (2-c)YAr^{-c} - \frac{\rho\omega^2 r^2}{2} - c\xi\bar{\Theta}_0\left(\log\frac{r}{b} + 1\right) \quad (3.9)$$

Equation (2.5) give:

$$T_{zz} = \left(\frac{1-c}{2-c}\right)\left(T_{\phi\phi} + T_{rr}\right) + 2k_1 - \frac{c\xi\Theta}{2-c} \quad (3.10)$$

where $k_1 = \frac{\mu(3-2c)}{(2-c)}e_{zz}$.

By substituting boundary conditions (2.10-2.11) in eqs. (3.7), (3.10), one gets:

$$A = a^{c}\left[1-\frac{c^{2}\xi\Theta_{0}}{(2-c)Y}-\frac{\rho\omega^{2}a^{2}c}{2(2-c)Y}\right] \tag{3.11}$$

$$\frac{\rho\omega^{2}}{2}=\frac{c\xi\Theta b^{-c}-\frac{2-c}{c}Y\left(b^{-c}-a^{-c}\right)}{a^{-c}b^{2}-a^{2}b^{-c}} \tag{3.12}$$

and $$k_{1}=\frac{c\xi\bar{\Theta}_{0}}{(2-c)\left(b^{2}-a^{2}\right)}\left[\frac{a^{2}}{4}-\frac{b^{2}}{4}-\frac{a^{2}}{2}\log\frac{a}{b}\right]-\left(\frac{1-c}{2-c}\right)\frac{\rho\omega^{2}\left(a^{2}+b^{2}\right)}{4} \tag{3.13}$$

Substituting the values of constant of integration A from equation (3.11) in equations (3.7)- (3.10) respectively, one get the transitional stresses as:

$$T_{rr}=\frac{(2-c)}{c}Y\left(1-\left(\frac{a}{r}\right)^{c}\left\{1-\frac{c^{2}\xi\Theta_{0}}{(2-c)Y}-\frac{\rho\omega^{2}a^{2}c}{2(2-c)Y}\right\}\right)-c\xi\Theta-\frac{\rho\omega^{2}r^{2}}{2}\,, \tag{3.14}$$

$$T_{\phi\phi}=\frac{(2-c)}{c}Y\left(1-\left(\frac{a}{r}\right)^{c}\left\{1-\frac{c^{2}\xi\Theta_{0}}{(2-c)Y}-\frac{\rho\omega^{2}a^{2}c}{2(2-c)Y}(1-c)\right\}\right)-\frac{\rho\omega^{2}r^{2}}{2}-c\xi\bar{\Theta}_{0}\left(1-\log\frac{r}{b}\right),$$

$$T_{zz}=\left(\frac{1-c}{2-c}\right)\left(T_{\phi\phi}+T_{rr}\right)+\frac{2c\xi\bar{\Theta}_{0}}{(2-c)\left(b^{2}-a^{2}\right)}\begin{bmatrix}\frac{a^{2}}{4}-\frac{b^{2}}{4}\\-\frac{a^{2}}{2}\log\frac{a}{b}\end{bmatrix}-\left(\frac{1-c}{2-c}\right)\frac{\rho\omega^{2}\left(a^{2}+b^{2}\right)}{2}-\frac{c\xi\Theta}{2-c}\,.$$

$$T_{\phi\phi}-T_{rr}=(2-c)Y\left(\frac{a}{r}\right)^{c}\left\{1-\frac{c^{2}\xi\Theta_{0}}{(2-c)Y}-\frac{\rho\omega^{2}a^{2}c}{2(2-c)Y}\right\}-c\xi\bar{\Theta}_{0} \tag{3.15}$$

Initial Yielding: It is found that the values of $\left|T_{\phi\phi}-T_{rr}\right|$ is maximum at $r=a$, which means that the yielding of the cylinder will take place at the internal surface of the cylinder and equation (3.15) become:

$$\left|T_{\phi\phi}-T_{rr}\right|=(2-c)Y\left\{1-\frac{c^{2}\xi\Theta_{0}}{(2-c)Y}-\frac{\rho\omega^{2}a^{2}c}{2(2-c)Y}\right\}-c\xi\bar{\Theta}_{0}\equiv Y_{1}\ \text{ (say yielding)} \tag{3.16}$$

Substituting the value of Y in terms of Y_1 in eq. (3.12), one get a relation between ω and temperature Θ_0 as:

$$\frac{\rho\omega^{2}}{2}=\frac{1}{\left(a^{-c}b^{2}-a^{2}b^{-c}\right)}\left[c\xi\Theta_{0}b^{-c}-\frac{2-c}{c}Y\left(b^{-c}-a^{-c}\right)\left\{\frac{1}{(2-c)}\left(Y_{1}+c^{2}\xi\Theta_{0}+\frac{\rho\omega^{2}a^{2}c}{2}+c\xi\bar{\Theta}_{0}\right)\right\}\right] \tag{3.17}$$

Fully Plastic State: The stresses for fully plastic state are obtained by taking $(C\to 0)$ in eqs. (3.14)-(3.17) becomes:

$$\left|T_{\phi\phi}-T_{rr}\right|=2Y-2\alpha E\bar{\Theta}_{0}\,,$$

$$T_{rr} = \left(2Y - 2\alpha E\bar{\Theta}_0\right)\log\frac{r}{a} - \frac{\rho\omega^2\left(a^2 - r^2\right)}{2},$$

$$T_{\phi\phi} = \left(2Y - 2\alpha E\bar{\Theta}_0\right)\left(\log\frac{r}{a} + 1\right) - \frac{\rho\omega^2\left(a^2 - r^2\right)}{2},$$

$$T_{zz} = \left(2Y - 2\alpha E\bar{\Theta}_0\right)\left(2\log\frac{r}{a} + 1\right) +$$

$$+\frac{2\alpha E\bar{\Theta}_0}{\left(b^2 - a^2\right)}\left[\frac{a^2}{4} - \frac{b^2}{4} - \frac{a^2}{2}\log\frac{a}{b}\right] + \frac{\rho\omega^2\left(a^2 - b^2 - 2r^2\right)}{4} - \alpha E\Theta,$$

$$\frac{\rho\omega^2}{2} = \frac{\left(2Y - 2\alpha E\bar{\Theta}_0\right)}{\left(b^2 - a^2\right)}\log\frac{b}{a},\ k_1 = \frac{\alpha E\bar{\Theta}_0}{2\left(b^2 - a^2\right)}\left[\frac{a^2}{4} - \frac{b^2}{4} - \frac{a^2}{2}\log\frac{a}{b}\right] - \frac{\rho\omega^2\left(a^2 + b^2\right)}{8} \tag{3.18}$$

when there is no thermal effect i.e. $\Theta_0 = 0$, eq. (3.18) become:

$$\left|T_{\phi\phi} - T_{rr}\right| = 2Y \equiv \dot{Y},\ T_{rr} = \dot{Y}.\log\frac{r}{a} - \frac{\rho\omega^2\left(a^2 - r^2\right)}{2},$$

$$T_{\phi\phi} = \dot{Y}\left(\log\frac{r}{a} + 1\right) - \frac{\rho\omega^2\left(a^2 - r^2\right)}{2},$$

$$T_{zz} = \dot{Y}\left(2\log\frac{r}{a} + 1\right) + \frac{\rho\omega^2\left(a^2 - b^2 - 2r^2\right)}{4},$$

$$\frac{\rho\omega^2}{2} = \frac{\dot{Y}}{\left(b^2 - a^2\right)}\log\frac{b}{a},\ \ k_1 = -\frac{\rho\omega^2\left(a^2 + b^2\right)}{8} \tag{3.19}$$

Equation (3.19) are same as given by Gupta [11, 12] for hollow rotating cylinder.

Asymptotic Solution through $P \to -1$

For finding the creep stresses, the transitional function is through the principal stress difference (see Seth [10, 11], Hulsurkar [16], Gupta and Dharmani [15], Gupta and Pankaj [18, 22], Pankaj Thakur [14, 21]) at the transition point $P \to -1$.We define the transition function R as:

$$R = T_{rr} - T_{\theta\theta} \equiv \frac{2\mu}{n^m}\left[\left\{1 - \beta^n\left(P+1\right)^n\right\}^m - \left(1 - \beta^n\right)^m\right] \tag{4.1}$$

Taking the logarithmic differentiating of eq. (3.1) with respect to β, one get:

$$\frac{d}{d\beta}\left(\log R\right) = mn\beta^{n-1}\frac{\left[\left(1-\beta^n\right)^{m-1} - \left\{1 - \beta^n\left(P+1\right)^n\right\}^{m-1}\left\{\left(P+1\right)^n + \left(P+1\right)^{n-1}\beta\frac{dP}{d\beta}\right\}\right]}{\left[\left\{1 - \beta^n\left(P+1\right)^n\right\}^m - \left(1-\beta^n\right)^m\right]} \tag{4.2}$$

Substituting the value of $dP/d\beta$ from eqn. (2.8) in eqn. (4.2), one gets:

$$\frac{d}{d\beta}(\log R)=\frac{(2-c)\left(1-\beta^{n}\right)^{m-1}mn\beta^{n-1}}{\left[\left\{1-\beta^{n}(P+1)^{n}\right\}^{m}-\left(1-\beta^{n}\right)^{m}\right]}+\frac{c\xi\bar{\theta}_{0}n^{m}}{2\mu\beta P\left[\left\{1-\beta^{n}(P+1)^{n}\right\}^{m}-\left(1-\beta^{n}\right)^{m}\right]}$$
$$-\frac{c}{\beta P}-\frac{c\rho\omega^{2}r^{2}n^{m}}{2\mu\beta P\left[\left\{1-\beta^{n}(P+1)^{n}\right\}^{m}-\left(1-\beta^{n}\right)^{m}\right]} \tag{4.3}$$

The asymptotic value of eqn. (4.3) as $P\to$-1, is given by:

$$\frac{d}{d\beta}(\log R)=\frac{(2-c)\left(1-\beta^{n}\right)^{m-1}mn\beta^{n-1}}{\left[1-\left(1-\beta^{n}\right)^{m}\right]}-\frac{c\xi\bar{\theta}_{0}n^{m}}{2\mu\beta\left[1-\left(1-\beta^{n}\right)^{m}\right]}+\frac{c}{\beta}+\frac{c\rho\omega^{2}r^{2}n^{m}}{2\mu\beta\left[1-\left(1-\beta^{n}\right)^{m}\right]} \tag{4.4}$$

Integrating of eqn. (4.4) gives:

$$R=T_{rr}-T_{\phi\phi}=A_{0}r^{-c}\left[1-\left(1-\beta^{n}\right)^{m}\right]^{2-c}\exp\left(f_{1}+f_{2}\right) \tag{4.5}$$

where A_0 is a constant of integration and

$$f_{1}=\frac{-n^{m}c\rho\omega^{2}}{2\mu}\int\frac{rdr}{\left\{1-\left(1-\beta^{n}\right)^{m}\right\}},\quad f_{2}=\frac{c\xi\,\bar{\Theta}_{0}n^{m}}{2\mu}\int\frac{dr}{r\left\{1-\left(1-\beta^{n}\right)^{m}\right\}}.$$

From eq. (2.6) and (4.5), one get:

$$T_{rr}=-\frac{\rho\omega^{2}r^{2}}{2}-A_{0}\int r^{-c-1}\left[1-\left(1-\beta^{n}\right)^{m}\right]^{2-c}\exp\left(f_{1}+f_{2}\right)dr+A_{1} \tag{4.6}$$

where A_1 is a constant of integration, which can be determine by boundary condition.

The asymptotic value of β as $P\to$-1 is *D/r*; *D* being a constant, therefore eqn. (4.6) becomes:

$$T_{rr}=-\frac{\rho\omega^{2}r^{2}}{2}-A_{0}\int r^{-c-1}\left[1-\left(1-D^{n}r^{-n}\right)^{m}\right]^{2-c}\exp\left(f_{1}+f_{2}\right)dr+A_{1} \tag{4.7}$$

where $f_{1}=\frac{-n^{m}c\rho\omega^{2}}{2\mu}\int\frac{rdr}{\left\{1-\left(1-D^{n}r^{-n}\right)^{m}\right\}}$ and $f_{2}=\frac{c\xi\,\bar{\Theta}_{0}n^{m}}{2\mu}\int\frac{dr}{r\left\{1-\left(1-D^{n}r^{-n}\right)^{m}\right\}}$.

From eq. (2.5), one gets:

$$T_{zz}=\left(\frac{1-c}{2-c}\right)\left(T_{rr}-T_{\phi\phi}\right)+2K_{1}-\frac{c\xi\Theta}{(2-c)} \tag{4.8}$$

where $K_{1}=\frac{\mu(3-c)}{(2-c)}e_{zz}$

Using boundary conditions (2.10) and (2.11) in eq. (4.7) and in eq. (4.8), one gets:

$$A_0 = \frac{-\rho\omega^2\left(b^2 - a^2\right)}{2\int_a^b r^{-c-1}\left[1-\left(1-D^n r^{-n}\right)^m\right]^{2-c}\exp\left(f_1 + f_2\right)dr},$$

$$A_1 = \frac{\rho\omega^2 a^2}{2} - \frac{\rho\omega^2\left(b^2 - a^2\right)\left[\int r^{-c-1}\left[1-\left(1-D^n r^{-n}\right)^m\right]^{2-c}\exp\left(f_1 + f_2\right)dr\right]_{r=a}}{2\int_a^b r^{-c-1}\left[1-\left(1-D^n r^{-n}\right)^m\right]^{2-c}\exp\left(f_1 + f_2\right)dr},$$

and $$K_1 = \frac{c\xi\,\bar{\Theta}_0}{(2-c)\left(b^2-a^2\right)}\left[\frac{a^2}{4} - \frac{b^2}{4} - \frac{a^2}{2}\log\frac{a}{b}\right] - \left(\frac{1-c}{2-c}\right)\frac{\rho\omega^2}{4}\left(b^2 + a^2\right). \tag{4.9}$$

Substituting the value of constants A_0, A_1 and K_1 from equations (4.9) in eqs. (4.5), (4.7) and (4.8), one get the creep transitional stresses as:

$$T_{rr} = -\frac{\rho\omega^2\left(r^2 - a^2\right)}{2} + \frac{\rho\omega^2\left(b^2 - a^2\right)\left[\int_a^r r^{-C-1}\left[1-\left(1-D^n r^{-n}\right)^m\right]^{2-c}\exp\left(f_1 + f_2\right)dr\right]}{2\int_a^b r^{-c-1}\left[1-\left(1-D^n r^{-n}\right)^m\right]^{2-c}\exp\left(f_1 + f_2\right)dr},$$

$$T_{\phi\phi} = T_{rr} + \frac{\rho\omega^2\left(b^2 - a^2\right)r^{-c}\left[1-\left(1-D^n r^{-n}\right)^m\right]^{2-c}\exp\left(f_1 + f_2\right)}{2\int_a^b r^{-c-1}\left[1-\left(1-D^n r^{-n}\right)^m\right]^{2-C}\exp\left(f_1 + f_2\right)dr},$$

$$\begin{aligned} T_{zz} &= \left(\frac{1-c}{2-c}\right)\left(T_{rr} + T_{\phi\phi}\right) + \frac{2c\xi\,\bar{\Theta}_0}{(2-c)\left(b^2-a^2\right)}\left[\frac{a^2}{4} - \frac{b^2}{4} - \frac{a^2}{2}\log\frac{a}{b}\right] \\ &\quad - \left(\frac{1-c}{2-c}\right)\frac{\rho\omega^2}{2}\left(b^2 + a^2\right) - \frac{c\xi\,\Theta}{(2-c)} \end{aligned}. \tag{4.10}$$

Equation (4.10) give the creep transitional stresses for a hollow rotating cylinder. These expressions correspond to only one stage of creep. If all the three stages of creep to be taken into account, we shall add the incremental values [11, 15] of ($T_{rr} - T_{\phi\phi}$). Thus equation (4.5) becomes:

$$T_{rr} - T_{\phi\phi} = A_0 r^{-3c}\prod_{m,n}\left[1-\left(1-D^n r^{-n}\right)^m\right]^{3-2c}.\exp\left(f_1 + f_2\right)$$

where *m*, *n* having three different sets of values each corresponding to one stage of creep.

For Steady State of Creep

Transitional creep stresses for secondary state of creep are obtained by putting $m = 1$ in eqn. (4.8), one gets:

$$T_{rr}=-\frac{\rho\omega^2\left(r^2-a^2\right)}{2}+\frac{\rho\omega^2\left(b^2-a^2\right)\left[\int_a^r r^{c(n-1)-2n-1}\exp\left(-c_1r^{n+2}+c_2r^n\right)dr\right]}{2\int_a^b r^{c(n-1)-2n-1}\exp\left(-c_1r^{n+2}+c_2r^n\right)dr},$$

$$T_{\phi\phi}=T_{rr}+\frac{\rho\omega^2\left(b^2-a^2\right)r^{c(n-1)-2n}\exp\left(-c_1r^{n+2}+c_2r^n\right)}{2\int_a^b r^{c(n-1)-2n-1}\exp\left(-c_1r^{n+2}+c_2r^n\right)dr},$$

$$T_{zz}=\left(\frac{1-c}{2-c}\right)\left(T_{rr}+T_{\phi\phi}\right)+\frac{2c\xi\,\bar{\Theta}_0}{\left(2-c\right)\left(b^2-a^2\right)}\left[\frac{a^2}{4}-\frac{b^2}{4}-\frac{a^2}{2}\log\frac{a}{b}\right]-\left(\frac{1-c}{2-c}\right)\frac{\rho\omega^2}{2}\left(b^2+a^2\right)-\frac{c\xi\Theta}{\left(2-c\right)}. \tag{5.1}$$

where $c_1=\dfrac{n\rho\omega^2c}{2\mu D^n\left(n+2\right)}$ and $c_2=\dfrac{c\xi\bar{\Theta}_0}{2\mu D^n}$. It is found that the value $\left|T_{rr}-T_{\phi\phi}\right|$ is maximum at $r = a$, therefore yielding of the cylinder starts at the internal surface and eqn. (5.1) becomes:

$$\left|T_{rr}-T_{\phi\phi}\right|=\frac{\rho\omega^2\left(b^2-a^2\right)a^{c(n-1)-2n}\exp\left(-c_1a^{n+2}+c_2a^n\right)}{2\int_a^b r^{c(n-1)-2n-1}\exp\left(-c_1r^{n+2}+c_2r^n\right)dr}\equiv Y_1 \tag{5.2}$$

where Y_1 is the yields stress.

For incompressible material the creep stresses by taking $(c\to 0)$, Seth [10]. The equations (5.1) and (5.2) reduces to:

$$T_{rr}=-\frac{\rho\omega^2\left(r^2-a^2\right)}{2}+\frac{\rho\omega^2\left(b^2-a^2\right)\left[\int_a^r r^{-2n-1}\exp\left(c_2r^n\right)dr\right]}{2\int_a^b r^{-2n-1}\exp\left(c_2r^n\right)dr},$$

$$T_{\phi\phi}=T_{rr}+\frac{\rho\omega^2\left(b^2-a^2\right)r^{-2n}\exp\left(c_2r^n\right)}{2\int_a^b r^{-2n-1}\exp\left(c_2r^n\right)dr},$$

$$T_{zz}=\frac{1}{2}\left(T_{rr}+T_{\phi\phi}\right)+\frac{2\alpha E\bar{\Theta}_0}{\left(b^2-a^2\right)}\left[\frac{a^2}{4}-\frac{b^2}{4}-\frac{a^2}{2}\log\frac{a}{b}\right]-\frac{\rho\omega^2}{4}\left(b^2+a^2\right)-\alpha E\Theta\,. \tag{5.3}$$

and $Y_1 = \dfrac{\rho\omega^2\left(b^2 - a^2\right)a^{-2n}\exp\left(c_2 a^n\right)}{2\int_a^b r^{-2n-1}\exp\left(c_2 r^n\right)dr}$, where $c_2 = \dfrac{3\alpha E\bar{\Theta}_0}{nyD^n}$.

As a particular case, transitional creep stresses for rotating cylinder without thermal effects are obtained by putting $\Theta_0 = 0$ in equation (5.1) one gets:

$$T_{rr} = -\frac{\rho\omega^2\left(r^2 - a^2\right)}{2} + \frac{\rho\omega^2\left(b^2 - a^2\right)\left[\int_a^r r^{c(n-1)-2n-1}\exp\left(-c_1 r^{n+2}\right)dr\right]}{2\int_a^b r^{c(n-1)-2n-1}\exp\left(-c_1 r^{n+2}\right)dr},$$

$$T_{\phi\phi} = T_{rr} + \frac{\rho\omega^2\left(b^2 - a^2\right)r^{c(n-1)-2n}\exp\left(-c_1 r^{n+2}\right)}{2\int_a^b r^{c(n-1)-2n-1}\exp\left(-c_1 r^{n+2}\right)dr},$$

$$T_{zz} = \left(\frac{1-c}{2-c}\right)\left(T_{rr} + T_{\phi\phi}\right) - \left(\frac{1-c}{2-c}\right)\frac{\rho\omega^2}{2}\left(b^2 + a^2\right). \tag{5.4}$$

For incompressible material $c \to 0$, equations (5.4) become:

$$T_{rr} = -\frac{\rho\omega^2\left(r^2 - a^2\right)}{2} + \frac{\rho\omega^2}{2}\left(b^2 - a^2\right)\frac{\left(r^{-2n} - a^{-2n}\right)}{\left(b^{-2n} - a^{-2n}\right)},$$

$$T_{\phi\phi} = T_{rr} - \frac{n\rho\omega^2\left(b^2 - a^2\right)r^{-2n}}{\left(b^{-2n} - a^{-2n}\right)}, T_{zz} = \frac{1}{2}\left(T_{rr} + T_{\phi\phi}\right) - \frac{\rho\omega^2}{4}\left(b^2 + a^2\right) \tag{5.5}$$

Expressions (4.4) and (4.5) are the same as obtained by Gupta [14]. The stresses for an elastic rotating hollow cylinder are obtained by putting $n = 1$ in eq. (5.5) as:

$$T_{rr} = \frac{\rho\omega^2}{2}\left[\left(a^2 + b^2\right) - \left(\frac{a^2 b^2}{r^2} + r^2\right)\right],$$

$$T_{\phi\phi} = \frac{\rho\omega^2}{2}\left[\left(a^2 + b^2\right) + \left(\frac{a^2 b^2}{r^2} - r^2\right)\right], \quad T_{zz} = \frac{\rho\omega^2}{2}\left[\frac{a^2 b^2}{r^2} - r^2\right] \tag{5.6}$$

These expression are the same as obtained by Rimrott and Luke 1961 at time $t = 0$. For plain strain case i.e. $e_{zz} = 0$, eq. (5.4) becomes:

$$T_{rr} = -\frac{\rho\omega^2}{2}\left[\left(r^2 - a^2\right) - \frac{\left(r^{-2n} - a^{-2n}\right)}{\left(b^{-2n} - a^{-2n}\right)}\left(b^2 - a^2\right)\right], \quad T_{\phi\phi} = T_{rr} - \frac{n\rho\omega^2\left(b^2 - a^2\right)r^{-2n}}{\left(b^{-2n} - a^{-2n}\right)},$$

$$T_{zz} = -\frac{\rho\omega^2}{2}\left[\left(r^2 - a^2\right) - \frac{\left(b^2 - a^2\right)}{\left(b^{-2n} - a^{-2n}\right)}\left\{(1-n)r^{-2n} - a^{-2n}\right\}\right]. \tag{5.7}$$

These expressions are same as obtained by Dev [9] provided we put $n = 1/N$, which Dev obtained by assuming the Norton's law and Von-Mises yield condition.

Results and Discussion

In Fig. 1 curves have been drawn between $\rho\omega^2/4Y$ and $\alpha E\Theta_0/2Y$ to give yielding through the whole of the cylinder (fully plastic state) for different wall thickness ratios. It can be seen that with the increase of temperature, the cylinder having smaller radii ratios requires smaller angular velocity to become fully plastic as compared to cylinder having higher radii ratios. In Figs. 2 curves have been drawn for radial, circumferential and axial stresses with respect to radii ratios r/a, and for various combination of $\alpha E\Theta_0/2Y$ and $\rho\omega^2/4Y$. The circumferential stress increases with the increase in temperature. With increase in thickness ratio stresses must be decreased. The strain in the longitudinal direction as a function of temperature and the corresponding rotational speed (from Fig. 1) is plotted in Fig. 3, for cylinder having different thickness ratios. In the absence of thermal effects, the axial contraction increases with the increase in thickness ratios of the cylinder, but with the inclusion of temperature effects, it can be seen that cylinder having thickness ratio 2 gives greater axial contraction as compared to cylinder having thickness ratios 3, 4, 7 and 10. It is of interest to note that, at high temperature, axial contraction become greater for $b/a = 2$, then decrease for $b/a = 3$, remain almost the same for $b/a = 4$, then again increases for cylinder having thickness ratios $b/a > 4$. To see the combined effects of rotation and temperature, this problems has been solved by using Simpson's rule in eq. (5.1) and (5.3). For mild steel various values from [17] can be taken: $Y = 3\times10^4$ 1b/in^2, $E = 3\times10^7$ 1b/in^2 and $\alpha = 7.5\times10^{-6}\, per^0F$ and $\rho\omega^2/2 = 4200$. In Figs. 4 and 5, curve have been drawn between the radial, circumferential and axial stresses for measure $n = 2$ and $n = 3$ respectively, with respect to radii ratio r/a. It has been seen that circumferential stress for incompressible material is maximum at the internal surface as compared to compressible material, which increase with the increase in temperature and measure n. It is noted in this context that Rimrott [3] showed similar results for plastic material without thermal effects, this is, if material tends to fracture by cleavage, it will begin as a sub-surface fracture close to the bore, because it is where the largest tensile stress occurs. This means that an increases in temperature increases the possibility of a fracture at the base at a lesser angular speed.

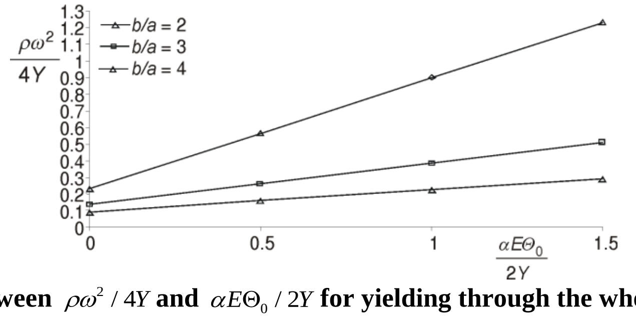

Fig. 1 Relation between $\rho\omega^2/4Y$ and $\alpha E\Theta_0/2Y$ for yielding through the whole cylinder

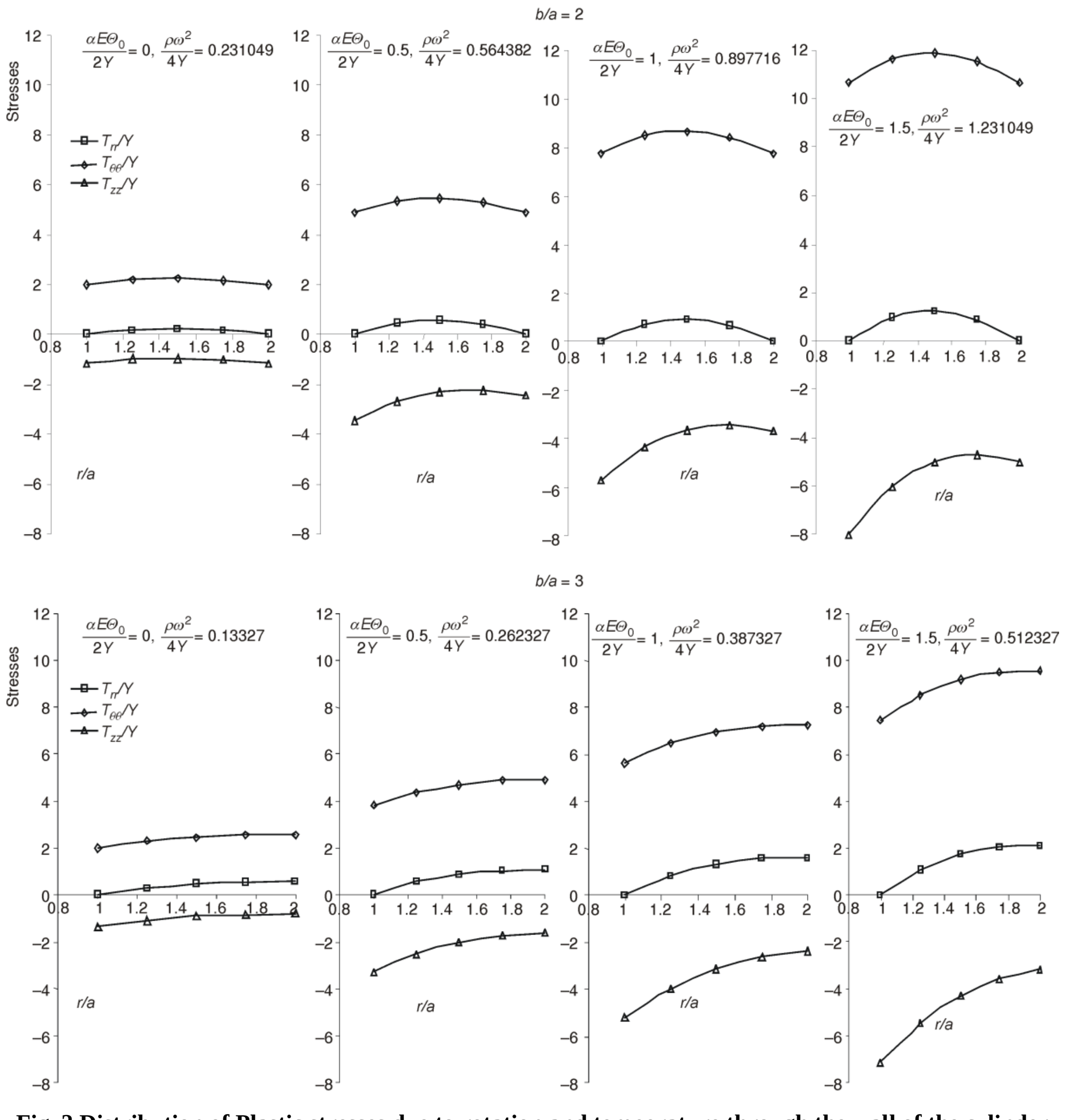

Fig. 2 Distribution of Plastic stresses due to rotation and temperature through the wall of the cylinder

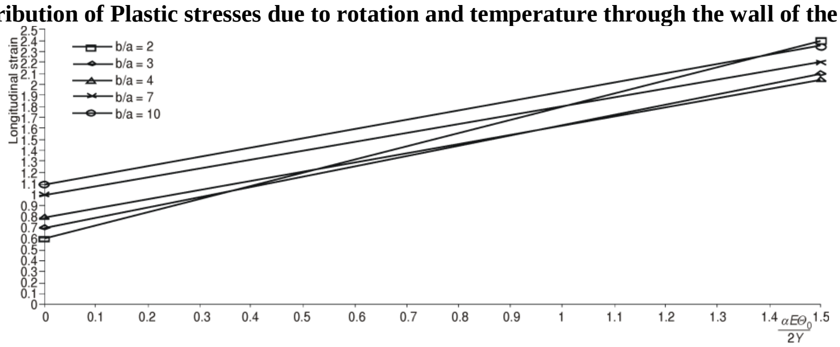

Fig. 3 Longitudinal strain verses temperature for various cylinder thickness ratios.

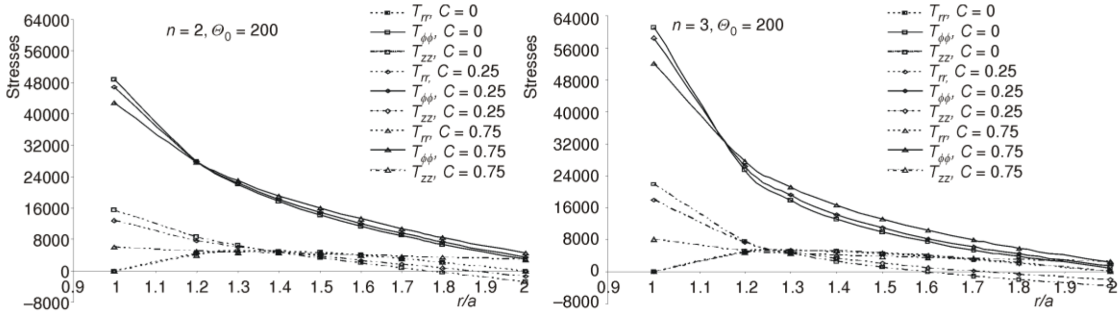

Fig. 4 Stress distribution in an isotropic rotating cylinder with respect to radii ratio r/a having Θ_0 = 200

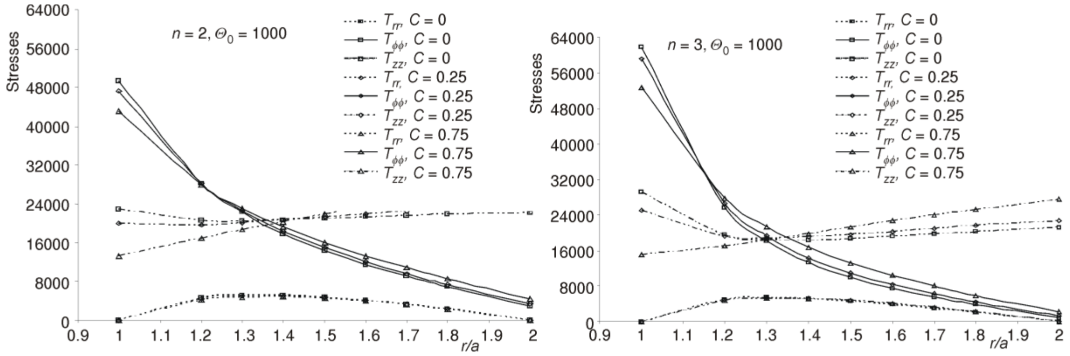

Fig. 5 Stress distribution in isotropic rotating cylinder with respect to radii ratio r/a having Θ_0 = 1000.

Conclusion

For elastic-plastic stage, it is seen that with the increase of temperature, the cylinder having smaller radii ratios requires lesser angular velocity to become fully plastic as compared to cylinder having higher radii ratios The circumferential stress becomes larger and larger with the increase in temperature. With increase in thickness ratio stresses must be decrease. For the creep stage, it is seen that circumferential stresses for incompressible materials maximum at the internal surface as compared to compressible material, which increase with the increase in temperature and measure n.

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Nomenclature

a,*b*- internal and external radii of the rotating cylinder, [m]

T_{ij} e_{ij} - stress [kgm^{-1}s^{-2}] strain rate tensors

C - compressibility factor,[-]

u,*v*,*w*,- displacement components,[m]

Y -Yield stress, [kg $m^{-1}s^{-2}$]

Greek letters

Θ - temperature,[^{0}F]

σ_r - Radial stress component (T_{rr}/Y),[-]

σ_ϕ - Circumferential stress ($T_{\phi\phi}/Y$),[-]

σ_z - Axial stress component (T_{zz}/Y),[-].

v - Poisson's ratio, [-]

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