A NEW METHOD FOR FIBER ORIENTATION DISTRIBUTION IN A PLANAR CONTRACTING TURBULENT FLOW

by

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A new Euler model is proposed to predict fiber orientation distribution at high Reynolds number in a dilute planar contraction. The model mainly accounts for the evolution of mean velocity and turbulence-induced rotational diffusion coefficient along the contraction. It is shown that the new model, as the function of local turbulent dissipation rate, fiber length, and fluid viscosity, can reflect the subtle change of fiber orientation accurately.

Key words: *rotational diffusion, fiber orientation, planar contraction, dilute turbulence*

Introduction

Turbulent fiber suspensions in a planar contraction have a relatively wide range of applications in the modern industry. There are mainly two methods to describe the motion of fibers in turbulent flow. One is the Lagrange method, in which the various forces are considered to calculate the fiber orientation [1-3]. However, the existence of fibers will affect the turbulent flow. Direct numerical simulation can be used to describe the interaction between fiber and fluid, for example, the lattice Boltzmann method [4-6]. For the Lagrange method Jeffery gave his pioneer work [7]. Based on Jeffery equation, Goldsmith *et al.* [8] derived the general first-order differential equations of fiber orientation in 3-D mean velocity gradient flows, which have been used to estimate the orientation by many researchers [9-16]. But in order to obtain an accurate prediction, a great mass of particles is needed so that the computer's performance may be challenged. And what's more, some instantaneous quantities cannot be computed from the particle simulation. The other is the Euler method, in which a probability distribution function $f(r, p, t)$, as eq. 1, is used to describe the fiber orientation state. In view of turbulent effect, the convection-dispersion type eq. 2 is given by Doi *et al.* [17]. In addition, according to the classical theory of turbulence and Reynolds statistical method the turbulent quantities can be expressed by means of phase-averaged approach. Thus Shen *et al.* [18] gave a general average probability distribution eq. 3 of fiber orientation in an incompressible turbulent flow.

Based on the approach of Shen *et al.* [18], there are two problems to solve for its application in the contraction flow, namely how to model the evolution of mean velocity and how to define diffusion coefficients along the contraction. In contrast to the works by Parsheh *et al.* [19], the following sections will focus on the two issues above.

$$\frac{\partial f}{\partial t} + u_j \frac{\partial f}{\partial x_j} = -\frac{\partial (f\dot{p}_j)}{\partial p_j}$$

(1)

$$\frac{\partial f}{\partial t} = D_p \nabla_r^2 f - \nabla_r \cdot (\omega f) + D_t \nabla^2 f - \nabla \cdot (\overline{V} f)$$

(2)

$$\frac{\partial \overline{f}}{\partial t} + \overline{u}_i \frac{\partial \overline{f}}{\partial x_i} + \frac{\partial (\overline{f} \, \overline{\dot{p}}_i)}{\partial p_i} = D_t \frac{\partial \overline{f}}{\partial x_i} + D_p \frac{\partial \overline{f}}{\partial p_i}$$

(3)

$$\overline{U}_x \frac{\partial \overline{f}}{\partial x} - 2 \frac{\partial \overline{f}}{\partial \varphi} \frac{\partial \overline{U}_x}{\partial x} \sin(2\varphi) - \overline{f} \cos(2\varphi) \frac{\partial \overline{U}_x}{\partial x} = D_t \frac{\partial^2 \overline{f}}{\partial x^2} + D_p \frac{4}{\sin^2(2\varphi)} \frac{\partial^2 \overline{f}}{\partial \varphi^2}$$

(4)

Mathematical modeling and solving

According to the Parsheh's experiment [19], 2-D axis-symmetric contraction flow is shown as fig. 1. The contraction is 550 mm in length, 179.2 mm in inlet height and 16 mm in outlet height, giving the contraction half angle $\beta = 8.4°$. The origin of co-ordinates lies at the contraction inlet, x denoting the streamwise direction and y the velocity gradient direction. The fibers are nominally 3.2 mm in length and 57 μm in diameter. The suspension's nL^3 value is 0.0053, which means dilute regime hypothesis can be applied. The macroscopic Re is based on the mean streamwise velocity, the local height and the kinematic viscosity, *i. e.* Re = $(U_0 h_0)/\nu$. The microscopic Re is based on the fiber diameter as the length scale, *i. e.* $Re_f = [(\partial U_x/\partial x)d_f^2]/\nu$, so tha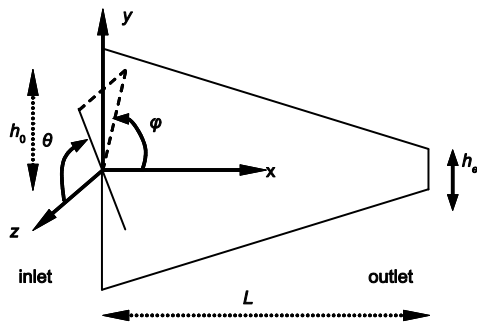t effect of fiber inertia is negligible. The grid resolution 3000 × 540 in x and y dimensions is considered for the contraction.

Figure 1. Contraction flow

Evolution of $\overline{f}$ equation in the contraction

Parsheh's experimental results showed that mean velocity components measurements in the contraction are in good agreement with the calculated values from simple quasi-one-dimensional potential flow equation, *i. e.* $U_x = U_0 h_0/(h_0 - 2x\,\text{tg}\,\beta)$ and $U_y = -2yU_0 h_0 \,\text{tg}\,\beta/(h_0 - 2x\,\text{tg}\,\beta)^2$. For the 2-D steady flow, there are $\partial f/\partial t = 0$ and $p = (\cos \varphi, \sin \varphi)^T$. When focusing the evolution of $\overline{f}$ along the center symmetry plane like the mostly existing investigation, namely $y = 0$, eq. (3) can be greatly simplified into eq. (4) thereby (referred to as the $\overline{f}$ method below). It is noted that the streamwise rate of strain, $\partial U_x/\partial x$, is the dominant term influencing fiber orientation in the contraction.

The orientation distribution $\overline{f}$ is determined by solving eq. (4) numerically. Defining $\overline{f}_j^n = f(\varphi_j, x_n)$, differential scheme is used to get the solution by means of discrete x and φ. Due to the symmetry of $\overline{f}$ with respect to value range of φ, here we only consider the interval $\varphi_0 \in [-\pi/2, \pi/2]$ while $x_0 \in [0, 550]$. Here the x-derivative is regarded as a time co-ordinate and the fiber orientation φ-derivative as a spatial co-ordinate. At the contraction inlet we assume that fibers are distributed homogeneously and use this initial condition as a Dirichlet boundary condition. For $\varphi_0 = -\pi/2$, we consider Neumann boundary conditions.

Fitting of diffusivity coefficient D_p

Since the initial fiber orientation is uniformly distributed and translational diffusion coefficient D_t is very small, here we only consider rotational diffusion D_p of fibers along the central streamline. Parsheh [20] experimentally showed that the dissipation length scale may be assumed a constant along the contraction. We define $D_p = [a(v/\varepsilon)^{1/2} + bL_f v\varepsilon)^{-1/4}]^{-1}$, which is a local value related to the fiber length L_f and the turbulent traits. The test data for comparison are from Parsheh *et al*. [19], *i. e*. Cases 1 and 2 as listed in tab. 1.

Table 1. Some flow parameters of suspension in the contraction for Cases 1 and 2

	ε_0 [m²s⁻³]	k_0 [m²s⁻²]	U_{x0} [ms⁻¹]	u_{rms0} [ms⁻¹]	Λ_0 [mm]	η_0 [mm]	Re_λ	Re	a	b
1	$3.05 \cdot 10^{-3}$	$1.034 \cdot 10^{-3}$	0.4375	$2.626 \cdot 10^{-2}$	10.91	0.124	48~122	85000	8.0	1.2
2	$2.44 \cdot 10^{-2}$	$4.138 \cdot 10^{-3}$	0.875	$5.252 \cdot 10^{-2}$	10.91	0.073	67~182	170000	2.0	1.2

The state ε_0 and k_0 at the inlet of the contraction is chosen to characterize the initial turbulent levels, then the local turbulence kinetic energy, k and turbulence dissipation rate ε, are estimated by CFD model to ensure outlet velocity nearly 5 m/s for case 1 and 10.0 m/s for case 2. The parameters a and b depend on the fitting result of orientation tensor a_{1111} of Cases 1 and 2, as shown in fig. 2. Here $C = h/h_0 = (h_0 - 2x\mathrm{tg}\,\beta)/h_0$ is the contraction ratio.
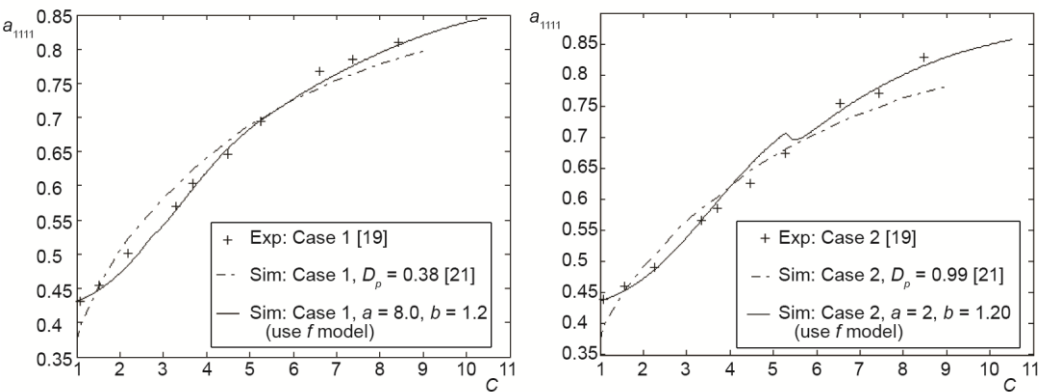
Figure 2. Development of a_{1111} along the contraction (experiment data [19] and results from the fitted constant D_p [21])

Discussion

Figure 2 shows that our fitting result agrees better with experimental ones. Though there is a slight leap of a_{1111} near $C = 2\sim3$ in Case 1 and $C = 5.4$ in Case 2 which happen earlier than experiment data near $C = 7$ in Case 2, it might capture the trend of a_{1111} near the outlet and this trend is more obvious with the increase of inlet velocity. The most likely source of the slight leap was the turbulent energy production near the outlet which results in a secondary orientation of the fiber. Here the difference in C position may be related to the translational diffusion neglected in calculations. And what's more, the higher inlet velocity and turbulent level are, the closer to the outlet the leap happens. It seems to be due to the contest between turbulence intensity from the inlet and those generated in the contraction.

Conclusions

A model is proposed to predict fiber orientation distribution in the contraction flow. Turbulence-induced rotational diffusion coefficient D_p is defined to vary with the local turbulent characteristics. From the fitting result of a_{1111}, local D_p can ensure the solution more accurate than the constant one. Maybe the constant D_p has the advantage of simplifying the calculation.

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