STUDY ON FLOW OF POWER-LAW FLUID THROUGH AN INFINITE ARRAY OF CIRCULAR CYLINDERS WITH IMMERSED BOUNDARY-LATTICE BOLTZMANN METHOD

by

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A direct forcing method for the simulation of particulate flows based on immersed boundary-lattice Boltzmann method is used to study the flow of power-law fluid through an infinite array of circular cylinders with cylinder separations of 20a (a is the cylinder radius) with laminar shedding behind cylinders. Time averaged drag coefficient, maximum of lift coefficient and Strouhal number are given out with the power-law index in the range of $0.4 \le n \le 1.8$ and Re in the range of $50 \le Re \le 140$.

Key words: *flow of power-law fluid, lattice Boltzmann equation, immersed boundary method, direct forcing method, laminar shedding*

Introduction

As we all know, the cross-flow of fluids past cylinders of circular and non-circular cross-sections is a fundamental flow and has pragmatic significance [1-4]. Such study is mostly about the Newtonian fluid. For the flow of Newtonian fluids over a circular cylinder, it is known that when the Reynolds number is greater than a critical value at Re ≈ 45-50, the wake region grows and ultimately becomes asymmetric and periodic in time, thereby leading to the onset of laminar vortex shedding. The laminar vortex shedding will keep until the Reynolds number reaches another critical value at Re ≈ 150-200. It has been shown that the same phenomenon happens for the flow of power-law fluids over a circular cylinder with different critical values of Reynolds number [5, 6].

Among the study on the cross-flow of power-law fluids through circular cylinders with laminar shedding ($50 \le Re \le 140$), most is about cross-flow through an unconfined circular cylinders [5, 6], or about cross-flow through a bundle of circular cylinders [7, 8], or about cross-flow through one or several circular cylinders confined in two parallel planes [9]. Though there is few works on the cross-flow of Newtonian fluids through an infinite array of parallel circular cylinders [10], there is no works on the cross-flow of power-law fluids through an infinite array of parallel circular cylinders.

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In this article, the cross-flow of power-law fluid through an infinite array of circular cyers with cylinder separations set to be 20*a* (*a* is the cylinder radius) is studied in the range of laminar shedding, which is shown in fig. 1 (U and p_a are velocity and atmospheric pressure far from cylinders). The power-law index is in the range of $0.4 \le n \le 1.8$ and Reynolds number is in the range of $50 \le \mathrm{Re} \le 140$, which are almost the same as that of Patnana *et al*. [5], whose results is for unconfined circular cylinder.

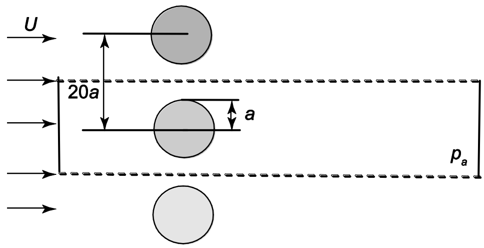

Figure 1. Flow configuration (real lines show the inlet and outlet; dashed lines show the mid-planes between two cylinders)

Method

In recent years, the lattice Boltzmann equation method (LBM) has been rapidly developed for fluid dynamic problems, such as multiphase flow, particulate flow, *etc*. [11-17]. Recently, the immersed boundary-lattice Boltzmann method (IB-LBM) was presented by Feng *et al*. [18] to simulate the motion of rigid particle in fluids, where a regular Eulerian grid is used for the flow domain and a Lagrangian grid is used for the particle's boundary. The force density is computed via a direct forcing scheme, and the flow field is then solved by the processes of the LBM. The detail of the direct forcing IM-LBM for incompressible fluid can be found in Feng *et al*. [18].

For power-law fluid, the apparent kinematical viscosity, ν_{ap} is determined as:

$$\nu_{ap}(\dot{\gamma}) = m|\dot{\gamma}|^{n-1} \tag{1}$$

where *m* is a parameter of power-law fluid, *n* – the power-law index, and $\dot{\gamma}$ – the shear rate, which can be calculated from the strain tensor **S**:

$$\dot{\gamma} = 2\sqrt{\sum_{\alpha,\beta=1}^{l} \mathbf{S}_{\alpha\beta}\mathbf{S}_{\alpha\beta}} \tag{2}$$

Then the local apparent viscosity ν_{ap} can be converted to a local apparent relaxation time τ_{ap}.

A important dimensionless number analogous to Reynolds number is $\mathrm{Re} = U^{2-n}D^n/m$, where *U* and *D* are characteristic velocity and length scales, respectively.

Study on flow of power-law fluid through an infinite array of widely-spaced circular cylinders

Considering power-law fluid cross-flows an infinite array of circular cylinders with cylinder separations set to be 20*a* with laminar shedding, the cross-flow of every circular cylinder can be thought to be similar to each other. So it is modeled to be power-law fluid cross-flowing a single circular cylinder in a zone with a width of 20*a* with flow parameters in upper and bottom boundary keeping identical to each other. The diameter of the circular cylinder *D* is chosen to be 41.2 lattices long, and the flow domain is chosen to be 30*D* × 10*D*. The cylinder is located at the point (5*D*, 5*D*). The velocity of fluid in the inlet is specified *U* in *x* direction, the stress-free condition is applied to the outlet boundary. A disturbance is added to velocity in inlet for some time to accelerate the emergence of laminar vortex shedding. Five different n (*i. e.*, 0.4, 0.6, 1.0, 1.4, and 1.8), and four different Re (*i. e.*, 50, 100, 120, and 140), are considered. Drag coefficient, $C_D = 2F_x/(\rho_0 U^2 D)$, lift coefficient, $C_L = 2F_y/(\rho_0 U^2 D)$, and Strouhal number, St = fD/U, of the frequency of vortex shedding behind the circular cylinder are concerned. The time evolution of C_D, C_L, and St for Re = 50 is shown in fig. 2, where the time is normalized by the character time D/U. When Re = 50, it can be seen that for small n (n = 0.4, n = 0.6 and n = 1.0), C_L is large; while for large n (n = 1.4 and 1.8), though Karman vortex appears with the disturbance of inlet velocity, C_L is much smaller and the vortex shedding is weak, while it seems that vortex will decay with time in these two cases. This is different from that of unbounded cylinder [5]. When Re = 100, Re = 120, and Re = 140, for all five n, C_L is large and the vortex shedding is strong.

The results of $\overline{C}_D$, C_{Lmax} and St to Re are shown in figs. 3, 4, and 5. When Reynolds number fixed to 50, $\overline{C}_D$, increases with increasing value of n as shown in fig. 3. When Rey-

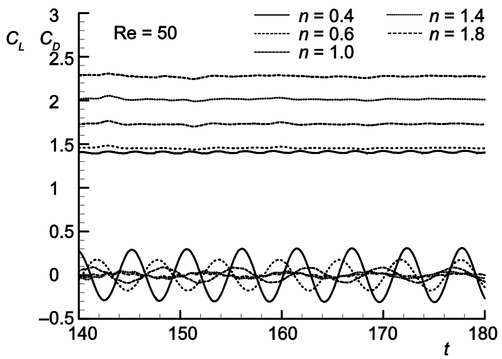

Figure 2. Drag coefficient, lift coefficient to time for Re = 50

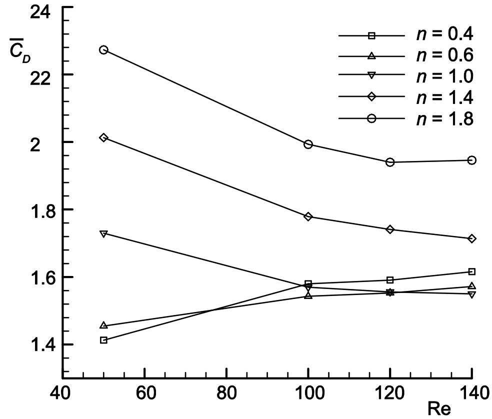

Figure 3. Time averaged drag coefficient to Re for different for power-law index

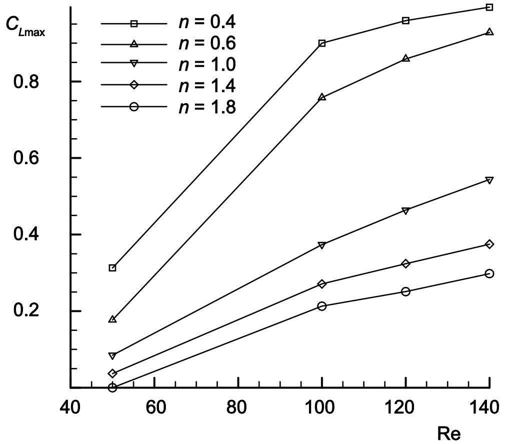

Figure 4. Maximum of lift coefficient to Re for different for power-law index

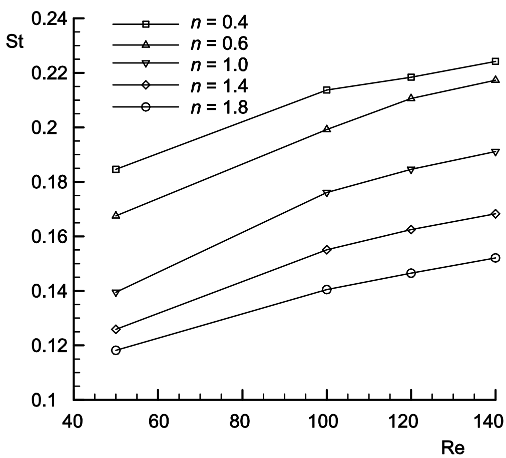

Figure 5. St of frequency of vortex shedding to Re for different power-law index

nolds number fixed to a value of 100, 120, and 140, $\bar{C}_D$, increases with increasing n from n = 1.0 to n = 1.8, while it changes little when $0.4 \le n \le 1.0$, as shown in fig. 3 which is different from the results of Patnana *et al*. for unbounded cylinder [5]. For a fixed value of Re, C_{Lmax}, and St decrease with increasing value of n, as shown in figs. 4 and 5. For a fixed n, with increasing Re, if $1.0 \le n$, $\bar{C}_D$, decreases, and if $n \le 0.6$, it increases, as shown in fig. 3. For a fixed n, C_{Lmax} and St increases with increasing Re as shown in figs. 4 and 5.

Conclusions

When Re = 50, the flow keeps steady for larger n, n = 1.4, and n = 1.8. When Re is fixed to 50, the time averaged drag coefficient increases with increasing power-law index (n). When Re is fixed to 100, 120, 140, the time averaged drag coefficient increases with increasing n from n = 1.0 to n = 1.8, but it changes little when $0.4 \le n \le 1.0$. And for a fixed Re, the maximum of lift coefficient and St number decreases with increasing n. For a fixed n, with increasing Re, if $1.0 \le n$, the time averaged drag coefficient decreases, and if $n \le 0.6$, it increases. And for a fixed n, the maximum of lift coefficient and St increases with increasing Re.

Acknowledgments

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