ONE-WAY COUPLING OF FIBER SUSPENSIONS THROUGH A ROTATING CURVED EXPANSION DUCT

by

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A numerical method based on the one-way coupling using the Jeffery equation is presented. The influence of the inlet velocity and the initial orientation on the evolution of fiber orientation is investigated. It is observed that the rotation mainly contributes to the pressure rise, and the flow structure is not obviously altered. Due to the one-way coupling, the effects of the inlet velocity and the rotating rate are insignificant.

Key words: *fiber suspension, Jeffery equation, orientation, rotating, curved duct*

Introduction

The flow-induced fiber orientation has been studied theoretically and experimentally by many workers. In practical engineering flows the flow field is first solved, then the fiber is inserted into the flow field, thus the fiber behavior is traced along the streamlines. Through direct integral force, the translation and rotation of an ellipsoid can be obtained, which is a promising alternative to the Jeffery equation. The theory had been further improved by [1]. Based on this theory, the Monte Carlo simulations were used to investigate the sedimentation of fiber suspensions [2]. And their simulations were qualitatively verified by the experiments [3]. With the slender body theory, the fiber orientation distribution in an evolving mixing layer were simulated by [4]. The fiber orientation distributions in the laminar and turbulent pipe flows were simulated by the same method [5]. Recently, this method has still been used in computing orientation distribution of fiber suspensions flowing through a contraction [6], a parallel plate channel containing a cylinder [7], an axisymmetric contraction [8], and a round turbulent jet [9]. Recently, the more advanced method, *i. e.*, the lattice Boltzmann method is conducted to simulate the fiber orientation within concentrated suspensions [10-12]. The fiber-fiber interaction can be accurately simulated by lattice Boltzmann method. However, this method requires much greater computational resources. For dilute and semi-dilute fiber suspensions, the one-

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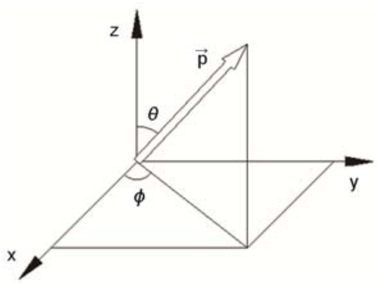

Figure 1. A schematic of unit vector p

-way coupling is more practical than the two-way coupling and the fiber-level simulations. Until now, the one-way coupling based on the real 3-D orientation has not been performed, and will be further utilized to investigate the fiber behavior within a rotating curved expansion duct.

Analytical solution of the Jeffery equation

The Jeffery equation is:

$$D\vec{\mathrm{p}}/Dt = \frac{1}{2}\boldsymbol{\omega}\cdot\vec{\mathrm{p}} + \frac{\lambda}{2}(\dot{\boldsymbol{\gamma}}\cdot\vec{\mathrm{p}} - \dot{\boldsymbol{\gamma}}:\vec{\mathrm{p}}\vec{\mathrm{p}}\vec{\mathrm{p}}) \tag{1}$$

where $\boldsymbol{\omega}$ is the vorticity tensor, and $\dot{\boldsymbol{\gamma}}$ – the deformation rate tensor. The fiber has length L and diameter d, $\lambda = (r_p^2 - 1)/(r_p^2 + 1)$ with $r_p = L/d$.

Two angular velocities, respectively, along the axis of θ and φ are implied in eq. (1), which have been deduced without considering the fiber shape factor. An adapted form of these angular velocities have been deduced by [13, 14]. In present work, the original angular velocities are deduced as follows:

$$\dot{\theta} = \frac{\lambda}{2}\sin 2\theta\left[\left(\frac{\partial v_x}{\partial x}\cos^2\varphi + \frac{\partial v_y}{\partial y}\sin^2\varphi\right) + \frac{1}{2}\sin 2\varphi\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)\right] \tag{2}$$

$$\dot{\varphi} = \frac{\lambda}{2}\left[\sin 2\varphi\left(\frac{\partial v_y}{\partial y} - \frac{\partial v_x}{\partial x}\right) + \cos 2\varphi\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)\right] + \frac{1}{2}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) \tag{3}$$

Analytical analysis

When $\Delta > 0$ (Δ is a discriminant in [10-12]), the analytical solution was given; however, when $\Delta < 0$, only the asymptotic solution was provided. The solution of eq. (2) is:

$$\theta = \mathrm{tg}^{-1}\left\{(\mathrm{tg}\,\theta_0)\exp\left[\int_0^{\tau}\frac{\lambda}{2}\sin 2\theta\left(\frac{\partial v_x}{\partial x}\cos^2\varphi + \frac{\partial v_y}{\partial y}\sin^2\varphi\right) + \right.\right.$$
$$\left.\left. + \frac{1}{2}\sin 2\varphi\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)\right]\mathrm{d}t\right\} \tag{4}$$

where θ_0 is the initial value.

Numerical simulation

To verify the analytical solutions, the Runge-Kutta method is performed. For simplicity, eq. (2) and eq. (3) are rewritten as $f(\theta,\varphi) = \dot{\theta}$, $g(\varphi) = \dot{\varphi}$. Then the Runge-Kutta method is implemented:

$$\begin{cases} K_1 = g(\varphi_n) \\ L_1 = f(\theta_n, \varphi_n) \end{cases}, \quad \begin{cases} K_2 = g\left(\varphi_n + \dfrac{t}{2}K_1\right) \\ L_2 = f\left(\theta_n + \dfrac{t}{2}L_1, \varphi_n + \dfrac{t}{2}K_1\right) \end{cases}, \quad \begin{cases} \varphi_{n+1} = \varphi_n + tK_2 \\ \theta_{n+1} = \theta_n + tL_2 \end{cases} \tag{5}$$

Following the steps given from eq. (5), and with a given initial orientation, the fiber orientation evolution can be obtained. Through numerical experiments, the fully agreement is observed and the analytical solutions are validated.

Analysis of orientation evolution

Numerical model and parameters

For a curved expansion duct the boundary-fitted non-orthogonal grid is obtained by the Laplace transformation. The basic flow equations are:

$$\nabla \cdot \vec{u} = 0 \tag{6}$$

$$\nabla(\rho u \vec{u}) = -\nabla p + \mu \nabla^2 \vec{u} - \rho\left[2\vec{\omega} \times \vec{u} + \vec{\omega} \times (\vec{\omega} \times \vec{r})\right] \tag{7}$$

where $\vec{u}$ is the velocity vector, p – the pressure, ρ – the density, μ – the dynamic viscosity, $-2\vec{\omega} \times \vec{u}$ – the Coriolis force, $-\vec{\omega} \times (\vec{\omega} \times \vec{r})$ – the centrifugal force, $\vec{\omega}$ – the angular velocity vector, and $\vec{r}$ – the radius vector. ∇ is the Cartesian differential operator. The FVM based on Ferziger *et al.* [15] is employed to solve flow. It is assumed that the suspension is dilute and homogeneous, where the fluid density ρ = 1.0 kg/m^3, the fluid viscosity $\mu = 10^{-4}$ Pa·s. The inlet velocity is set to the tangent direction along the wall. When U = 0.2 m/s, the inlet Reynolds number Re = $\rho UR/\mu$ = 100, where inlet radius R = 50 mm. The one-way coupling is adopted, where the fiber orientation evolution along streamlines is figured out by considering many fibers starting from their specific initial orientations. To obtain detailed orientation evolution, each element is divided into 100 sub-elements. And taking the time step to be the smallest grid length divided by the local flow velocity. Then the fibers are injected with an initial orientation at the inlet. By this means, the fiber orientation evolution can be obtained step by step.

The influence of rotating rate

The impact of the rotating rate is regarded. The inlet velocity is U = 0.2 m/s, the initial orientation is (π/4, π/4), and r_p = 10. As shown in fig. 2, only the flow field is varied to observe the orientation evolution. The rotating rate is low so that the Re would satisfy the laminar regime. Thus the rotation mainly contributes to the pressure rise, and the flow structure is not obviously altered. However, there are small changes in the detailed flow structure. But as it can be seen that the rotation impact on the fiber orientation evolution is not distinct under the laminar regime.

The influence of the initial orientation

The impact of the initial orientation is regarded. The inlet flow velocity is 0.2 m/s, the rotating rate is 0.2 1/s, and $r_p = 10$. The fiber orientation evolution at three different initial orientations are examined. The results do not show an obvious tendency against the initial orientation. Along the convex-wall, the fiber is eventually close to the flow-plane. And with the in-plane angle increasing, the fiber also tends to the flow-plane along the concave-wall, but the trend is less obvious.

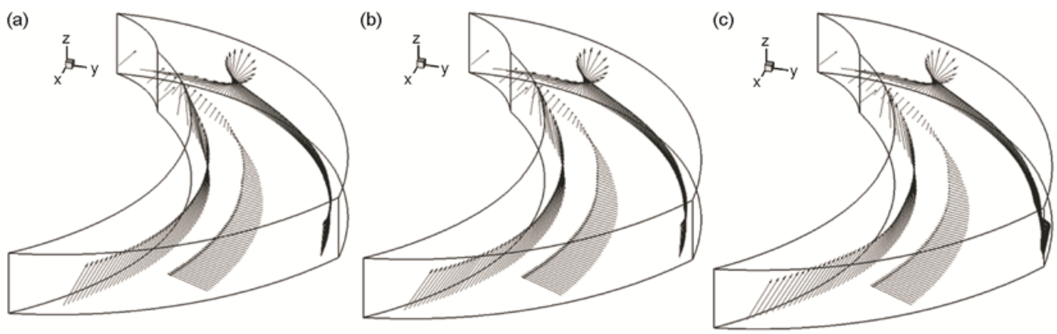

Figure 2. Fiber orientation at different rotating rates: (a) 0.1 [s^{-1}]; (b) 0.2 [s^{-1}]; (c) 0.3 [s^{-1}]

Along the central-line, although the initial fiber orientation evolution is distinct, the tendency at the downstream is almost identical. Due to the very large in-plane angle ($\pi/3$), the fiber rotates to the opposite direction at the initial stage, which leads to its distinct orientation evolution from others.

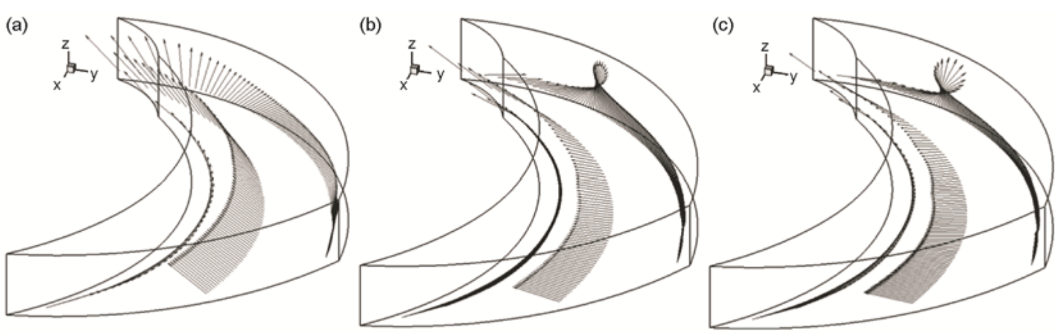

Figure 3. Fiber orientation evolution with different initial orientations (a) ($\pi/4$, $-\pi/2$); (b) ($\pi/4$, $-\pi/3$); (c) ($\pi/4$, $-\pi/6$)

Conclusions

The 3-D solution of the Jeffery equation is analytically explored. The one-way coupling based on the Jeffery equation is developed to examine the fiber orientation evolution within a rotating curved expansion duct. Distinct orientation behavior between the near-wall-regions and the central-regions is observed. The fiber orientation flips much quickly at the vicinity of the entrance. The rotation mainly contributes to the pressure rise. The influence of rotating rate is nearly negligible. In the central-region, the in-plane orientation is oblique to the streamline.

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