SEMI-ANALYTICAL METHOD FOR SOLVING NON-LINEAR EQUATION ARISING OF NATURAL CONVECTION POROUS FIN

by

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Original scientific paper
DOI: 10.2298/TSCI1205303P

In the present study, the problem of non-linear model arising in heat transfer through the porous fin in a natural convection environment is presented and the homotopy perturbation method is employed to obtain an approximate solution, which admits a remarkable accuracy.

Key words: *porous fin, natural convection, homotopy perturbation method, Darcy's model*

Introduction

There are few phenomena in different fields of science occurring linearly. Most of problems and scientific phenomena such as heat transfer are inherently of non-linearity. We know that except a limited number of these problems, most of them do not have exact solutions. Therefore, these non-linear equations should be solved approximately either numerically or analytically. In the numerical method, stability and convergence should be considered so as to avoid divergence or inappropriate results. Time consuming is another problem of numerical techniques. In analytical methods, the perturbation method is widely used, but in most cases to find a suitable small parameter is difficult. Therefore, many different methods have recently introduced such as the Exp-Function method [1], the Adomian's decomposition method [2], the homotopy perturbation method (HPM) [3-6], the variational iteration method (VIM) [7-10], and the homotopy analysis method [11-20].

In this paper, we present a proper procedure based on HPM to find the approximate solutions of non-linear differential equations governing porous fin. Results demonstrate that HPM is simple and convenient.

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Governing equation

As shown in fig. 1, a rectangular fin profile is considered. The dimensions of the fin are length L, width W and thickness t. The cross-section area of the fin is constant. This fin is porous to allow the flow of infiltrate through it. The following assumptions are made to solve this problem. The porous medium is isotropic and homogenous, The porous medium is saturated with singlephase fluid, The surface radiant exchange is neglected, Physical properties of both fluid and solid matrix are constant, The temperature inside fin is only function of X, There is no temperature variation across the fin thickness, The solid matrix and fluid are assumed to be at local thermal equilibrium with each other, The interactions between the porous medium and the clear fluid can be simulated by the Darcy formulation.

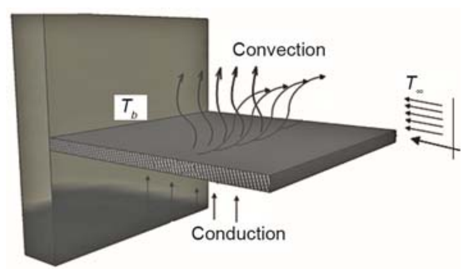

Figure 1. Schematic diagram of porous fin profile

The energy balance of the slice segment of the fin of thickness Δx requires that:

$$q(x) - q(x + \Delta x) = \dot{m} c_p \left[T(x) - T_\infty \right] \tag{1}$$

The mass flow rate of the fluid passing through the porous material can be written as:

$$\dot{m} = \rho v_w \Delta x W \tag{2}$$

From the Darcy's model we have:

$$v_w = \frac{\mathrm{g} k \beta}{\nu} \left[T(x) - T_\infty \right] \tag{3}$$

Substitutions of eq. (2) and (3) into eq. (1) yields:

$$\frac{q(x) - q(x + \Delta x)}{\Delta x} = \frac{\rho c_p \mathrm{g} k \beta w}{\nu} \left[T(x) - T_\infty \right]^2 \tag{4}$$

As, $\Delta x \to 0$ eq. (4) becomes:

$$\frac{\mathrm{d}q}{\mathrm{d}x} = \frac{\rho c_p \mathrm{g} k \beta w}{\nu} \left[T(x) - T_\infty \right]^2 \tag{5}$$

From Fourier‘s Law of conduction, we have:

$$q = -k_{eff} A \frac{\mathrm{d}T}{\mathrm{d}x} \tag{6}$$

where A is the cross-sectional area of the fin $A = (wt)$ and k_{eff} – the effective thermal conductivity of the porous fin given by $k_{eff} = \varphi k_f + (1 - \varphi) k_s$. Substitution eq. (6) into eq. (5) gives:

$$\frac{d^2T}{dx^2}-\frac{\rho c_p g k \beta}{t\, k_{eff}\, \nu}\left[T(x)-T_\infty\right]^2=0 \tag{7}$$

Hence, with applying energy balance equation at steady-state condition, and introducing non-dimensional temperature function, where, $\theta = [T(x) - T_\infty]/(T_b - T_\infty)$ and $X = x/L$ into eq. (7) we have:

$$\frac{d^2\theta}{dX^2}-S_h\,\theta(X)^2=0 \tag{8}$$

$$\theta(1)=1, \quad \theta'(0)=0 \tag{9}$$

where S_h = (Da Ra/k_r)$(L/t)^2$ is a porous parameter.

Application of homotopy perturbation method

In this section, we employ HPM to solve eq. (8) subject to boundary conditions (9). We can construct homotopy function of eq. (8) as described in [6]:

$$H(\theta,\ p)=(1-P)\left[\theta''(X)-g_0(X)\right]+p\left[\theta''(X)-S_h\,\theta(X)^2\right]=0 \tag{10}$$

where $p = \in [0, 1]$ is an embedding parameter. For $p = 0$ and $p = 1$ we have:

$$\theta(X,0)=\theta_0(X), \quad \theta(X,1)=\theta(X) \tag{11}$$

Note that when p increases from 0 to 1, $\theta(X, p)$ varies from $\theta_0(X)$ to $\theta(X)$. By substituting:

$$\theta(X)=\theta_0(X)+p\,\theta_1(X)+p^2\,\theta_2(X)+\cdots=\sum_{i=0}^{n}p^i\theta_i(X), \quad g_0=0 \tag{12}$$

Into eq. (10) and re-arranging the result based on powers of p-terms, we have:

$$p^0: \quad \theta''_0(X)=0, \qquad \theta_0(1)=1, \quad \theta'_0(0)=0 \tag{13}$$

$$p^1: \quad \theta''_1(X)-S_h\theta_0(X)^2=0, \qquad \theta_1(1)=0, \quad \theta'_1(0)=0 \tag{14}$$

$$p^2: \quad \theta''_2(X)-2S_h\theta_0(X)\theta_1(X)=0, \qquad \theta_2(1)=0, \quad \theta'_2(0)=0 \tag{15}$$

$$\vdots$$

Solving eqs. (13–15) with boundary conditions, we have:

$$\theta_0(X)=1 \tag{16}$$

$$\theta_1(X)=0.5000\,S_h\,X^2-0.5000S_h \tag{17}$$

$$\theta_2(X) = 0.08333333 S_h^2 x^4 - 0.5000000 S_h^2 x^2 + 0.41666666 S_h^2 \\ \vdots \tag{18}$$

The solutions $\theta(X)$ were too long to be mentioned here, therefore, they are shown graphically. The solution of this equation, will be as follows ($S_h = 0.02$ for example).

$$\begin{aligned} \theta(X) &= \sum_{i=0}^{9} \lim_{p\to 1} p^i \theta_i(X) = \\ &= +3.971902855 10^{-23} x^{18} + 1.430531060 10^{-20} x^{16} + 5.671690645 10^{-18} x^{14} + \\ &+2.194326998 10^{-15} x^{12} + 8.310515092 10^{-13} x^{10} + 3.021506772 10^{-10} x^{8} + \\ &+1.068033296 10^{-7} x^{6} + 0.00003235930767 x^4 + 0.009804233606 x^2 + \\ &+ 0.9901633001 \end{aligned} \tag{19}$$

Results and discussion

In this paper, the homotopy perturbation method such as analytical technique is employed to find an analytical solution of the non-linear fin problem. For validation all these results are compared with the BVP. The main goal of this article is to show the simplicity and power of HPM.

Table 1. The results of analytical method and Numerical methods for $\theta(X)$ for S_h = 0.09

X	$\theta(X)$		
	HPM	NUM	Error of HPM
0	0.9580905354	0.9580905400	0.0000000046
0.10	0.9585036666	0.9585036704	0.0000000038
0.20	0.9597437732	0.9597437774	0.0000000042
0.30	0.9618129969	0.9618130011	0.0000000043
0.40	0.9647149174	0.9647149213	0.0000000039
0.50	0.9684545682	0.9684545716	0.0000000035
0.60	0.9730384581	0.9730384612	0.0000000032
0.70	0.9784745996	0.9784746020	0.0000000024
0.80	0.9847725440	0.9847725462	0.0000000022
0.90	0.9919434235	0.9919434260	0.0000000025
1.00	1.0000000000	1.0000000000	0.0000000000

Figures 2-5 show the temperature distribution in the porous fin for different value of S_h. Comparing figs. 2-5 gives closer results to numerical solution. It is observed that the HPM is very effective, simple, and more accurate method. Moreover, in order to show the effectiveness of HPM, numerical comparison with approximate solutions proposed are tabulated in tab. 1. It is interesting to note that the results obtained by the homotopy perturbation method are very close to the numerical results.

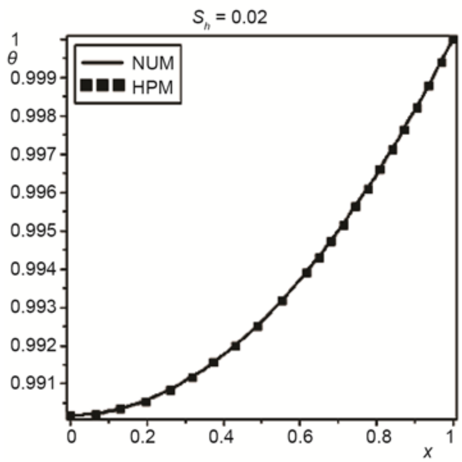

Figure 2. The comparison between the numerical and HPM solution when S_h = 0.02

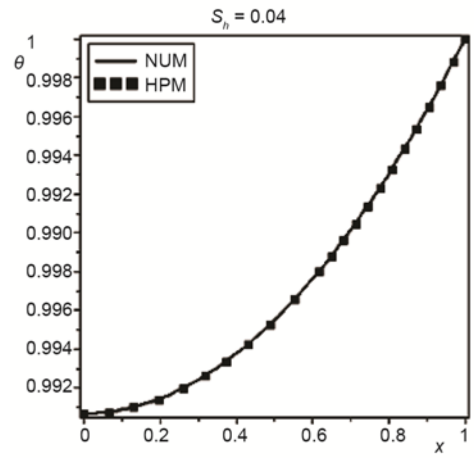

Figure 3. The comparison between the numerical and HPM solution when S_h = 0.04

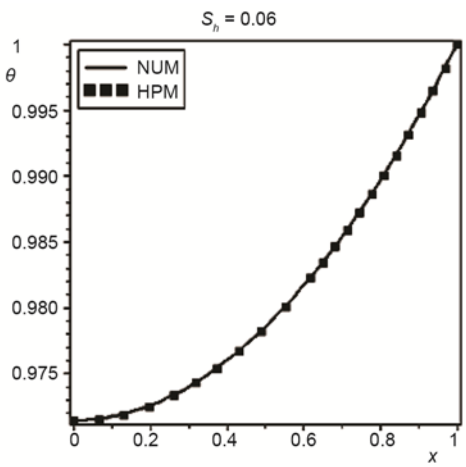

Figure 4. The comparison between the numerical and HPM solution when S_h = 0.06

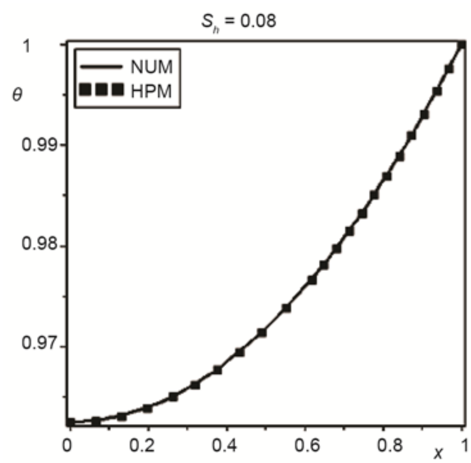

Figure 5. The comparison between the numerical and HPM solution when S_h = 0.08

Conclusions

In this paper, we present a proper procedure based on HPM to study temperature distribution of non-linear differential equations governing on porous fin. Also, energy balance and Darcy's model are used to formulate the heat transfer equations. A numerical method is carried out to verify the validation of the method. The obtained results show that this method is very convenient and effective.

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Paper submitted: August 12, 2012
Paper revised: September 7, 2012
Paper accepted: September 12, 2012