STUDY ON REVERSE-QUADTREE ADAPTIVE GRID TECHNIQUE

by

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Short paper
DOI: 10.2298/TSCI1205515H

The fast multipole method is universally adopted for solving the convection equation in the vortex method. In this paper, a reverse-quadtree adaptive grid technique is proposed in order to improve the quadtree adaptive grid technique in the fast multipole method. Taking flow past a cylinder as an example, the results indicate the reverse-quadtree scheme can save more calculation time than the quadtree scheme when the particle population is large enough.

Key words: *reverse-quadtree adaptive grid technique, fast multipole method, vortex method*

Introduction

As a Lagrange method for simulating Navier-Stokes equations directly, vortex method has been extensively adopted in wind engineering and aerodynamic [1], and even in simulations of 3-D turbulence without any turbulence model. In 1973, Chorin [2] first proposed the discrete vortex method (DVM) in which Navier-Stokes equations can be solved by dividing into a convection equation and a diffusion equation, and then these equations can be solved respectively by using an *N*-body method and a random walk method. Degond *et al.* [3] developed DVM in dealing with the viscous term subsequently. Although gradually accepted by scholars in computational fluid, the low resolution is a bottle neck of DVM. Afterwards, with the improvements by Shiels [4] and Huang [5], core-spreading vortex method reached greater precision and the vortex method increasingly developed to be a mature method in computational fluid [6]. When solving the convection equation, we are actually dealing with an *N*-body problem with the computational amount of $O(N^2)$ magnitude. With the evolution of a flow filed, the calculation amount will certainly get a harsh increment, so a fast algorithm is essential for the vortex method application. Chen [7] introduced the binary encoding technology which reduced the computational amount and improved the efficiency of Van Dommelen's scheme. However, the quadtree division scheme needs enormous arrays for storing cell codes, subordination between parent cells and child cells, and codes of childless cells. Based on Chen's scheme, we designed a reverse quadtree adaptive

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grid division scheme which builds arrays as little as possible, and present some specific analyses on computing efficiency.

Improvement of fast multipole method

Governing equations

The 2-D impressible flow is governed by the vorticity transport equation:

$$\frac{\partial \omega}{\partial t} + u_x \frac{\partial \omega}{\partial x} + u_y \frac{\partial \omega}{\partial y} = \nu \nabla^2 \omega \tag{1}$$

where u_x and u_y are components of the flow velocity $\vec{\mathrm{V}}$, v is the kinematic viscosity coefficient, and $\omega = \nabla \times \vec{\mathrm{V}}$ is the vorticity. As ψ is the flow function, the velocity can be expressed as $u_x = \partial\psi/\partial y$ and $u_y = -\partial\psi/\partial y$, and the continuity equation is:

$$\nabla^2 \psi = -\omega \tag{2}$$

Take Gauss distribution as the normalizing function and the vorticity field can be represented by N Lagrangian, scalar-valued particles, and the vorticity at any point can be written as:

$$\omega(\vec{\mathrm{x}}, t) = \sum_{j=1}^{N} \frac{\Gamma_j}{\pi \sigma_j^2} \exp\left(-\frac{\left\| \vec{\mathrm{x}} - \vec{\mathrm{x}}_j \right\|^2}{\sigma_j^2} \right) \tag{3}$$

where Γ_j, σ_j, and $\vec{\mathrm{x}}_j$ denotes circulation, size and the center of the j-th vortex elements, respectively.

In fast multipole method (FMM), the calculation domain is first divided into cells of several levels with a reverse quadtree adaptive grid technique, then all cells in each level are encoded and relevant Laurent coefficients are calculated. If two particles' distance is short we must calculate the velocity with Biot-Savart law (point-to-point), otherwise we may get the velocity by using the Laurent series (box-to-point).

Reverse-quadtree adaptive grid

A reverse quadtree adaptive grids division scheme (fig. 1) is designed.

(1) Find a minimum square domain which can completely overlap the whole particles, and denote this domain as a cell of the 0-th level.

(2) Select a control number N_{ctrl}, if the particle population in the cell of the zeroth level is larger than N_{ctrl}, the cell of the zeroth level is divided into 4 cells of the 1^{st} level equally. If the particle population in any cell of the 1^{st} level is still larger than N_{ctrl}, then divide every cell of the 1st level into 4 cells of the 2^{nd} level equally, and repeat this division step till the particle population in each cell of the k-th level is no more than N_{ctrl}.

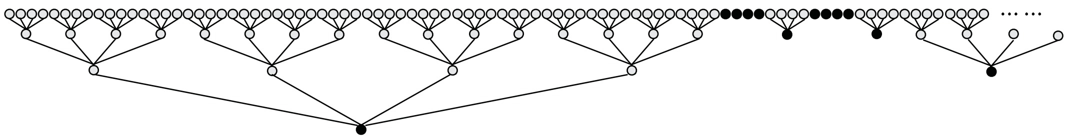

Figure 1. Reverse-quadtree technique for the adaptive grids division

(3) If 4 cells of the $(k+1)$-th level are divided from a cell of the k-th level, then the cell of the k-th level is denoted as the parent of the four cells of the $(k+1)$-th level.

(4) Suppose that the flow field is divided into 4^L cells of the L-th finest totally. Each cell of the L-th level has a x code and a y code $(x, y \in 0 \sim 2^L - 1)$ and (x, y) is the 2-D designation of that cell of the L-th level.

(5) Transfer the 2-D code x, y into binary digits, then we can produce a new binary digit by interlacing the digits and transform the new digit to a decimal one (remarked as F), then F is the 1-D designation of the cell of the L-th level based on the reverse-quadtree technology and (F, k) denotes a cell of the k-th level with the start code F.

(6) Check the total particle population in $(4^{L-L+1}k, L)$, $(4^{L-L+1}k + 4^{L-L}, L)$, $(4^{L-L+1}k + 2\cdot 4^{L-L}, L)$, $(4^{L-L+1}k + 3\cdot 4^{L-L}, L)$, $(k = 0, 1, 2, \ldots, 4^{L-1})$, if the population is smaller than N_{ctrl}, then remark the cell $(4^{L-L+1}k, L-1)$ as a childless cell.

(7) Check the total particle population in $(4^{L-(L-i)+1}k, L-i)$, $(4^{L-(L-i)+1}k + 4^{L-(L-i)}, L-i)$, $(4^{L-(L-i)+1}k + 2\cdot 4^{L-(L-i)}, L-i)$, $(4^{L-(L-i)+1}k + 3\cdot 4^{L-(L-i)}, L-i)$, $(k = 0, 1, 2, \ldots, 4^{(L-i)-1})$, if the population is smaller than N_{ctrl}, then remark the cell $(4^{L-(L-i)+1}k, L-i-1)$ as a childless cell.

(8) Let $i = i + 1$, and repeat step (7) till the total particle population in every 4 adjacent childless cells is larger than N_{ctrl}.

Dividing the flow field from leaves to the branches, then the root is the key point of this reverse-quadtree scheme, which only records the childless cells.

Laurent coefficients of each cell

The Laurent coefficients C_k of each cell should be obtained before using the box-to-point formula. The more particles are in a cell, the heavier the computational amount on C_k is. The Laurent coefficient C_k of a cell can be directly obtained from calculations on the Laurent series of its child cells $(C'_{k,1}, C'_{k,2}, C'_{k,3}, C'_{k,4})$ (fig. 2), and the velocity at Z_n can be expressed as:

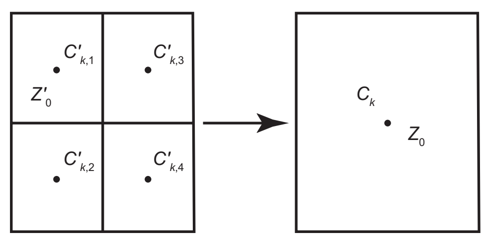

Figure 2. Laurent series of parent cells

$$\vec{V}_n \approx \sum_{k=1}^{P}\sum_{j=1}^{k}\frac{m_j^k C_j}{(Z_n - Z_0')^k} \tag{4}$$

$$C_k' = \sum_{j=1}^{k} m_j^k C_j \tag{5}$$

where $m_k^k = 1, \quad m_{k-l}^k = m_{k-l+1}^k \mathrm{d}Z(k-l)/l$.

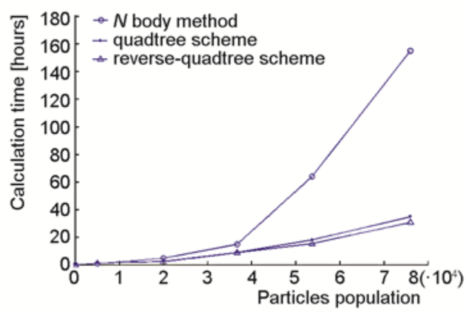

Figure 3. The relationship between calculation time and particles population

Numerical examples

Take the flow past a cylinder for examples. Suppose that there is a cylinder in an infinite domain with the radius $R = 0.5$, the kinematic viscosity of the flow is $\nu = 1.82\cdot10^{-3}$ and the free stream velocity is $\vec{V}_\infty = 1.0 + 0i$ (Re = 550). The control size of the discrete particles is $\sigma_{\max} = (4\nu dt)^{-1/2}$, and the initial size of the particles is $\sigma = (2\nu dt)^{-1/2}$, the time step $dt = 0.01$; when calculating Laurent series, the number of terms in expansion is set as $P = 13$, the particle control number in each grid is $N_{ctrl} = 50$, and the maximum grid level is 11. The diffusion equation is solved by using the core-spreading vortex method [1], and the convection equation is solved by using the reverse-quadtree adaptive grid technique.

As shown in fig. 3, when the particle population is smaller, the advantage of reverse-quadtree scheme is not very significant. However, when the particle population is large enough, the reverse-quadtree scheme saves more calculation time than the quadtree scheme.

Conclusions

A reverse-quadtree adaptive grid technique is designed, which can improve the quadtree adaptive grid technique. The reverse-quadtree scheme should save more calculation time than the quadtree scheme when the particle population is large enough.

Nomenclature

C_k – Laurent coefficient, [–]
Γ_j – circulation of the j-th vortex element, [m^{2}s^{-1}]
σ_j – size of the j-th vortex element, [m]
u_x – x-direction component of velocity, [ms^{-1}]
u_y – y-direction component of velocity, [ms^{-1}]
ω – vorticity, [1s^{-1}]
$\vec{V}$ – flow velocity, [ms^{-1}]
$\vec{x}_i$ – center position of the j-th vortex element, [–]

Acknowledgment

This work was supported by the National Natural Sciences Foundation of China (No.11272042).

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Paper submitted: July 30, 2012

Paper revised: September 1, 2012

Paper accepted: September 13, 2012