SIMULATION OF FLOW PAST TWO TANDEM CYLINDERS USING DETERMINISTIC VORTEX METHOD

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The vortex method is a direct numerical simulation method for solving the Navier-Stokes equations. In order to reveal the influence of Reynolds number and distances between the cylinders, the incompressible flow past a pair of tandem cylinders is solved on the base of the vortex method. The results show that for the flow past two tandem cylinders, there is a critical distance of the tandem cylinders. Over the critical distance, the flow field will have a sudden change, and the drag coefficient, lift coefficient and Strouhal number will also change dramatically. The critical distance will diminish as the Reynolds number rises.

Key words: *two tandem cylinders, flow around circular cylinder, deterministic vortex method, vortex shedding*

Introduction

The problem of flow past a pair of tandem cylinders is very common in engineering. Some experiments and numerical simulations have been carried out to study the flow past a pair of tandem cylinders. Slaoutiand *et al.* [1] used random vortex method to simulate numerically the flow past tandem cylinders. The drawback of random vortex method is that the simulation accuracy is restricted by the total number of vortex elements, and the error would increase greatly with time growing. Relative to the random vortex method, the deterministic vortex method enjoys higher precision.

At present, the deterministic vortex method includes core scatter vortex method and particle strength exchange method (PSE) [2]. Huang *et al.* [3, 4] established a model of flow past ablation blunt with moving boundary by deterministic vortex method. But the deterministic vortex method is mainly used in simulation of flow past a single object. The numerical simulation of flow past multiple objects has rarely been reported. This work will numerically simulate the flow past two tandem cylinders with different distances in Re = 200 and Re = 300, respectively, by using PSE.

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Particle strength exchange method

Two-dimensional incompressible flow field vorticity equation is:

$$\frac{\partial \omega}{\partial t} + u_x \frac{\partial \omega}{\partial x} + u_y \frac{\partial \omega}{\partial y} = \nu \nabla^2 \omega \tag{1}$$

$$\nabla^2 \psi = -\omega \tag{2}$$

where ψ is the stream function, ν – the kinematic viscosity coefficient of fluid, and ω – the vorticity,

$$u_x = -\sum_{i=1}^{N} \frac{\Gamma_i}{\pi \sigma_i^2} \frac{y - y_i}{\|\mathbf{x} - \mathbf{x}_i\|^2} \left[1 - \exp\left(-\frac{\|\mathbf{x} - \mathbf{x}_i\|^2}{\sigma_i^2} \right) \right] \tag{3}$$

$$u_y = \sum_{i=1}^{N} \frac{\Gamma_i}{\pi \sigma_i^2} \frac{x - x_i}{\|\mathbf{x} - \mathbf{x}_i\|^2} \left[1 - \exp\left(-\frac{\|\mathbf{x} - \mathbf{x}_i\|^2}{\sigma_i^2} \right) \right] \tag{4}$$

$$\Gamma_i^n = \Gamma_i^{n-1} + \frac{2\nu}{\sigma_{ij}^2} \sum_{j \in D_i} (S_i \Gamma_j - S_j \Gamma_i) \eta_{ij} (\mathbf{x}_i - \mathbf{x}_j) \Delta t \tag{5}$$

Based on the no-slip boundary conditions, we get for the vorticity density γ:

$$\gamma(s) - \frac{1}{\pi} \oint_s \frac{\partial}{\partial \mathbf{n}} \log \|\mathbf{x}(s) - \mathbf{x}(s')\| \gamma(s') \,\mathrm{d}s' = -2 \frac{\partial \psi_{ext}}{\partial \mathbf{n}} \mathbf{x}(s) \tag{6}$$

where ψ_{ext} stands for wake flow function, s for the object boundary, $\mathbf{n}$ for the unit normal vector on the boundary. New vorticity diffusing to the flow field satisfies equation:

$$\omega_\gamma(x, y, \delta t) = \sum_{i=1}^{M} \frac{\gamma_i \exp\left(-\frac{x^2}{4\nu\delta t} \right)}{2\sqrt{\pi \nu \delta t}} \left[\operatorname{erf}\left(\frac{\frac{l_i}{2} - y}{\sqrt{4\nu\delta t}} \right) + \operatorname{erf}\left(\frac{\frac{l_i}{2} + y}{\sqrt{4\nu\delta t}} \right) \right] \tag{7}$$

where l is the length of vortex sheet, M – the number of vortex sheets, and (x, y) are the co-ordinates of a point in the flow field, which is related to the vortex sheet.

The calculation process of particle strength exchange method is:

– initialize the flow field;
– solve equations (6) and (7), and then calculate the strength of new particles;
– solve equation (5), and then calculate the viscous diffusion;
– solve equations (3) and (4) to get the velocity of the particle, and then calculate the new vorticity field, and
– repeat steps b-d. That is to calculate the next time step.

Numerical examples

Based on the particle strength exchange method flow past two tandem cylinders, we can calculate by using FORTRAN codes. Solve the problem of flow past two tandem cylinder in Re = 200 and Re = 300, respectively, and choose different ratios of *L*/*D* from 1.5 to 5.0 to simulate. Table 1 shows the C_d, C_L, and St at Re = 200.

Table 1. Flow past two tandem cylinder at Re = 200

L/*D*	C_d		C_L		St
	Front	Later	Front	Later	
1.5	0.70	–0.08	0.05	0.30	0.162
2.0	0.83	–0.05	0.03	0.24	0.154
3.0	0.79	0.11	0.04	0.21	0.127
4.0	0.88	0.54	0.78	2.02	0.161
5.0	0.93	0.62	0.83	1.84	0.164

Figure 1 shows the flow vorticity contours in Re = 200 and Re = 300 at different cylinder distances (*L*/*D* = 1.5 ~ 5.0). It can be seen that when the distance between two cylinders is short, regardless of Re = 200 or Re = 300, the shear layer of the front cylinder attaches to the later cylinder, and vortex shedding occurs at the later cylinder. However, when the distance between two cylinders exceeds a critical value, vortex shedding begins to appear at the front cylinder, and shedding vortex attaches to the later one, at the same time, the shedding vortex at the later cylinder changes. The critical value associates with the flow Reynolds number. When Re = 200, the critical value of *L*/*D* is from 3.5 to 4.0, while Re = 300, the critical value is from 2.5 to 3.0. The critical distance decreases as the Reynolds number increases.

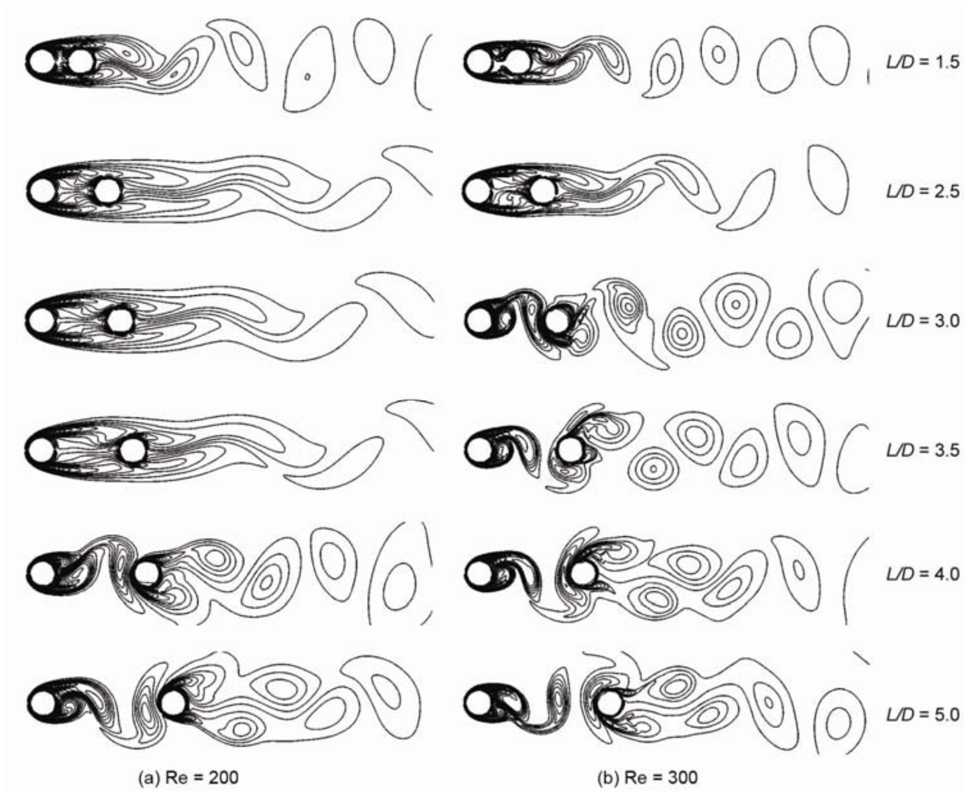

Figure 1. Vorticity contours at different *L*/*D*

When the distance between two cylinders is less than the critical value, the drag coefficient of the two cylinders almost remains unchanged (fig. 2). Because vortex shedding does not occur at the front cylinder, there is few shedding vortex at the later cylinder. When the distance exceeds the critical value, the drag coefficients of the two cylinders begin to fluctuate, and the oscillation frequency of the drag coefficients is twice as much as the oscillation frequency of the lift coefficients. The oscillation frequency of the drag coefficients of the two cylinders are the same, and also the coefficient frequency of the lift coefficients.

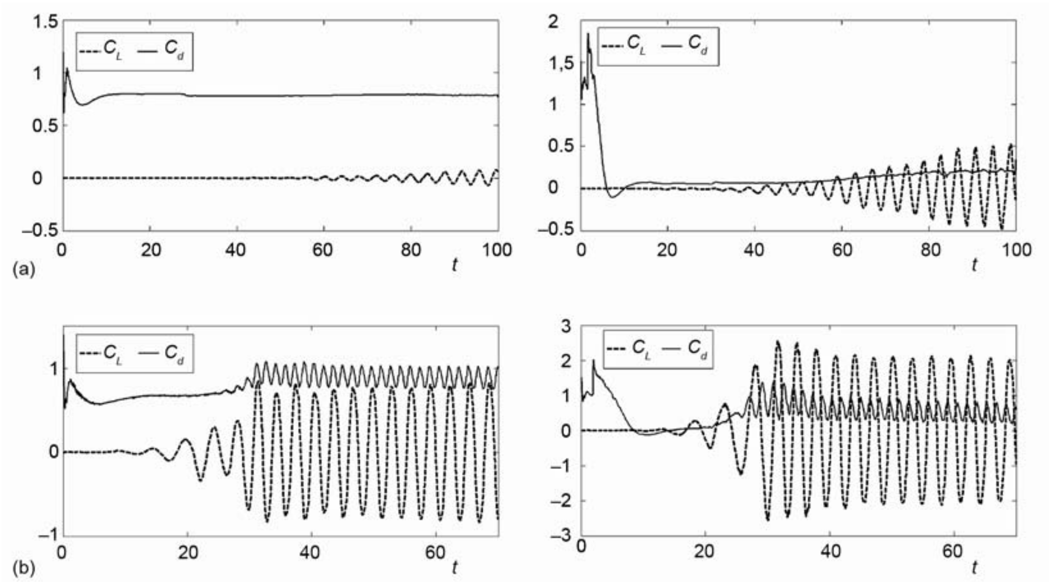

Figure 2. Variation curve of C_d, and C_L with time; (a) front cylinder (left), later cylinder (right) when *L/D* = 3.5, (b) front cylinder (left), later cylinder (right) when *L/D* = 4.0

Conclusions

Based on this vortex method, the incompressible flow past a pair of tandem cylinders is solved by using FORTRAN codes under the condition that Re = 200 and Re = 300, respectively, and get the regular variation of fluid field with the different distances between the cylinders. The results show that for the flow past two tandem cylinders, there is a critical distance between the tandem cylinders. Over the critical distance, the flow field will have a sudden change, and the drag coefficient, lift coefficient and Strouhal number will also change dramatically. The critical distance will diminish as the Reynolds number rises.

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Nomenclature

Re – Reynolds number, [–]
C_d – drag coefficient, [–]
C_l – lift coefficient, [–]
St – Strouhal number, [–]
L – distance between the centers of two cylinders, [m]
D – cylinder diameter, [m]

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