SHORT-TERM AND LONG-TERM THERMAL PREDICTION OF A WALKING BEAM FURNACE USING NEURO-FUZZY TECHNIQUES

by

Hamed Dehghan BANADAKI[a*]***, Hasan Abbasi NOZARI***[b]***, and Mahdi Aliyari SHOOREHDELI***[c]

[a] Department of Electrical Engineering, Ashkezar Branch, Islamic Azad University, Yazd, Iran
[b] Department of Electrical Engineering, Joybar Branch, Islamic Azad University, Joybar, Iran
[c] Department of Mechatronics, Faculty of Electrical Engineering, K. N. Toosi University, Tehran, Iran

Original scientific paper
DOI: 10.2298/TSCI120410210B

The walking beam furnace is one of the most prominent process plants often met in an alloy steel production factory and characterised by high non-linearity, strong coupling, time delay, large time-constant and time variation in its parameter set and structure. From another viewpoint, the walking beam furnace is a distributed-parameter process in which the distribution of temperature is not uniform. Hence, this process plant has complicated non-linear dynamic equations that have not worked out yet. In this paper, we propose one-step non-linear predictive model for a real walking beam furnace using non-linear black-box subsystem identification based on locally linear neuro-fuzzy model. Furthermore, a multi-step predictive model with a precise long prediction horizon (i. e., ninety seconds ahead), developed with application of the sequential one-step predictive models, is also presented for the first time. The locally linear model tree which is a progressive tree-based algorithm trains these models. Comparing the performance of the one-step linear neuro-fuzzy model predictive models with their associated models obtained through least squares error solution proves that all operating zones of the walking beam furnace are of non-linear sub-systems. The recorded data from Iran Alloy Steel factory is utilized for identification and evaluation of the proposed neuro-fuzzy predictive models of the walking beam furnace process.

Key words: *non-linear prediction, walking beam furnace, locally linear neuro-fuzzy, locally linear model tree, least-squares error*

Introduction

Literature review

Walking beam furnace (WBF) is one of the most crucial components of a steel production factory, which is a complex process with sequential activities demanding technical supports such as model-based fault diagnosis, predictive control, reconfiguration, maintenance, repair, and other operations. Moreover, the WBF consists of several operating zones, which have profound effects on the production rate, strip quality, and the stability of operation. In order to develop energy-saving techniques for WBF process, initially, it is vital to create a reliable model of it. Although due to high non-linearity, large time delay, large time-con-

* Corresponding author; e-mail: hmd.dehghan@gmail.com

stant and various uncertain factors, the non-linear modelling of WBF based on prediction techniques has become a challenging problem, extracting an appropriate non-linear predictive model of the WBF process appears to be indispensable for non-linear model-based predictive control and condition monitoring/diagnosis trials purposes.

Recently, a few attempts have been made to make use of data-driven techniques to identify an appropriate thermal predictive model of the WBF process. Dynamic modelling of the quality control of a real large-scale continuous annealing process based on generalized growing-and-pruning radial basis function (RBF) neural network was presented in [1]. In [2], a non-linear predictive model of a WBF was constructed based on thermal equations, which describe the behavior of furnace under the state of rolling line and fuel flux provided by control loop model. An adaptive feed forward neural network model for predicting the temperature of the slabs in the soaking zones of the WBF whilst the slabs were still in the furnace was proposed in [3]. In [4], online sub-system modelling of a real continuous annealing process was investigated, and a novel dynamic generalized growing-and-pruning radial basis function network was established to generate the quality model of the process. Non-linear simulator model of a real furnace with a modular structure was proposed based on recurrent neuro-fuzzy models for the first time [5]. In [6], a multivariable system identification method to estimate the parameters of a linear predictive model for a walking beam re-heating furnace according to ARX-procedure was described. Particular dynamic network architecture namely internal feedback neuron network (IFNN) is presented in [7], which is trained using a specially derived gradient-based training algorithm. For validating the dynamics capturing capability of this network, it was used to predict an ignition hood furnace used in steel industries. A MIMO radial basis function model of the WBF process whose structure is originally based on an improved sequential-learning algorithm is presented in [8]. This algorithm employs an improved growing-and-pruning algorithm based on the notion of the significance of hidden neurons in which an extended Kalman filter improves the learning accuracy. In [9], a dynamic multilayer perception model of a real WBF trained by PSO algorithm was proposed. Evolutionary algorithms such as genetic algorithm have been employed to estimate the parameters of the mathematical model of walking hearth-type reheating furnace from measured temperature in each zone of reheating process and then the acquired model was used in optimal control of the fuel consumption [10]. An online just-in-time local modelling technique applicable to a large amount of database has been proposed in [11]. This technique that makes the retrieval of neighboring data more efficient by using stepwise selection and quantization is applied to long period prediction of the variables of a dynamic industrial furnace with several deeply-intertwined physical phenomena. In [12], a corrective neural network was proposed with a long-term learning method to improve the accuracy of rolling-force prediction in hot-rolling mill. The learning algorithm was devised so as the proposed neural network could cope with the difficulties such as low thickness accuracy at the first-coil which exist over conventional short-term learning. Since in practice, the rolling information of roughing mill cannot be fed back dynamically to reheating furnace in a hot rolling process. Thus, owing to such shortcoming, the energy consumption enlarges in this process. To tackle this problem, fuzzy neural network (FNN) is used to deal with the feedback of rolling information, and then real-time compensation of furnace temperature [13]. Neural networks are also applied as components of hybrid neuro/analytical process models of the technological processes such as rolling mill process [14]. Such models are introduced to be the keys to fit the general physical models to the needs of the automation of a specific mill.

Apart from the state-of-the-art, to our knowledge, no research has been devoted to cope with the construction of short-term and long-term piecewise linear neuro-fuzzy predictive models of a real WBF. In the present research, the short-term and long-term LLNF predictive models of a WBF process with innovative coupled and integrated configurations are presented for the first time. All the former intelligent approaches exploited for the temperature prediction of WBF process were merely based upon the artificial neural networks (ANN). However, the main shortcoming of ANN is that processes cannot be expressed in them because they are usually considered as black boxes. Neuro-fuzzy modelling can be regarded as a grey-box technique bridging neural networks and qualitative fuzzy models in which system is expressible in fuzzy rules with using fuzzy modelling. The most common neuro-fuzzy systems are based on two types of fuzzy models, Takagi-Sugeno (TS) and Mamdani, combined with ANN learning algorithms. However, the TS-type neuro-fuzzy model is preferable when the accuracy of the model represents the foremost concern [15]. The LLNF model exploited in the present study are also simply interpretable as TS-type neuro-fuzzy systems, which can yield accurate model of the WBF under consideration.

Case study: walking beam furnace

Iran Alloy Steel Company (IASCO) located at 30 km far from Yazd, the city in center of Iran, was founded in 1999 and is one of the largest steel production factories in Middle East and Iran. The factory consists of several parts such as steel production units, thermal and complementary operations, heavy and light rolling [5]. In the present study, the temperature prediction of the WBF operating in light rolling unit of this factory is taken into account. The WBF is an essential part of a rod mill plant where a billet is heated to the required rolling temperature so that it can be milled to produce wire. The WBF are generally classified into two groups namely batch furnace and continuous one. In batch furnace, the charged steel sections remain in a fixed position on the hearth until it is heated to rolling temperature, whereas, in continuous furnace, which is also under investigation in the present study, the charged steel section moves through the furnace and is heated to rolling/forging temperature as it plods through the furnace. In most cases, the WBF have three zones including pre-heating, heating, and soaking. However, some new furnaces have more heating and soaking zones.

The WBF process, which is functioning in rod mill plant of Iran Alloy Steel Company, is a kind of continuous furnace in which slabs are heated. In the furnace, all slabs are heated to reach a pre-defined discharging temperature and a balance of temperature distribution throughout the slabs. The distribution and movement of the slabs in the furnace are subjected to the heating capacity of the furnace and state of rolling line such as rolling pacing. The maximum temperatures in such furnaces are limited to 1250 °C. Our WBF of interest encompasses five zones namely pre-heating, heating zone 1, heating zone 2, soaking zone, and one recuperative zone. The slabs move throughout the WBF from the pre-heating zone to soaking zone. A typical structure of the walking beam furnace is provided in fig. 1.

Excluding the recuperative zone, which is a non-thermal area and the slabs, are heated by waste gas in it, the other zones are considered as control-area zones. The roles of pre-heating zone and two other heating zones are to heat the slabs repeatedly. The main aim of soaking zone is to adjust the temperature gradient so that the interior temperature and exterior temperature of the slabs can reach a balance. All control-area zones are equipped with their own burners mounted on their roofs. Each burner set has a flow meter sensor to sense the flow of air and gas that goes to flow-control loop to set the flow and air rate of each burner set. For sensing the temperature of each zone there are two sets of thermocouples mounted on the right and left internal walls of each zone that their maximum value at each instant goes for temperature-control loop, fig. 1.

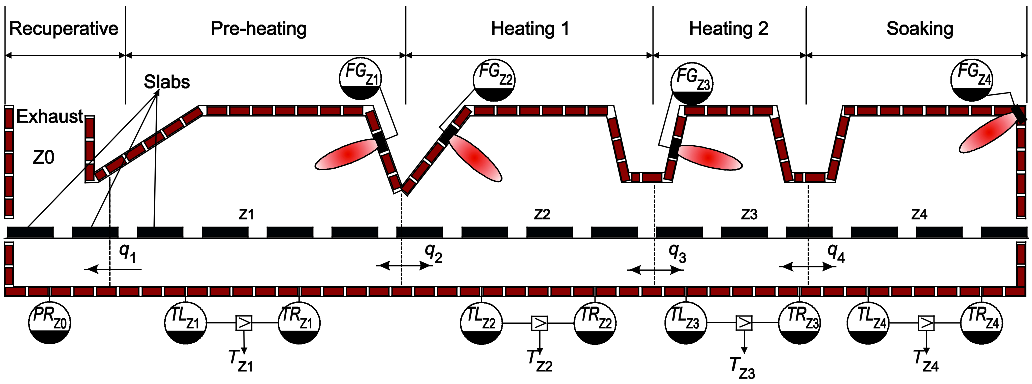

Figure 1. Schematic of an industrial walking beam furnace with the monitored sensors

Moreover, there are pressure sensors in recuperative zone for sensing the furnace pressure. Technical description of the variables and tools used in fig. 1 is given in tab. 1.

Table 1. Technical equipments and variables description of the walking beam furnace

	Pre heating			Heating 1			Heating 2			Soaking			Rec.
Variable name	TL_{Z1}	TR_{Z1}	FG_{Z1}	TL_{Z2}	TR_{Z2}	FG_{Z2}	TL_{Z3}	TR_{Z3}	FG_{Z3}	TL_{Z4}	TR_{Z4}	FG_{Z4}	PR_{Z0}
Unit	[°C]	[°C]	[Nm^3h^{-1}]	[°C]	[°C]	[Nm^3h^{-1}]	[°C]	[°C]	[Nm^3h^{-1}]	[°C]	[°C]	[Nm^3h^{-1}]	[m bar]
P&ID name	TIC 10603L	TIC 10603R	FIC 10600	TIC 10604L	TIC 10604R	FIC 10602	TIC 10605L	TIC 10605R	FIC 10604	TIC 10606L	TIC 10606R	FIC 10606	PIC 10600
Sensors no.	4	4	8	4	4	12	4	4	8	4	4	8	2
Burners no.	8			12			8			8			0

Methodology

Data pre-processing for prediction purpose

The procedure of data-driven system identification based on prediction technique can be briefed in four phases [16]:

- collect uncorrupted input-output data from the nominal operating condition of the process,
- select appropriate model structure,
- estimate the model parameters, and
- model validation.

One of the most important assumptions to get valid information from an input and an output is that the changes happened in the output are solely affected by the system input and not disturbance or noise, *etc.* [17]. Hence, data pre-processing is required in order to extract suitable data from the available data. Thus, one has to initially examine the original raw data**,** polish them to eliminate trends and outlying values and apply filtering to enhance important frequency ranges**.** In this and next sub-sections, the procedure of data pre-processing appropriate for extracting valid data will be discussed. To identify a predictive model of a process plant, the collected data should be sufficiently reliable to represent various dynamics of the process. Owing to safety and limited access to real WBF operating in the factory, we could not inject various excitation signals with different frequencies into the process. Although system identification theory puts stress on using influential signals such as pseudo-random binary signal (PRBS) to stimulate all dynamics of the process, we have to exploit the available data from normal operating situations which leads to a passive prediction approach rather than an active one [17, 18].

The first step to prepare a predictive model is gathering data from different operating zones of WBF process. The data sets of the different zones are recorded in data access system (at the control room of the factory). These experimental data are collected for the duration of 23 days (*i. e.*, from 7th to 29th of July 2010) with a sampling interval of 5 seconds. It must be noted that the sampling rate was selected such that the measured data becomes persistently excited. Furthermore, the 50% of data samples are used as training set, while, the remainder of them are left aside as checking data (*i. e*., 20% as test set and 30% as validity set). After consultation with the process experts and operators of the factory, we made an effort to eliminate faulty-operational data points from recorded data samples and also, the inputs of the process which do not vary over time should not be considered in the predictive model, since they remain constant during prediction [19]. Additionally, due to uncontrolled effects such as noise and disturbance acting on the WBF process, one should try using some pre-processing techniques presented in identification references [17] such as filtering and normalization.

Data filtering

Filtering of the abrupt changes in data is very important in pre-processing phase of the process prediction. These kinds of ordinary abrupt changes that often occur in practice may be due to the malfunction of the sensors or data acquisition cards. They may yield some numerical problems in measuring and recording variables as well [5]. Such changes in data-points may also happen because the sensor is turned off when it needs to be substituted or repaired. These sudden changes have a lot of energy in high frequency rang that degrades estimation of the models parameters or validity rate [20]. Additionally, the WBF process suffers severely from strong disturbance and noise that may result in model-reality mismatches. To tackle these problems, the acquired original signals are passed through a low band pass filter that can erase noise signal from the original one. It must be noted that a suitable filter should not change or affect the original signal shape whilst abolishing noise or disturbing signals. Figure 2(a) depicts the effects of the low pass filter on the original flow signal $FG_{Z4}(t)$. As seen, low band pass filter can perform promisingly in removing noise signal and shaving the abrupt peaks from the original signal and simultaneously retain the shape of the original signal. The transfer function of the designed filter with the cut-off frequency of 70 Hz is also given in eq. (1):

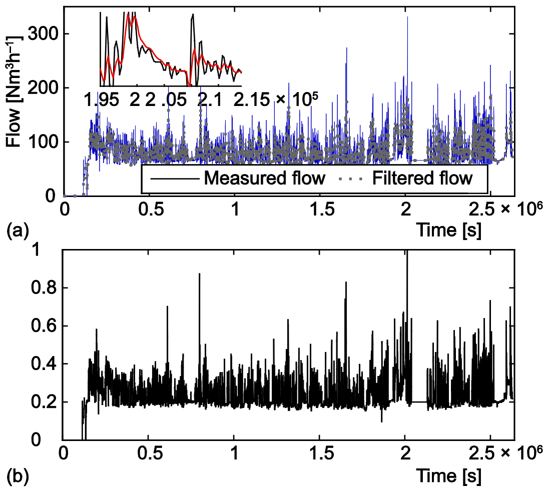

Figure 2. Effects of data pre-processing on gas flow signal of zone 4; (a) low pass filter, (b) normalization *(for color image see journal web-site)*

$$T_f = \frac{0.25z + 0.75}{Z} \tag{1}$$

Data normalization

While some of the input signals feeding into predictive model have high magnitudes, their associated local connection weights in a neuron of the LLNF network (or ANN) may affect the corresponding output more than other connection weights. Furthermore, since the inputs and outputs signals have different ranges, they may yield error in data quantization and badly prediction of the process under consideration [19]. In order to deal with these problems, the numeric signals should have the magnitudes close to each other. An appropriate way to approach this aim is normalization of the experimental data passed through the filter. It is noteworthy that the notion of normalization should be taken into account as an essential part of data pre-processing and its inclusion to the pre-processing phase improves the accuracy of predictive model in practice. The original signal S can be mapped into normalized signal S_N by eq. (2):

$$S_\mathrm{N} = \frac{S - S_\mathrm{min}}{S_\mathrm{max} - S_\mathrm{min}} \tag{2}$$

where S_max, and S_min are maximum and minimum values of the signal to be normalized, respectively. Figure 2(b) represents the effects of the normalization on the filtered signal derived from FG_{Z4} by using eq. (2). It is seen that the pre-filtered signal has been bounded between 0 and 1 after normalization. Thus, by applying eq. (2) to the remainder of the process signals, a set of uniform signals in terms of magnitude which are ranged between 0 and 1 will be obtained. Additional statistical details concerning the original collected database of the WBF process from the factory are also reported in tab. 2.

Table 2. Statistical characteristics of the recorded database of WBF before pre-processing

	Inputs				Outputs				
Variable	FG_{Z1}	FG_{Z2}	FG_{Z3}	FG_{Z4}	T_{Z1}	T_{Z2}	T_{Z3}	T_{Z4}	PR_{Z0}
Standard deviation	108.7	159.0	104.4	27.9	154.9	146.8	104.4	119.2	11.7
Mean value	176.8	215.4	175.2	73.03	919.4	957.6	1125.8	1112.2	100.5
Minimum value	0	0	55	0	270	312	615	559	63
Maximum value	385	424	526	331	1020	1050	1200	1200	116

Non-linear modelling based on the prediction technique

Non-linear dynamic process plants can be modelled by both prediction and simulation techniques. Note that further description concerning the simulation technique suitable for data-driven process modelling is beyond the scope of this paper and a comprehensive introduction about non-linear simulator models and also their particular characteristics can be found in [5, 18]. Predictive models may use static (memory-less) models within their architectures (*i. e.* ANN). Since static models are not adequately capable of capturing the dynamics of non-linear process plants, extending memory to the static configuration of such networks seems to be necessary. The predictive structure differs from the recurrent or simulation one in terms of the way that dynamism has been introduced to the static model [18]. Note that due to the extraordinary characteristics of the WBF process including non-uniform temperature distribution through the furnace and open inter-connections between operating zones, which may yield intrinsic temperature interactions, all the formulations of proposed prediction methodology are merely concerned with the WBF throughout this paper.

Proposed one-step predictive model of WBF using LLNF networks

The general configuration of a one-step predictive model suitable for non-linear dynamic modelling of an individual operating zone of the WBF is depicted in fig. 3. For the sake of simplicity, the structure of the one-step LLNF predictive model of the soaking zone which will be employed to predict T_{Z4}(k) is merely shown in fig. 3. The structures of the one-step predictive models of the other zones follow such configuration as well. Furthermore, it must be noted that the integration of the one-step predictive models of all zones forms the overall one-step predictive model of the WBF process.

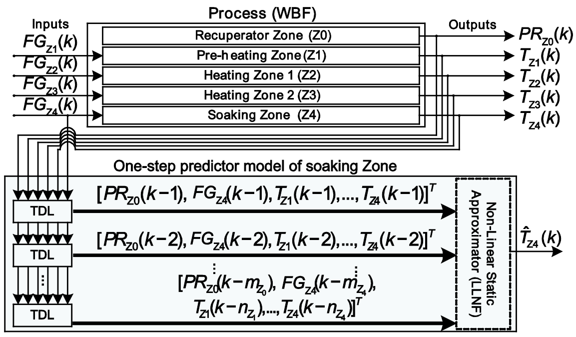

Figure 3. Configuration of one-step LLNF predictive model for a single zone (soaking zone)

The proposed structure of the one-step predictive model utilizes the banks of time delay line (TDL) filters in which filters are typically chosen as unit time-delays to add external dynamism (memory) to static LLNF model. That is, the TDL filters are used to generate the delayed inputs and outputs. As seen, the presented MISO predictive model has two different sets of input channels: previous real process inputs and the previous real process output which are injected into the static LLNF model and finally the one-step future output $\hat{T}_{Zi}(k)$ will be predicted. It is worth stressing that due to the open connections of the zones, fig. 1; highly interactions may exist between the temperature behavior of the different zones. In other words, the variations of the temperatures in some zones, probably affect the temperature of the connected and even non-connected zones and these temperature interactions should not be ignored in the temperature prediction of this process plant. Thus, fig. 3 also describes that how possible temperature interactions, which may exist among all zones of the WBF, are simply included in the one-step predictive model of a single zone by feeding the actual outlet temperature signals of the other operating zones back to it.

Proposed multi-step predictive model of WBF using LLNF networks

A multi-step predictive model should be able to predict the behavior of system for long future horizons (*i. e.*, *h*-steps ahead). The typical structure of a multi-step predictive model based on the sequential one-step predictive models with the horizon of *h* is presented in fig. 4.

In order to obtain the predicted output values of any operating zone at each moment, the one-step LLNF predictive model presented in the previous section should be used *h* times. As it is unambiguously seen, the sequential identical LLNF models where the output of any of them (except the last one) provides one of the inputs for the next LLNF model are used to establish the overall multi-step predictive model. This multi-horizon neuro-fuzzy predictive model obtains the predicted output values of the system (*i. e.*, $\hat{T}_{Zi}(k+l)\, i = 1 \sim 4$, $l = 1 \sim h$) using previous and current values of the process output and inputs. It must be also noted that

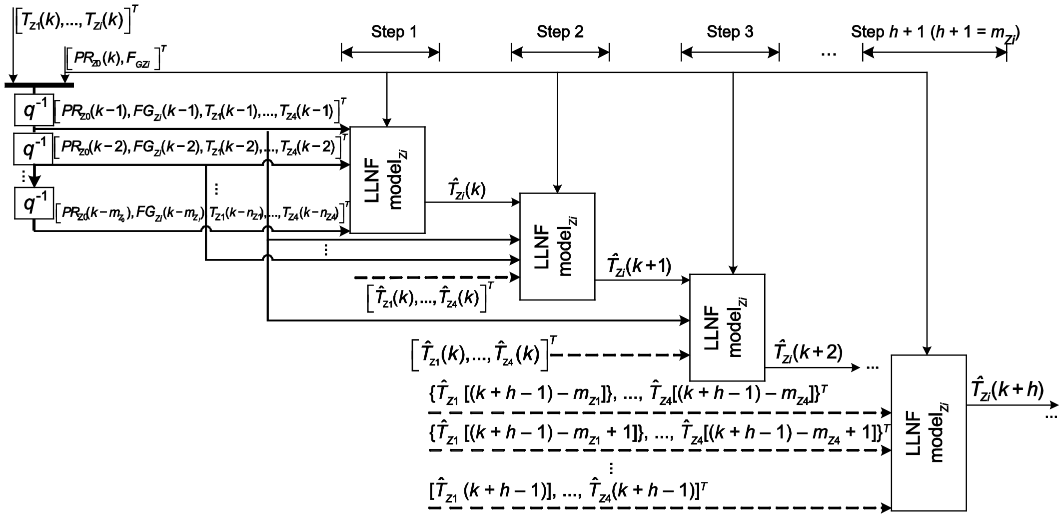

Figure 4. Configuration of *h*-step ahead predictive model based on LLNF network for a single zone

for an *h*-step ahead predictive model, the last prediction step is fully based on model output values if $h > n_{Zi}$. Hence, for long prediction horizons, the difference between prediction and simulation techniques fades and the last one-step predictive model will be turned completely into the recurrent/simulator model [21].

As discussed in previous section, since inordinate temperature interactions exist among the temperature behavior of different operating zones of the WBF, and also due to non-uniform distribution of temperature within the furnace, the structure of the proposed multi-step predictive model of this process plant is highly temperature-coupled so as the developed model could accurately capture the dynamics of the non-linear WBF process specially for long future horizons.

Modelling of the WBF based on linear prediction technique

The procedure of MISO linear modelling of dynamic processes from input-output sequences is described in the section *Non-linear modelling based on the prediction technique*. According to the process identification theory, simplest solutions should be initially tried to build the model of a process when no prior knowledge concerning its intrinsic characteristics is available (such idea is also called as black-box process identification). It has been mathematically proven that the least-squares error (LSE) method is the optimum modelling method for linear process plants [8, 22]. Thus, in a black-box identification approach, the LSE solution is firstly preferred to be tried over any non-linear ones. (*i. e.*, if the LSE method does a descend job and cannot produce promising prediction results, then, non-linear methods are the dominant alternatives). A finite sequence of the input-output variables of the WBF, which were observed by a constant sampling interval, is considered. If dynamic linear relations exist among these variables, they can be described by eq. (3):

$$\begin{aligned}\hat{T}_{Zi}(k) &= \sum_{l=1}^{n_{Zi}} a_l T_{Zi}(k-l) + \sum_{j=1}^{4} \sum_{l=1(j\neq i)}^{mt_{Zj}} b_{lj} T_{Zi}(k-l) + \\ &+ \sum_{l=1}^{m_{Z0}} b_{l4} PR_{Z0}(k-l) + \sum_{l=1}^{mf_{Zi}} b_{l5} FG_{Zi}(k-l)\end{aligned} \tag{3}$$

which describes one-step MISO linear predictive model of the *i*-th zone whose parameters are a_l, b_{lj} ,and m_{Z0}, mf_{Zi}, mt_{Zi} (i = 1, …, 4) are the numerator orders of the *i*-th zone's model and n is the denominator order where usually has the same value as the numerators [21]. All parameters of the model can be estimated by least-squares method since the model error is linear in parameters [20, 21].

Locally linear neuro-fuzzy network as predictive model

In the section *Non-linear modelling based on the prediction technique*, the general structures of one-step and multi-step predictive models were explained. In order to identify predictive models for WBF process, the LLNF network is utilized and LOLIMOT learning algorithm is employed to find the best structure and parameters of the network. The main reasons of using LLNF models trained by LOLIMOT algorithm are, low computational complexity, noise robustness due to regularization effect, high accuracy, and online adaptation [21].The configuration of the LLNF model suitable for one-step prediction is presented in fig. 5. Each neuron realizes a local linear model (LLM) and an associated validity function that determines the validity area of the LLM. The LLNF model is simply interpretable as TS-type neuro-fuzzy model inasmuch as each neuron represents one rule, and the validity function represent the rule premise and the LLM represent the rule consequents.

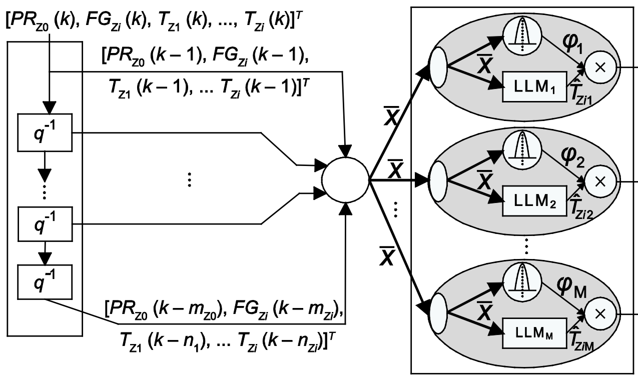

Figure 5. Typical structure of the one-step neuro-fuzzy predictive model; combination of the static LLNF network and TDL filters

As it was discussed in the section *Non-linear modelling based on the prediction technique*, to create a one-step predictive model, fig. 5, the delayed inputs of the process as well as the past samples of the LLNF model output should be fed into the model as the inputs [23]. Hence, for dynamic LLNF network, the input vector of model is given as:

$$\bar{x} = [PR_{Z0}(k),\ FG_{Zi}(k),\ TR_{Zi}(k),\ T_{Z1}(k),\ T_{Z2}(k),\ T_{Z3}(k),\ T_{Z4}(k)]^T \tag{4}$$

where PR_{Z0}(k), FG_{Zi}(k), and TR_{Zi}(k) contain the previous values of *i*-th zone generated by TDL such that:

$$PR_{Z0}(k) = [PR_{Z0}(k-1),\ PR_{Z0}(k-2), \dots ,\ PR_{Z0}(k - m_{Z0})] \tag{5}$$

$$FG_{Zi}(k) = [FG_{Zi}(k-1),\ FG_{Zi}(k-2), \dots ,\ FG_{Zi}(k - mf_{Zi})] \tag{6}$$

$$TR_{Zi}(k) = [T_{Zi}(k-1), T_{Zi}(k-2), \dots , T_{Zi}(k - mt_{Zi})] \tag{7}$$

and, $T_{Zi}(k)$ contains the delayed samples of the walking bean furnace output:

$$T_{Zj}(k) = [T_{Z1}(k-1), ..., T_{Zj}(k-m_{Zj})]^T, \qquad j \neq i \tag{8}$$

where m_{Z0}, mf_{Zi}, and mt_{Zi} (i = 1,…,4) are the numerator orders of the i-th zone and n_{Zi} denote the denominator of i-th zone. Hence, the global output of the model is calculated as the weighted summation of the output of all LLM:

$$\begin{aligned}\hat{T}_{Zi}(k) = \sum_{j=1}^{M} \sum_{l=1(l \neq i)}^{4} \Big[& b_{j1} PR_{Z0}(k-1) + b_{j2} PR_{Z0}(k-2) + ... + b_{jm_{Z0}} PR_{Z0}(k-m_{Z0}) + \\ & + b_{j(m_{Z0}+1)} FG_{Zi}(k-1) + b_{j(m_{Z_0}+2)} FG_{Zi}(k-2) + \ldots + b_{j(m_{Z_0}+mf_{Zi})} FG_{Zi}(k-mf_{Zi}) + \\ & + b_{j(m_{Z0}+mf_{Zi}+1)} T_{Zl}(k-1) + b_{j(m_{Z_0}+mf_{Zi}+2)} T_{Zl}(k-2) + \ldots + b_{j(m_{Z_0}+mf_{Zi}+mt_{Z_l})} T_{Zl}(k-mt_{Zl}) - \\ & - a_{j1} T_{Zi}(k-1) - a_{j2} T_{Zi}(k-2) - \ldots - a_{jn_{Zi}} T_{Zi}(k-n_{Zi}) + \zeta_j \Big] \varphi(\overline{x})\end{aligned} \tag{19}$$

where $b_{j(m_{Z_0}+mf_{Z_i}+mt_{Z_l})}$ and $a_{jn_{zi}}$ represent the numerator and denominator coefficients, respectively, which are estimated employing the weighted least-squares solution [21]. ζ_j is the offset and $\varphi_j(\overline{x})$ – the operating-point dependent weighting factor of the j-th LLM.

The validity functions on $\overline{x}$ with which LLNF network interpolates between different LLM are usually chosen as normalized Gaussians. Thus, they form a partition of unity as:

$$\sum_{j=2}^{M} \varphi_j(\overline{x}) = 1 \tag{10}$$

Considering the axis-orthogonal Gaussians, the validity functions are given by:

$$\varphi_j(\overline{x}) = \frac{\mu_j(\overline{x})}{\sum_{j=1}^{M} \mu_j(\overline{x})} \tag{11}$$

where $\mu_j(\overline{x})$ is defined as:

$$\mu_j(\overline{x}) = \exp\left\{ -\frac{1}{2}\left[\left(\frac{x_1 - c_{j1}}{\sigma_{j1}}\right)^2 + \ldots + \left(\frac{u_D - c_{jD}}{\sigma_{jD}}\right)^2 \right]\right\}, \; D = m_{Z0} + m_{fZi} + n_{Zi} + \sum_{l=1(l \neq i0)}^{4} m_{T_{Zl}} \tag{12}$$

where c and σ are the center co-ordinate and the dimensional individual standard deviation, respectively.

The merit of local linear neuro-fuzzy modelling is that a complicated process model is divided into a number of smaller and thus simpler sub-problems, which are solved independently by identifying simple linear models [21, 23, 24]. The vital factor for the success of such an approach is the division strategy for the original complex problem that this will be accomplished by an algorithm so-called LOLIMOT (locally linear model tree). LOLIMOT is an incremental tree-structure algorithm that partitions the input-space by axis-orthogonal splits [21]. This algorithm comprises of an outer loop in which the rule premise structure is determined and a nested inner loop in which the rule consequent parameters are optimized by local LSE estimation. The latter loop can be summarized as a five-step algorithm [21, 24].

(1) Start with an initial model of the process based on LSE estimation.
(2) Find the worst performing LLM which has the maximum local cost function *e. g.*, MSE.
(3) The worst LLM found in step 2 is broken into two halves axis-orthogonally. Partitions in all dimensions are tried and for each partition, the following steps are performed:

(3-a) construction of the multi-dimensional validity functions for both hyper-rectangles,

(3-b) locally estimation of the rule consequent parameters for both newly generated neurons,

(3-c) calculation of the global cost functions for the current overall model of the process.

(4) Find the best division; the best of the alternatives checked in Step 3, and increment the number of LLM: M → M+1.

(5) Test for convergence. If the termination criterion is met then stop; otherwise, go to Step 2.

Note that for the termination criterion in the fifth step of LOLIMOT algorithm various options exist including a maximal model complexity that is a maximal number of LLM, statistical validation tests, or information criteria [23]. It is also worth noting that the LOLIMOT algorithm provides the best linear or non-linear model of a process with maximum generalization capability and performs well in short-term and long-term prediction applications.

Prediction procedure relies on the inputs containing enough information and dynamism. In order to determine the proper inputs of the LLNF network, it is necessary to know that each input has its own particular duration of effect on the output. In this study, selection of dynamic orders of the one-step LLNF predictive models is carried out by a blend of trial-and-error procedure during prediction and prior inkling about the WBF [5].

The best numbers of dynamic used for prediction after several trials are listed in tab. 3.

Table 3. The best number of LLNF orders for inputs and outputs of the WBF

		Number of LLNF orders			
		TG_{Z1}	TG_{Z2}	TG_{Z3}	TG_{Z4}
Variables	FG_{Z1}	3	-	-	-
	FG_{Z2}	-	2	-	-
	FG_{Z3}	-	-	4	-
	FG_{Z4}	-	-	-	2
	TG_{Z1}	2	1	-	-
	TG_{Z2}	1	4	1	-
	TG_{Z3}	-	1	3	1
	TG_{Z4}	-	-	1	3
	PR_{Z0}	2	1	1	3

Results and discussions

In the present section, experimental results of the linear and non-linear modelling of the WBF based on proposed linear and non-linear prediction methods are presented. Regarding long-term prediction, a long horizon of ninety seconds is brought into account and the results are portrayed in the rest of this paper. In the case of local linear modelling, selection of rules/neurons number represents the great concern. Selecting large number of rules may yield over parameterization and model complexity problems. In our research, the number of neurons for all the MISO models was determined based on the root mean squared error (RMSE) curve [18]. A typical RMSE curve for pre-heating zone is provided in fig. 6. In this regard, the optimal neuron/rule number was determined by increasing the neurons until more neurons did not diminish the RMSE for the test data remarkably. It is unambiguously seen that as the neuron number enlarged from one to three, the RMSE values for both train and test data sets lessened slightly.

But whilst the number of rules increased past 3, there were no significant improvements in the RMSE for the test data. Hence, an LLNF network with four rules was selected based on this RMSE curve. The same procedure was also executed to select the optimal rules number of other LLNF sub-models. The number of rules/neurons obtained through the RMSE curves for all LLNF predictive sub-models are brought up in tab. 4.

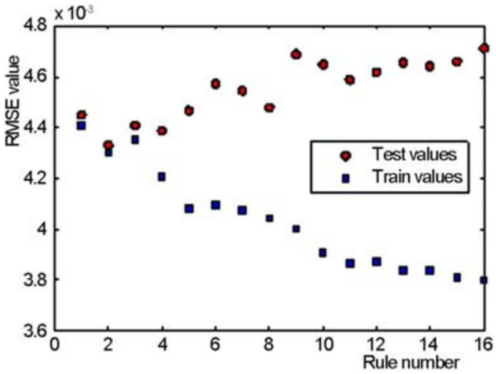

Figure 6. Training and test error variations *vs.* the rules number for temperature of heating zone 2

Table 4. Rules number of different LLNF predictive models for different prediction horizons

Prediction horizon [s]	LLNF models			
	Zone 1	Zone 2	Zone 3	Zone 4
5	3	5	2	4
10	6	10	4	8
15	9	15	6	12
20	12	20	8	16
⋮	⋮	⋮	⋮	⋮
90	54	90	36	72

To evaluate the accuracy of the predictive models, two criteria are defined namely PA_1 and PA_2:

$$PA_1 = \sqrt{\frac{1}{Q}\sum_{j=1}^{Q}\left[T_{Zi}(j)-\hat{T}_{Zi}(j)\right]^2}, \qquad i=1,\ldots,4$$
$$PA_2 = 100\left[1-\frac{\left\|\hat{T}_{Zi}(j)-T_{Zi}(j)\right\|_2}{\left\|T_{Zi}(j)-\bar{m}(T_{Zi})\right\|_2}\right], \qquad i=1,\ldots,4 \tag{13}$$

where $\|\cdot\|_2$ denotes Euclidean distance, Q is the number of the data samples that are considered for prediction, $\bar{m}$ – the mean value of the signal T_{Zi}, $\hat{T}_{Zi}$, and T_{Zi} are the predicted and real outputs associated to the i-th operating zone of the furnace, respectively. Note that the criterion PA_1 is also so-called as RMSE and the criterion PA_2 indicates the percentage that the predicted output fits the real one.

One-step prediction errors obtained based on normalized data and their associated histograms for temperature prediction of zone 1 to zone 4 are depicted in fig. 7. Taking into account the one-step prediction errors and their corresponding histograms, one can simply infer that the pre-acquired neurons and dynamic numbers are adequate to enable each one-step neuro-fuzzy predictive model to forecast its related zone’s outlet temperature accurately. Moreover, on the basis of all histograms’ data, it is obvious that in all four one-step predictive models the most accumulations of errors occur around zero, that is, the prediction error which also portrays the prediction accuracy is roughly close to zero. It is noted that whether the prediction error is close to zero or not is the most crucial factor that affects the trustworthy of any non-linear model-based fault diagnosis approach in which a one-step non-linear predictive model of the process is required to generate the indicator signals namely “residuals” that show the occurrence of any defect throughout the process [23].

Since the WBF process has four heating zones, four one-step neuro-fuzzy predictive models are developed in nominal operating conditions. Hence, the proposed entire one-step predictive model of the WBF comprises four temperature-coupled LLNF networks. That is, due to non-uniform distribution of temperature throughout the furnace as well as thermal interactions between operating zones, the outlet temperatures of the other zones are also fed

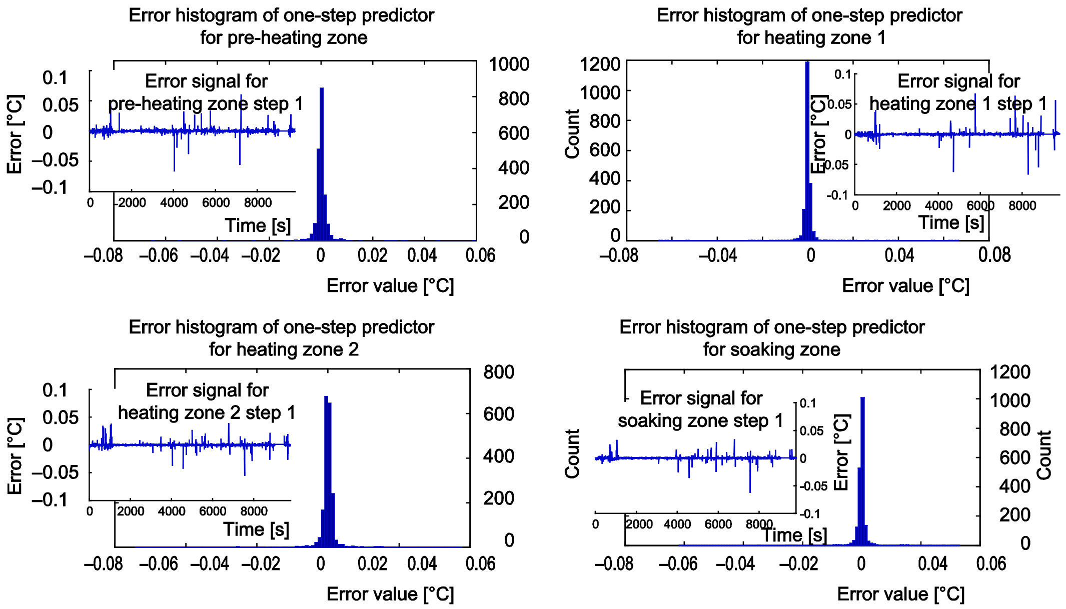

Figure 7. One-step prediction errors based on normalized data and their associated histograms for all zones

back to the one-step predictive model of an individual zone. Furthermore, it must be noted that for constructing the *h*-step LLNF predictive model of a single zone, *h* identical pre-identified one-step predictive models of that zone are arranged in a sequential configuration, fig. 4. After creating the *h*-step MISO neuro-fuzzy predictive models of all operating zones based on their associated one-step predictive models, a full-scale MIMO neuro-fuzzy predictive model of the WBF with the horizon of *h* is provided.

Figure 8 represents the actual responses of the outlet temperatures of two selected zones (*i. e.*, pre-heating zone and heating zone 1) and their associated responses of the one-step linear and LLNF predictive models. With a glance, one can verify that LSE estimation does a descend job and has severe problems in the thermal modelling of this zone, whereas, the LLNF model is more accurate in the sense that its thermal response highly resembles to the response of its related operating zone. Such a case can be observed specially in the time

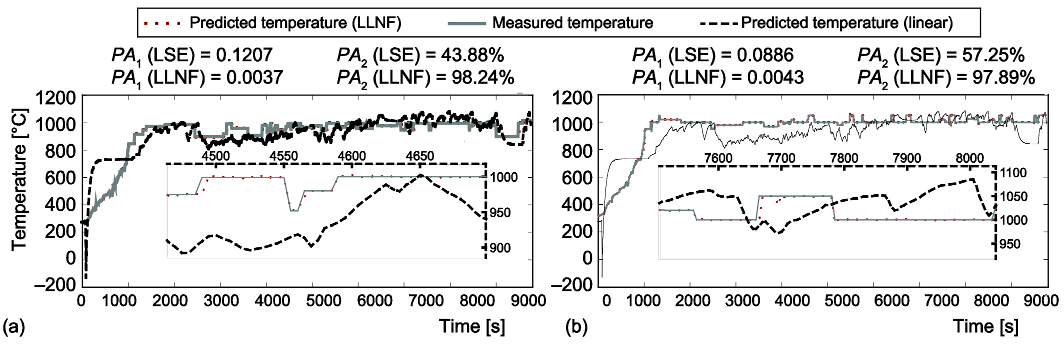

Figure 8. Performance of the one-step LSE and LLNF based predictive models; (a) pre-heating zone, (b) heating zone 1

interval between 4500 seconds and 4700 seconds, with reference to fig. 8(a), where the real thermal response of the pre-heating zone was successfully tracked by the response of its associated one-step LLNF predictive model, whereas the response of the linear model had some tracking problems at the same time interval and especially in intervals in which a set of fluctuations are observed within the thermal response of the pre-heating zone, (*e. g.*, 7000 s and 9000 s). Taking into account the optimality of the least-squares solution for modelling of linear process plants, all prediction results concerning the temperature prediction of all operating zones of the WBF prove that all zones belongs to the category of non-linear systems, tab. 5.

Table 5. Prediction accuracy of for different linear and non-linear predictor models

	Pre-heating zone		Heating zone 1		Heating zone 2		Soaking zone	
	PA_1 [°C]	PA_2 [%]	PA_1 [°C]	PA_2 [%]	PA_1 [°C]	PA_2 [%]	PA_1 [°C]	PA_2 [%]
LSE	0.1207	43.88	0.0886	57.25	0.1642	11.62	0.0612	68.32
Step 1	0.0037	98.24	0.0043	97.89	0.0037	97.99	0.0030	98.40
Step 2	0.0249	88.37	0.0246	88.10	0.0231	87.52	0.0226	88.31
Step 3	0.0352	83.61	0.0347	83.24	0.0327	82.36	0.0319	83.49
Step 4	0.0987	54.12	0.0921	55.62	0.1009	45.72	0.0783	59.48
Step 5	0.1008	53.14	0.0946	54.40	0.1029	44.56	0.0804	58.42
Step 6	0.1029	52.16	0.0971	53.22	0.1050	43.53	0.0825	57.36
Step 7	0.1046	51.39	0.0993	52.15	0.1062	42.85	0.0842	56.43
Step 8	0.1063	50.60	0.1015	51.11	0.1075	42.16	0.0861	55.49
Step 9	0.1081	49.76	0.1037	50.05	0.1090	40.39	0.0880	54.50
Step 10	0.1100	48.89	0.1059	48.98	0.1107	40.44	0.0899	53.53
Step 11	0.1119	48.01	0.1080	47.91	0.1125	39.48	0.0917	52.57
Step 12	0.1137	47.17	0.1100	47.00	0.1145	38.42	0.0936	51.63
Step 13	0.1155	46.32	0.1120	46.05	0.1164	37.39	0.0954	50.69
Step 14	0.1172	45.55	0.1140	45.14	0.1183	36.41	0.0972	49.71
Step 15	0.1190	44.75	0.1158	44.27	0.1200	35.46	0.0989	48.87
Step 16	0.1209	43.84	0.1177	43.38	0.1219	34.47	0.1007	47.96
Step 17	0.1229	42.93	0.1195	42.49	0.1238	33.46	0.1025	47.01
Step 18	0.1250	41.98	0.1215	41.58	0.1255	32.54	0.1044	46.09

In reference to the explanation made in section *Non-linear modelling based on the prediction technique*, four MISO one-step linear predictor models, four MISO one-step non-linear LLNF predictor models, and four MISO multi-step non-linear LLNF predictor models with the prediction horizon ranged from 5 seconds to 90 seconds have been generated to forecast the temperature of different zones of the WBF. It must be noted that creating such thermal predictive model of the WBF represents the great concern in steel production industries for a wide range of thermal purposes. Figure 9 shows the measured and the long-term predictive outputs of LLNF model concerning the pre-heating zone. As illustrated in fig. 9, one-step prediction of this output exhibits the best response compared to the longer-term predictions in the sense that an almost perfect fit is achieved over the whole operational domain. Then, as the prediction horizon increases during the long-term predictions of this zone, it is observed that the LLNF predictive model of this zone gives more inferior predictive responses containing unpromising values of predictions that could be due to the systematic errors generated in the short-term predictors in previous stages of prediction.

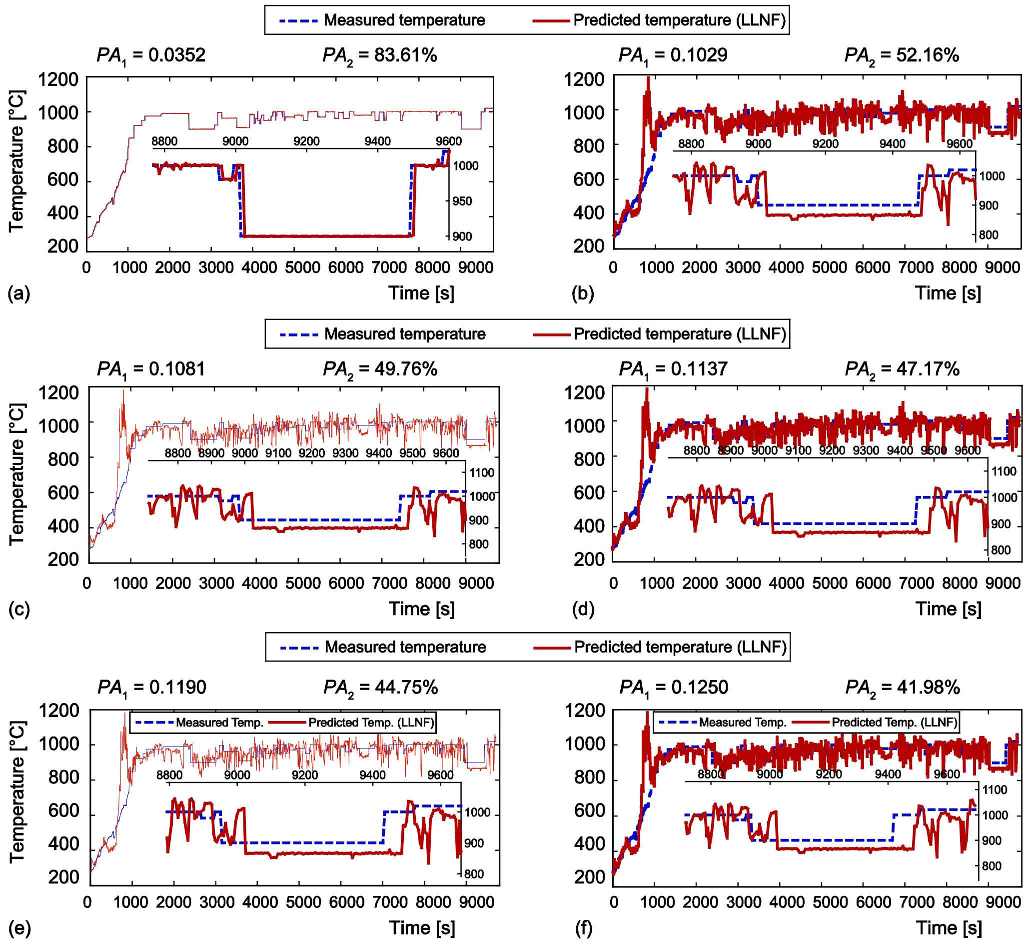

Figure 9. Performances of the multi-step LLNF predictive models of pre-heating zone for different future horizons; (a) 15 seconds, (b) 30 seconds, (c) 45 seconds, (d) 60 seconds, (e) 75 seconds, and (f) 90 seconds prediction

Moreover, for the sake of simplicity and due to lack of adequate space to add all prediction portrayals concerning all zones in this paper, some prediction performances for the remained zones *i. e.*, heating zone 1, heating zone 2, and soaking zone are selected to show in fig. 10. In this figure, sub-figures are chosen as pairs such that one can visibly observe the accuracy deteriorations of the predictive model over the longer-term predictions. For example, with reference to figs. 10(a) and (b) and as a result of prediction-horizon growing, the response of the predictive model for 10 seconds future instants (*i. e.*, short-term) outperform the response of the predictive model for 50 seconds future instants (*i. e.*, long-term) in the sense that it provides a better fit against the measured response of the zone 2. More details regarding the prediction accuracies of all operating zones over the whole prediction horizons are reported in tab. 5.

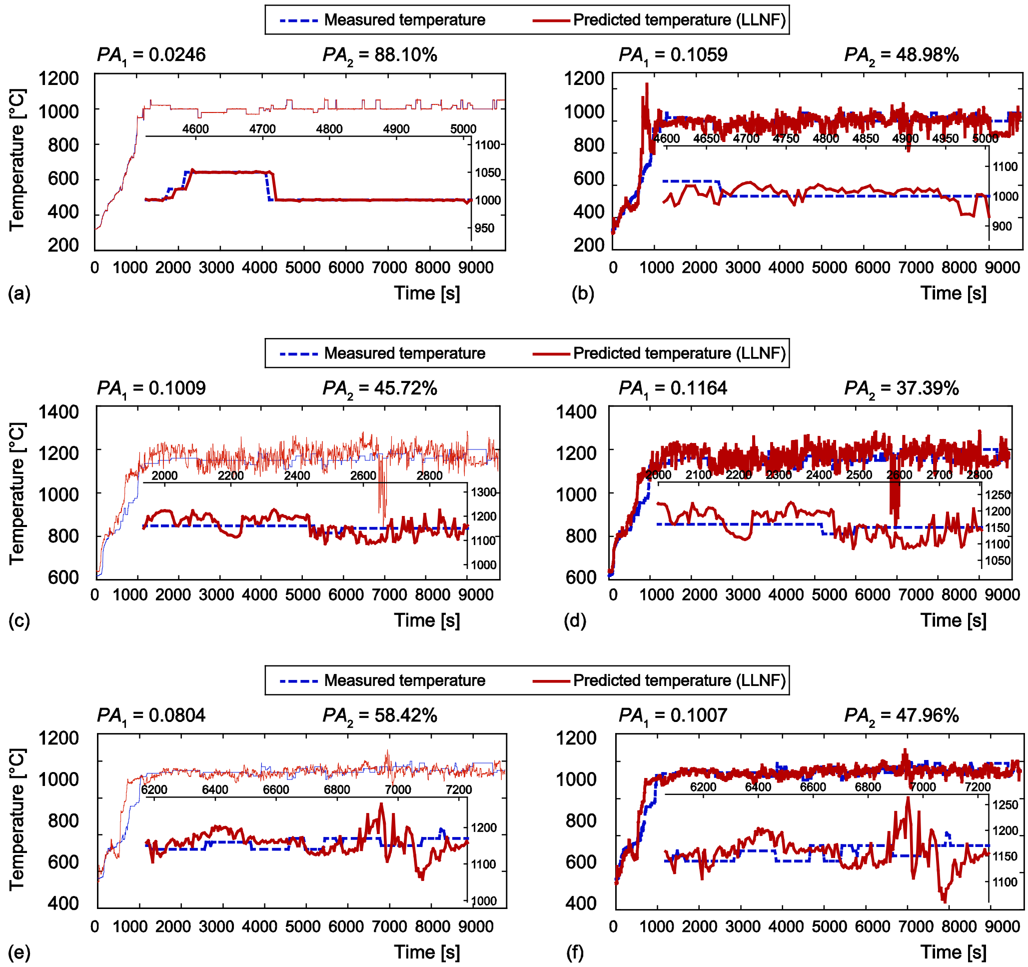

Figure 10. Performances of the multi-step LLNF predictive models of different zones for the selected future horizon; (a) 10 seconds, (b) 50 seconds prediction for heating zone 1, (c) 20 seconds, (d) 65 seconds prediction for heating zone 2, (e) 25 seconds, and (f) 80 seconds prediction for soaking zone

In reference to tab. 5, it is obvious that the linear predictive models provided the worst modelling results in the sense that their prediction accuracy values obtained through both modelling evaluation criteria are noticeably lesser than those ones produced by non-linear LLNF predictive models. In addition, by enlarging the prediction horizon from 5 second to ninety seconds (*i. e.* a long prediction horizon), the associated prediction errors based on both prediction accuracy criteria have been deteriorated. The main reason behind this discernible phenomenon may be that for the long-term prediction of an output (*i. e.*, where *h* is greater than 1), the previous values of the considered output is required, but since there is no recorded/measured data for these values, the predicted values at the previous steps are used for the prediction of this output instead of the real unavailable data. Here, it is also observed that an increase in prediction horizon especially from five to 20 enlarges the modeling error. It is also noted that the most accuracy deterioration occurred during the thermal prediction of zone 4, while, the least happened for the thermal prediction of zone 3. Moreover, it must be taken into consideration that the proposed long-term predictive model is derived from a short-term (one-step ahead) predictor by simply iterating the predictor, thus, such long-term predictions are limited in their accuracy by observational and dynamic noise and by the sensitivity to initial conditions of the dynamical system [25]. These long-term repeated predictors are also limited by systematic errors in the short-term predictors which can be due to under or over fitting data. Hence, these systematic errors generated in the short-term predictors are accumulated increasingly as the prediction horizon enlarges (such situations can be observed in tab. 5). Note that the present research regards a feasibility study and from an engineering point of view, since the prediction accuracy declines dramatically by the increase of the prediction horizon from five to 20, it is not recommended to utilize the long-term predictive model to forecast the future horizons greater than 5.

Conclusions

In this paper, identification of non-linear short-term and long-term predictor models by locally linear neuro-fuzzy (LLNF) modelling technique for an actual MIMO process namely walking beam furnace (WBF) is discussed and proposed for the first time. The specially configured long-term neuro-fuzzy predictive models with accuracy of ninety-second prediction horizons for the outputs of all operating zones of WBF have been presented. Taking into account the experimental results, one-step neuro-fuzzy predictive models demonstrate the highest prediction accuracy compared to predictive models of farther horizons in the sense that they can effectively create a high-fidelity replica of their associated operating zones. Furthermore, the comparative study made between LSE (optimum modelling method for linear systems) and LLNF prediction methods confirms that all operating zones of the WBF process belong to the class of non-linear systems. Throughout the paper, also, it is elaborated that a variety of technical remarks should be taken into consideration in intelligent prediction of a MIMO process by means of any intelligent tool based on experimental data, meanwhile, any negligence of these points during prediction procedure may result in constructing unreliable predictive models. Some of the main remarks in creating intelligent nominal predictive model of a MIMO process are as follows.

- Faulty operational data points have to be ignored when a predictive model of a process under nominal operating conditions is going to be identified. In addition, the inputs of the process, which do not vary over time, should not be considered in the predictive model, since they remain constant during prediction.
- Outliers (section *Data filtering*) usually contaminate measured data or states for a non-linear dynamic system (including all process plants). Identifying and removing outliers will make the data more trustworthy and reliable. Filtering is a suitable alternative that guarantees this matter.
- The magnitudes of the different signals should be close to each other. Normalization of all the data is an appropriate technique, which fulfil such requirements (section *Data normalization*).
- No matter which intelligent tool is used, even though only one of the outputs of a MIMO process plant is going to be predicted, the values of other outputs are also needed to be included in predictor model (outputs of the dynamic systems are usually coupled).

After this work is accomplished, a database of typical WBF faults will be utilized in a model-based condition monitoring system whose requirements can be defined by the factory of beneficiary, developing this way a prototype module for condition monitoring and fault diagnosis of the WBF.

Acknowledgments

The authors want to thank the Iran Alloy Steel Company for providing us with the experimental data for this project. Special thank also goes to S. Rajabi for her kind help in preparing this paper.

Nomenclature

c – centre co-ordinate
FG_{Z1} – gas flow of zone 1, [NM^3h^{-1}]
FG_{Z2} – gas flow of zone 2, [NM^3h^{-1}]
FG_{Z3} – gas flow of zone 3, [NM^3h^{-1}]
FG_{Z4} – gas flow of zone 4, [NM^3h^{-1}]
h – prediction horizon
k – data sample number
$\bar{m}$ – mean value
m_{z0} – dynamic depth of pressure input
m_{zi} – dynamic depth of i-th input
PA_1 – first predictive accuracy function
PA_2 – second predictive accuracy function
PR_{z0} – furnace pressure, [mbar]
Q – number of data samples
S – original signal
$S_{\min}$ – minimum of original signal
$S_{\max}$ – maximum of original signal
S_N – normalized signal
T_f – transfer function of designed filter
TL_{Z1} – left temperature of zone 1, [°C]
TL_{Z2} – left temperature of zone 2, [°C]
TL_{Z3} – left temperature of zone 3, [°C]
TL_{Z4} – left temperature of zone 4, [°C]
TR_{Z1} – right temperature of zone 1, [°C]
TR_{Z2} – right temperature of zone 2, [°C]
TR_{Z3} – right temperature of zone 3, [°C]
TR_{Z4} – right temperature of zone 4, [°C]
$\hat{T}_{Zi}$ – i-th one-step future output
u – input signal
$\bar{x}$ – validity function
y – output signal
Z – time delay in discrete domain
z – time, [s]

Greek symbols

ζ_j – offset of least-squares solution
φ_j – j-th operating-point
σ – individual standard deviation

Abbreviations

ANN – artificial neural network
LLM – local linear model
LLNF – locall linear neuro-fuzzy
LOLIMOT – local linear model tree
LSE – least square error
MIMO – multi input multi output
MISO – multi input single output
PRBS – pseudo-random binary signal
RMSE – root mean squared error
TDL – time delay line
TS – Takagi-Sugeno
WBF – walking beam furnace

References

[1] Shaoyuan, Li., *et al*., Dynamic Temperature Modeling of Continuous Annealing Furnace Using GGAP-RBF Neural Network, *Neurocomputing, 69* (2006), 4-6, pp. 524-536
[2] Zhang, B., *et al*., The Modeling and Control of a Reheating Furnace, *Proceedings*, American Control Conference, 2002, Anchorage Alas., USA, pp. 3823-3828
[3] Laurinen, P., Roning, J., An Adaptive Neural Network Model for Predicting the Post Roughing Mill Temperature of Steel Slabs in the Reheating Furnace, *Materials Processing Technology, 168* (2005), 3, pp. 423-430
[4] Chen, Q., *et al*., Furnace Temperature Modeling for Continuous Annealing Process Based on Generalized Growing and Pruning RBF Neural Network, *Advances in Neural Networks, 3174,* (2004), pp. 755-760
[5] Banadaki, H. D., *et al*., Non-Linear Simulator Model Identification of a Walking Beam Furnace Using Recurrent Local Linear Neuro-Fuzzy Network, *International Journal of Control and Automation, 4* (2011), 4, pp. 123-134
[6] Kusters, A., van Ditzhuijzen, G. A. J. M., MIMO System Identification of a Slab Reheating Furnace, *Proceedings,* Third IEEE Conference on Control Applications, 1994, pp. 3097-1563
[7] Gobbak, A. K., *et al.,* H., Internal Feedback Neuron Networks for Modeling of an Industrial Furnace, *Neural Networks, International Conference,* 1997, Vol 2, pp. 700-705

[8] Liao, Y., *et al*., Modeling of Reheating-Furnace Dynamics Using Neural Network Based on Improved Sequential-Learning Algorithm, *Computer Aided Control System Design*, 2006, pp. 3175-3181
[9] Xuegang, S., *et al*., Dynamic Modeling of Reheat-Furnace Using Neural Network Based on PSO Algorithm, *International Conference on Mechatronics and Automation,* 2009, pp. 3097-3101
[10] Pongam, T., *et al*., Open-Loop Identification of the Mathematical Model of the Reheating Furnace Walking Hearth Type in Manufacturing Process, *International Conference on System Modeling and Optimization,* 2012, Vol. 23, pp. 24-30
[11] Ogawa, M., *et al*., Long-Term Prediction of Industrial Furnace by Extended Sequential Prediction Method of LOM, *SICE Annual Conference,* 2010, pp. 1490-1493
[12] Leea, D., Leeb, Y., Application of Neural-Network for Improving Accuracy of Roll-Force Model in Hot-Rolling Mill, *Control Engineering Practice, 10* (2002), 4, pp. 473-478
[13] Kaiju, Z., *et al*., Fuzzy Neural Network's Application in Furnace Temperature Compensation Based on Rolling Information, *IFAC World Congress,* 2005, Vol. 16, Part 1, p. 1722
[14] Schlanga, M., *et al*., Current and Future Development in Neural Computation in Steel Processing, *Control Engineering Practice, 9* (2001), 9, pp. 975-986
[15] Roger Jang, J., *et al*., *Neuro-Fuzzy and Soft Computing: A Computational Approach to Learning and Machine Intelligence*, Prentice Hal Inc., Engelwood cliffs, N. J., USA, 1997
[16] Razavi-Far, R., *et al*., Model-Based Fault Detection and Isolation of a Steam Generator Using Neuro-Fuzzy Networks, *Neurocomputing, 72* (2009), 13, pp. 2939-2951
[17] Sadeghian, M., Fatehi, A., Identification of Non-linear Predictor and Models of a Cement Rotary Kiln by Locally Linear Neuro-Fuzzy Technique, World Academy of Science, *Engineering and Technology, 34* (2009), Oct., pp. 1121-1127
[18] Nozari, H. A., *et al*., Intelligent Non-Linear Modelling of an Industrial Winding Process Using Recurrent Local Linear Neuro-Fuzzy Networks, *Journal of Zhejiang University – Science C, 13* (2012), 6, pp. 403-412
[19] Mohammadzaheri, M., Lei, C., Intelligent Modeling of MIMO Non-linear Dynamic Process Plants for Predictive Control Purposes, *Proceedings,* 17th World Congress the International Federation of Automatic Control, Seoul, 2008, pp. 12401-12406
[20] Nelles, O., Local Linear Model Tree for on-Line Identification of Time Variant non-Linear Dynamic Systems, *International Conference on Artificial Neural Networks,* 1996, Vol. 1112, pp. 115-120
[21] Nelles, O., *Non-Linear System Identification*, Springer Inc., 2001
[22] Ljung, L., *System Identification Theory for the User*, Prentice Hall Inc., Engelwood Cliffs, N. J., USA, 1987
[23] Nozari, H. A., *et al.*, Model-Based Robust Fault Detection and Isolation of an Industrial Gas Turbine Prototype Using Soft Computing Techniques, *Neurocomputing, 91* (2012), Ang., pp. 29-47
[24] Nelles, O., Isermann, R., Basis Function Networks for Interpolation of Locally Linear Models, *Proceedings,* IEEE Conference on Decision and Control, Kobe, Japan, 1996, Vol. 1, pp. 470-475
[25] Judd, K., Small, M., Towards Long-Term Prediction, *Journal of Physica D: Nonlinear Phenomena, 136* (2000), 1, pp. 31-44

Paper submitted: April 10, 2012
Paper revised: May 10, 2012
Paper accepted: October 14, 2012