EVOLUTION OF PARTICLE SIZE DISTRIBUTION IN AIR IN THE RAINFALL PROCESS VIA THE MOMENT METHOD

by

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Population balance equation is converted to three moment equations to describe the dynamical behavior of particle size distribution in air in the rainfall. The scavenging coefficient is expressed as a polynomial function of the particle diameter, the raindrop diameter and the raindrop velocity. The evolutions of particle size distribution are simulated numerically and the effects of the raindrop size distribution on particle size distribution are studied. The results show that the raindrops with smaller geometric mean diameter and geometric standard deviation of size remove particles much more efficiently. The particles which fall in the "greenfield gap" are the most difficult to be scavenged from the air.

Key words: *particles in air in the rainfall process, scavenging coefficient, distribution of particle, numerical simulation*

Introduction

Our surroundings are filled with aerosol particles. The respiratory diseases are not only related to the particle mass concentration but also the particle size and number concentration. Therefore, it is necessary to remove aerosols from the air. Rainfall process is one of the most effective approaches to remove aerosols, in which raindrops collide with aerosols and then collect them. The removing process is affected by external factors, and in the rainfall process, the most interesting thing is how the particle size distribution (PSD) changes as time progresses and how PSD is affected by different raindrop size distribution (RSD). In order to answer the questions, the population balance equation (PBE) for particles is introduced. Slinn [1] obtained a semi-empirical formula of collision efficiency which has been widely used. Andronache [2, 3] concluded that the below-cloud scavenging (BCS) coefficients of aerosols by rainfall depends mainly on the aerosol size distribution parameters and on rainfall intensity. Later, Andronache *et al.* [4] developed a more complicated model to predict the scavenging coefficient. The research mentioned above was focused on getting the relation between the scavenging coefficient and the external factors. In the preset study the PBE is solved with referring to the wet scavenging process, and the scavenging coefficient is expressed as a polynomial function.

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Theory

The equation describing the removal of particles in air by collision with raindrops is [5]:

$$-\frac{\partial n(d_p,t)}{\partial t} = \int_0^\infty \frac{\pi D_d^2}{4} U(D_d) n(D_d) E(d_p,D_d) \mathrm{d}D_d n(d_p,t) \tag{1}$$

where $n(d_p, t)$ is the particle size distribution in air, $U(D_d)$ – the velocity of a falling raindrop with diameter D_d and can be expressed as follows:

$$\begin{cases} U(D_d) = 30.75 D_d^2 \cdot 10^6 & D_d < 100\ \mu\mathrm{m} \\ U(D_d) = 38 D_d^2 \cdot 10^3 & 100\ \mu\mathrm{m} < D_d < 1000\ \mu\mathrm{m} \\ U(D_d) = 133.046 D_d^{0.5} & D_d > 1000\ \mu\mathrm{m} \end{cases} \tag{2}$$

The collision efficiency is given as [1]:

$$E(d_p,D_d) = \frac{1}{\mathrm{Re}\cdot\mathrm{Sc}}[1 + 0.4\sqrt{\mathrm{Re}}\sqrt[3]{\mathrm{Sc}} + 0.16\sqrt{\mathrm{Re}}\sqrt{\mathrm{Sc}}] +$$

$$+ 4\frac{d_p}{D_d}\left[\frac{\mu_a}{\mu_w} + 1 + \sqrt{\mathrm{Re}}\ \frac{d_p}{D_d}\right] + \sqrt{\left(\frac{\mathrm{St} - S^*}{\mathrm{St} - S^* + \frac{2}{3}}\right)^3} \tag{3}$$

where Re = $D_d U(D_d)\rho_a/(2\mu_a)$ is the Reynolds number, Sc = $\mu_a/(\rho_a D_{diff})$ – the Schmidt number with the diffusion coefficient $D_{diff} = k_b T C_c/(3\pi\mu_a d_p)$, St = $2\tau U(D_d)/D_d$ – the Stokes number of particles with relaxation time $\tau = \rho_p d_p^2/(18\mu_a)$, and $S^* = [1.2 + \ln(1 + \mathrm{Re})/12]/[1 + \ln(1 + \mathrm{Re})]$ – a dimensionless parameter. Here, μ_w is the viscosity of a water drop, k_b – the Boltzmann's constant, T – the absolute temperature of air, and C_c – the Cunningham slip correction factor and can be approximated as follows [6]:

$$C_c = 1 + 2.493\frac{\lambda}{d_p} + 0.84\frac{\lambda}{d_p}\exp\left(-0.435\frac{d_p}{\lambda}\right) \cong 1 + 3.34\frac{\lambda}{d_p} \tag{4}$$

where λ is the molecular mean free path. The terms on the right-hand side of eq. (3) represent the effects of Brownian diffusion, interception and inertial impaction, respectively. In eq. (1) the particle size distribution can also be approximated by :

$$\begin{aligned} n(D_d) &= \frac{N_d}{\sqrt{2\pi}\ln\sigma_{dg}}\exp\left[-\frac{\ln^2(D_d/D_{dg})}{2\ln^2(\sigma_{dg})}\right]\frac{1}{D_d} \\ n(d_p) &= \frac{N_p}{\sqrt{2\pi}\ln\sigma_{pg}}\exp\left[-\frac{\ln^2(d_p/d_g)}{2\ln^2(\sigma_{pg})}\right]\frac{1}{d_p} \end{aligned} \tag{5}$$

where N_d, D_{dg}, and σ_{dg} are the number concentration, geometric mean diameter and geometric standard deviation of raindrop, respectively. The k-th moment of raindrops and particles are :

$$\xi_k = \int_0^\infty D_d^k n(D_d)\mathrm{d}D_d = \xi_0 D_{dg}^k \exp\left(\frac{k^2}{2}\ln^2\sigma_{dg}\right)$$
$$m_k = \int_0^\infty d_p^k n(d_p)\mathrm{d}d_p = m_0 d_{pg}^k \exp\left(\frac{k^2}{2}\ln^2\sigma_{dp}\right) \quad (6)$$

The final expression of scavenging coefficient and governing moment equation are [7]:

$$\Lambda(d_p) = \gamma_1\xi_1(d_p^{-1} + Ad_p^{-2}) + \gamma_2\xi_{7/4}\left(d_p^{-2/3} + \frac{2Ad_p^{-5/3}}{3}\right) + \gamma_3\xi_{7/4}\left(d_p^{-1/2} + \frac{Ad_p^{-3/2}}{2}\right) +$$
$$+\gamma_4\xi_{3/2}d_p + \gamma_5\xi_{1/2}d_p^2 + \gamma_6\xi_{5/4}d_p^2 + \gamma_7\xi_{5/2} + \gamma_8\xi_{11/4}d_p^{-1} \quad (7)$$

$$-\frac{\partial m_k}{\partial t} = \gamma_1\xi_1(m_{k-1} + Am_{k-2}) + \gamma_2\xi_{7/4}\left(m_{k-2/3} + \frac{2Am_{k-5/3}}{3}\right) + \gamma_3\xi_{7/4}\left(m_{k-1/2} + \frac{Am_{k-3/2}}{2}\right) +$$
$$+\gamma_4\xi_{3/2}m_{k+1} + \gamma_5\xi_{1/2}m_{k+2} + \gamma_6\xi_{5/4}m_{k+2} + \gamma_7\xi_{5/2}m_k + \gamma_8\xi_{11/4}m_{k-1} \quad (8)$$

$$\gamma_1 = \frac{k_b T}{6\mu_a}, \quad \gamma_2 = \left(\frac{0.4\cdot 130\pi}{4}\right)\left(\frac{2\mu_a}{130\rho_a}\right)^{1/2}\left(\frac{k_b\rho_a T}{3\pi\mu_a^2}\right)^{2/3},$$

$$\gamma_3 = \left(\frac{0.16\cdot 130\pi}{4}\right)\left(\frac{2k_b T}{3\cdot 130\pi\mu_a}\right)^{1/2}, \quad \gamma_4 = \frac{130\pi\mu_a}{\mu_w}, \quad \gamma_5 = 130\pi,$$

$$\gamma_6 = 130\pi\left(\frac{130\rho_a}{2\mu_a}\right)^{1/2}, \quad \gamma_7 = \frac{130\pi}{4}, \quad \gamma_8 = -\left(\frac{0.9\cdot 130\pi}{4}\right)\left(\frac{18\mu_a}{2\cdot 130\rho_p}\right)^{1/2}, \quad A = 3.34\lambda$$

According to the definition of m_k, m_0 is the total particle number concentration, and $(\pi/6)\, m_3$ is the total volume of particles. We solve the first three moment equations and to get the geometric mean particle diameter d_{pg}, and the geometric standard deviation σ_{pg}.

Results and discussions

The Runge-Kutta method is employed to solve eq. (8) with $k = 0$, 1, and 2. The evolutions of PSD are simulated numerically and the effects of RSD on PSD are studied in nine cases. The values of the initialization parameter are listed in tab. 1.

Figures 1 and 2 show the evolution of particle number concentration and instantaneous PSD. For case 6 and 8 the geometric standard deviation of raindrop diameter, σ_{dg}, is same but the geometric mean raindrop diameter, D_{dg}, is different.

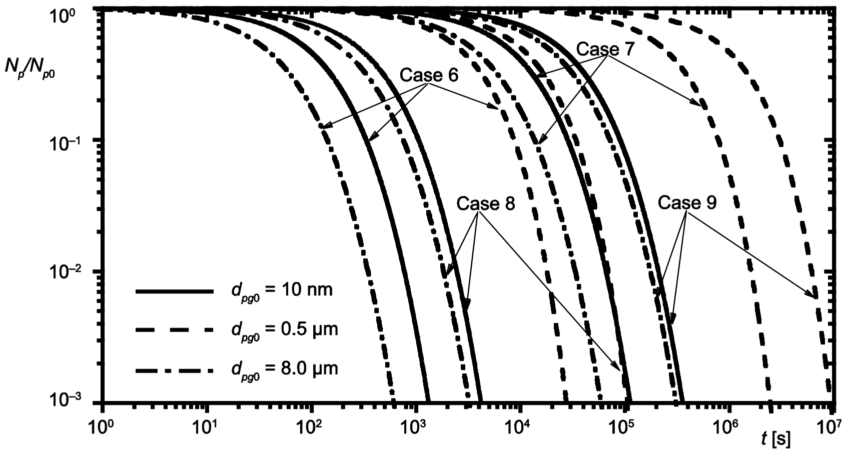

Figure 1. Effect of RSD on particle number concentration for cases 6-9

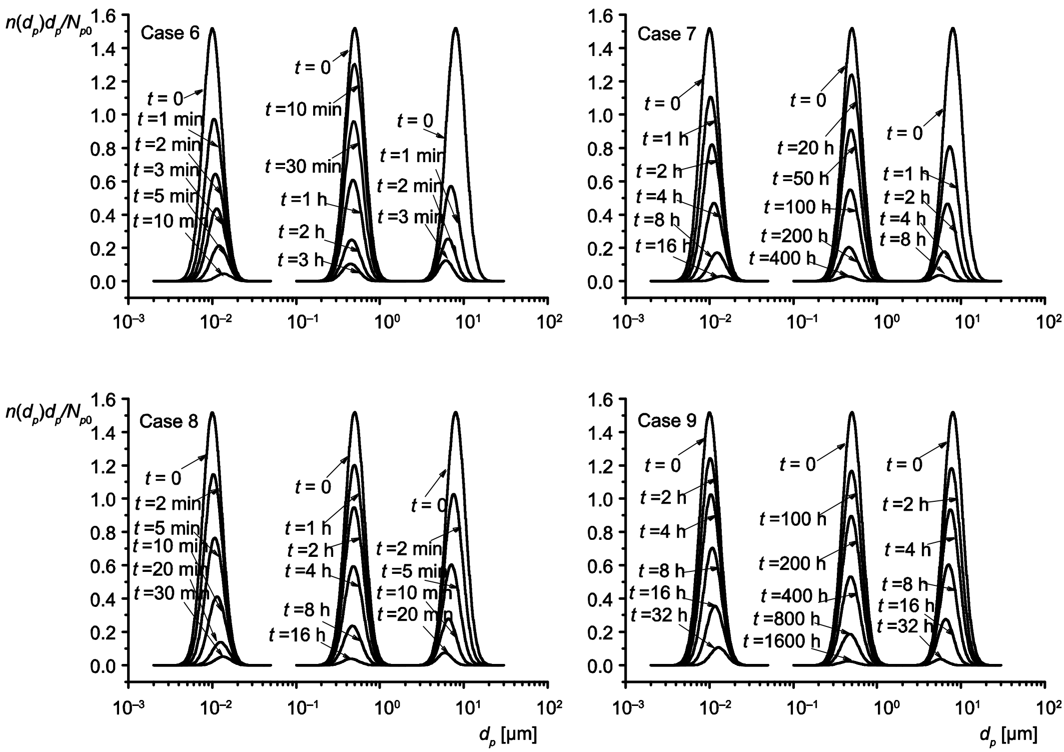

Figure 2. Evolution of PSD under the effect of RSD as time progresses for cases 6-9

As shown in fig. 2 the particles with d_{pg0} = 10 nm have been almost removed within 10 minutes in case 6, while it needs about 30 minutes in case 8. The particle number concentration decreases faster in case 6 than that in case 8. For particles with d_{pg0} = 0.5 μm and d_{pg0} = 8 μm in cases 6 and 8, the same tendency can be observed, which demonstrates that the RSD in case 6 can scavenge particles with much higher efficiency than that in case 8.

Table 1. The values of the initialization parameter for cases 1-9

Case	N_d m^{-3}	D_{dg} mm^{-1}	σ_{dg}	N_{p0} m^{-3}	d_{pg0} μm^{-1}	σ_{pg0}
1	$1.0 \cdot 10^5$	0.1	1.2	$1.0 \cdot 10^6$	0.001	1.2/1.5/1.8
2	$1.0 \cdot 10^5$	0.1	1.2	$1.0 \cdot 10^6$	0.01	1.2/1.5/1.8
3	$1.0 \cdot 10^5$	0.1	1.2	$1.0 \cdot 10^6$	0.1	1.2/1.5/1.8
4	$1.0 \cdot 10^5$	0.1	1.2	$1.0 \cdot 10^6$	5.0	1.2/1.5/1.8
5	$1.031 \cdot 10^7$	0.2	1.1	$1.0 \cdot 10^6$	0.01/0.5/8.0	1.3
6	$9.569 \cdot 10^4$	0.2	1.5	$1.0 \cdot 10^6$	0.01/0.5/8.0	1.3
7	$6.601 \cdot 10^5$	0.5	1.1	$1.0 \cdot 10^6$	0.01/0.5/8.0	1.3
8	$6.124 \cdot 10^3$	0.5	1.5	$1.0 \cdot 10^6$	0.01/0.5/8.0	1.3
9	$6.786 \cdot 10^4$	0.5	1.5	$1.0 \cdot 10^6$	0.01/0.5/8.0	1.3

Conclusions

Particles dominated by Brownian diffusion are removed more easily. The particle number concentration decreases much more rapidly when d_{pg} is smaller. In the scavenging process, the particle geometric mean diameter increases when d_{pg0} < 20 nm, and decreases when $d_{pg0} \geq 1$ μm, but changes a little when 20 nm < d_{pg0} < 1 μm. The PSD tends to be mono-disperse, and decays much faster for large σ_{pg0}. The RSD with small D_{dg} can remove particles with much higher efficiency than that with large D_{dg}, and the RSD with small σ_{dg} can collect particles more easily than that with large σ_{dg}.

Acknowledgments

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