FLUCTUATING HYDRO-MAGNETIC NATURAL CONVECTION FLOW PAST A MAGNETIZED VERTICAL SURFACE IN THE PRESENCE OF THERMAL RADIATION

by

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The effect of radiation on fluctuating hydro-magnetic natural convection flow of viscous, incompressible, electrically conducting fluid past a magnetized vertical plate is studied; when the magnetic field and surface temperature oscillates in magnitude about a constant non zero mean. The numerical solutions have been obtained for different values of radiation parameter, magnetic Prandtl number, magnetic force parameter, Prandtl number, and surface temperature in terms of amplitude and phase angle of coefficients of skin friction, rate of heat transfer, and current density at the surface of the plate. Moreover, the effects of these parameters on transient coefficients of skin friction, rate of heat transfer and current density have been discussed. The finite difference method for primitive variable transformation and asymptotic series solution for stream function formulation has been used to obtain the numerical solution of the boundary layer flow field.

Key words: *hydro-magnetic, fluctuating, natural convection, magnetized plate, current density, heat transfer, skin friction*

Introduction

The original motivation for the present work is that to know something about the interaction of radiation effect in the energy equation past a magnetized vertical heated surface, when the magnetic field and surface temperature oscillates in magnitude about a constant non zero mean. In previous work Lighthill [1] first noticed the unsteady forced flow of viscous incompressible fluid past a flat plate and circular cylinder with small amplitude oscillation in free stream. The numerical solution of unsteady flow past a semi-infinite plate due to small fluctuation in free stream velocity has been considered by Ackerberg *et al.* [2]. Merkin [3] studied the free convection boundary layer flow on an isothermal horizontal circular cylinder in viscous fluid flow and obtained results by using Blausis and Gortler series expansion method and finite series method. Rott *et al.* [4], and Lam *et al.* [5] extended the work done by Lighthill [1] and investigated the same results analytically and by using series method. Schoenhals *et al.* [6], and

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Blakenship *et al.* [7] performed the solution of free convection boundary layer flow past a vertical plate, when the plate is subject to transverse mechanical vibration. The corresponding problem of longitudinal vibration has been analyzed by Eshgy *et al.* [8]. Menold *et al.* [9] have been studied the phase relation of surface temperature, heat flux, and shear stress along the surface of a vertical plate with oscillating surface temperature. Further contribution to the unsteady free convection flow with oscillating surface temperature along a vertical plate was given by Nanda *et al.* [10]. Muhari *et al.* [11], and Verma [12] examined the effect of oscillation in the context of the surface temperature on the unsteady free convection flow from a horizontal plate. Kelleher *et al.* [13] reported the free convection boundary layer flow past a vertical heated plate to the surface temperature oscillation. The basic steady flow along a magnetized plate has been investigated by [14, 15]. Later [16-22] studied the magnetohydrodynamics (MHD) boundary layer flow when a uniform magnetic field in the stream direction is applied.

Keeping in view, the above literature survey, our aim is to investigate the fluctuating hydro-magnetic natural convection flow of electrically conducting viscous incompressible and optically thick fluid past a magnetized vertical surface radiative flux by using finite difference method and perturbation technique.

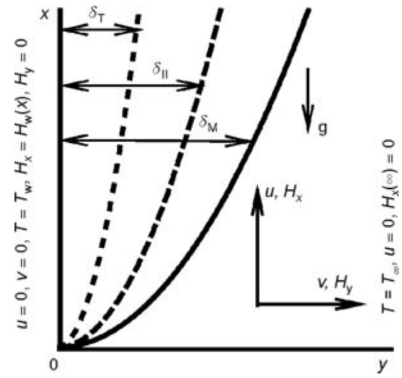

Figure 1. The co-ordinate system and flow configuration

Mathematical analysis and governing equations

We consider a unsteady 2-D MHD natural convection flow of an electrically conducting, viscous, and incompressible fluid past a periodically oscillated heated and magnetized vertical plate by including radiation effects in the energy equation. The flow configuration and the co-ordinate system are shown in fig. 1.

We have taken x-axis along the surface and y-axis is normal to it. In fig. 1, δ_M, δ_T, and δ_H stands for momentum, thermal, and magnetic field boundary layer thicknesses, respectively. The dimensionless momentum, magnetic, and energy boundary layer equations with the influence of radiative heat flux are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial \tau}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\theta+\frac{\partial^2 u}{\partial y^2}+S\left(B_x\frac{\partial B_x}{\partial x}+B_y\frac{\partial B_x}{\partial y}\right) \tag{2}$$

$$\frac{\partial B_\mathrm{x}}{\partial x}+\frac{\partial B_\mathrm{y}}{\partial y}=0 \tag{3}$$

$$\frac{\partial B_\mathrm{x}}{\partial \tau}+u\frac{\partial B_\mathrm{x}}{\partial x}+v\frac{\partial B_\mathrm{x}}{\partial y}-B_\mathrm{x}\frac{\partial u}{\partial x}-B_\mathrm{y}\frac{\partial u}{\partial y}=\frac{1}{\mathrm{Pm}}\frac{\partial^2 B_\mathrm{x}}{\partial y^2} \tag{4}$$

$$\frac{\partial \theta}{\partial \tau}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{4}{3R_\mathrm{d}}\frac{\partial}{\partial y}\left\{[1+(\theta_w-1)\theta]^3\frac{\partial \theta}{\partial y}\right\}\right) \tag{5}$$

The boundary conditions to be satisfied:

$$u(x, y = 0, \tau) = 0, \quad v(x, y = 0, \tau) = 0, \quad B_x(x, y = 0, \tau) = B(t), \quad B_y(x, y = 0, \tau) = 0,$$
$$\theta(x, y = 0, \tau) = \theta(\tau), \quad u(x, y = \infty, \tau)\ 0, \quad B_x(x, y = \infty, \tau) = 0, \quad \theta(x, y = \infty, \tau) = 0 \tag{6}$$

where $B(\tau)$ and $\theta(\tau)$ are the components of velocity and temperature which can be defined as:

$$B(\tau) = 1 + \varepsilon e^{it}, \quad \theta(\tau) = 1 + \varepsilon e^{it} \tag{7}$$

where ε is very small amplitude oscillation. Here, Pr, S, Pm, R_d, $\mathrm{Gr_L}$, and θ_w are Prandtl number, magnetic force parameter, magnetic Prandtl number, radiation parameter, Grashof number, and surface temperature (ratio to wall and ambient fluid temperature), respectively, which are defined as:

$$\mathrm{Pr} = \frac{\nu}{\alpha}, \quad S = \frac{\bar{\mu} B_0^2}{\rho U_0^2}, \quad \mathrm{Pm} = \frac{\nu}{\gamma}, \quad R_\mathrm{d} = \frac{K\alpha_\mathrm{R}}{4\sigma T_\infty^3}, \quad \mathrm{Gr_L} = \frac{g\beta\Delta T L^3}{\nu^2}, \quad \theta_\mathrm{w} = \frac{T_\mathrm{w}}{T_\infty}$$

where μ, ν, γ, and L are, respectively, the dynamic fluid viscosity, kinematics viscosity, magnetic diffusion, and characteristic length. We will write u, v, B_x, B_y, and θ as the sum of steady and oscillating components as purposed by Chawla [15]:

$$\begin{gathered} u = u_0(x, y) + \varepsilon e^{i\tau} u_1(x, y), \quad v = v_0(x, y) + \varepsilon e^{i\tau} v_1(x, y), \\ B_x = H_{x0}(x, y) + \varepsilon e^{i\tau} H_{x_1}(x, y), \quad B_y = H_{y_0}(x, y) + \varepsilon e^{i\tau} H_{y_1}(x, y), \\ \theta = \theta_0(x, y) + \varepsilon e^{i\tau}\theta_1(x, y) \end{gathered} \tag{8}$$

here $u_0(x, y)$, $v_0(x, y)$, $H_{x_0}(x, y)$, $H_{y_0}(x, y)$, $\theta_0(x, y)$, and $u_1(x, y)$, $v_1(x, y)$, $H_{x_1}(x, y)$, $H_{y_1}(x, y)$, and $\theta_1(x, y)$ are the steady and fluctuating parts of the flow variables. By using eq. (8) into eqs. (1)-(6) and by collecting terms of the first power of ε we have the following set of equations:

$$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0 \tag{9}$$

$$u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial v_0}{\partial y} = \frac{\partial^2 u_0}{\partial y^2} + S\left(H_{x_0} \frac{\partial H_{x_0}}{\partial x} + H_{y_0} \frac{\partial H_{x_0}}{\partial y} \right) + \theta_0 \tag{10}$$

$$\frac{\partial H_{x_0}}{\partial x} + \frac{\partial H_{x_0}}{\partial y} = 0 \tag{11}$$

$$u_0 \frac{\partial H_{x_0}}{\partial x} + v_0 \frac{\partial H_{x_0}}{\partial y} - H_{x_0} \frac{\partial u_0}{\partial x} - H_{y_0} \frac{\partial u_0}{\partial y} = \frac{1}{\mathrm{Pm}} \frac{\partial^2 H_{x_0}}{\partial y^2} \tag{12}$$

$$u_0 \frac{\partial \theta_0}{\partial \hat{x}} + v_0 \frac{\partial \theta_0}{\partial y} = \frac{1}{\mathrm{Pr}} \left(\frac{\partial^2 \theta_0}{\partial y^2} + \frac{4}{3R_\mathrm{d}} \frac{\partial}{\partial y} \left\{ [1 + (\theta_w - 1)\theta_0]^3 \frac{\partial \theta_0}{\partial y} \right\} \right) \tag{13}$$

The corresponding boundary conditions are:

$$\begin{gathered} u_0(x, y=0) = 0, \quad v_0(x, y=0) = 0, \quad H_{x_0}(x, y=0) = 1, \quad H_{y_0}(x, y=0) = 0, \\ \theta_0(x, y=0) = 1, \quad u_0(x, y=\infty) = 0, \quad H_{x_0}(x, y=\infty) = 0, \quad \theta_0(x, y=\infty) = 0 \end{gathered} \tag{14}$$

and

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial \bar{y}} = 0 \tag{15}$$

$$iu_1 + u_0 \frac{\partial u_1}{\partial x} + u_1 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} + v_1 \frac{\partial u_0}{\partial y} =$$

$$= \frac{\partial^2 u_1}{\partial y^2} + S\left(H_{x_0} \frac{\partial H_{x_1}}{\partial x} + H_{x_1} \frac{\partial H_{x_1}}{\partial x} + H_{y_0} \frac{\partial H_{x_1}}{\partial x} + H_{y_1} \frac{\partial H_{x_0}}{\partial y} \right) + \theta_1 \tag{16}$$

$$\frac{\partial H_{x_1}}{\partial x} + \frac{\partial H_{y_1}}{\partial y} = 0 \tag{17}$$

$$iH_{x_1} + u_0 \frac{\partial H_{x_1}}{\partial x} + u_1 \frac{\partial H_{x_0}}{\partial y} + v_0 \frac{\partial H_{x_1}}{\partial y} + v_1 \frac{\partial H_{x_0}}{\partial y} -$$

$$-H_{x_0} \frac{\partial u_1}{\partial x} - H_{x_1} \frac{\partial u_0}{\partial x} - H_{y_0} \frac{\partial u_1}{\partial x} - H_{y_1} \frac{\partial u_0}{\partial y} = \frac{1}{\mathrm{Pm}} \frac{\partial^2 H_{x_1}}{\partial y^2} \tag{18}$$

$$i\theta_1 + u_0 \frac{\partial \theta_0}{\partial x} + u_1 \frac{\partial \theta_0}{\partial x} + v_0 \frac{\partial \theta_1}{\partial y} + v_1 \frac{\partial \theta_0}{\partial y} =$$

$$= \frac{1}{\mathrm{Pr}} \left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{4}{3R_\mathrm{d}} \frac{\partial}{\partial y} \left\{ [1 + (\theta_\mathrm{w} - 1)\theta_0]^3 \frac{\partial \theta_1}{\partial \hat{y}} + 3[1 + (\theta_\mathrm{w} - 1)\theta_0]^2 \Delta\theta_1 \frac{\partial \theta_0}{\partial \hat{y}} \right\} \right) \tag{19}$$

The corresponding boundary conditions are:

$$u_1(x, 0) = 0, \quad v_1(x, 0) = 0, \quad H_{x_1}(x, 0) = 1, \quad H_{y_1}(x, 0) = 0, \quad \theta_1(x, 0) = 1 \tag{20}$$

We can find the solution of steady part functions, u_0, v_0, H_{x_0}, H_{y_0}, and θ_0 by using eqs. (9)-(13). By using these solutions in eqs. (15)-(19), we can quit from the situation of non-linearity and can find the solution of fluctuating flow for momentum, energy, and magnetic field equations.

Solution methodology

We now turn to get the numerical solutions of the problem, for this purpose we will use two methods namely (1) primitive variable transformation for finite difference method and (2) stream function formulation for asymptotic series solutions near and away from the leading edge of the plate.

Primitive variable transformation

To get the set of equations in convenient form for integration, we use the following symmetry of transformations for the dependent and independent variables.

Transformations for steady and unsteady case

$$u_0 = \sqrt{x} U_0(X, Y), \quad v_0 \frac{1}{\sqrt[4]{x}} V_0(X, Y), \quad Y = \frac{1}{2} \frac{1}{\sqrt[4]{x}} y$$
$$H_{x_0} = \sqrt{x} \phi_{1\mathrm{s}}(X, Y), \quad H_{y_0} = \frac{1}{\sqrt[4]{x}} \phi_{2\mathrm{s}}(X, Y), \quad \theta_0 = \overline{\theta}_0(X, Y) \tag{21}$$

By using (21) into eqs. (9)-(13) with boundary conditions (14) we have:

$$\frac{1}{2}U_0 + X\frac{\partial U_0}{\partial X} - \frac{1}{4}Y\frac{\partial U_0}{\partial Y} + \frac{\partial V_0}{\partial Y} = 0 \tag{22}$$

$$\frac{1}{2}U_0^2 + XU_0\frac{\partial U_0}{\partial X} + \left(V_0 - \frac{1}{4}YU_0\right)\frac{\partial U_0}{\partial Y} = $$
$$= \theta_0 + \frac{\partial^2 U_0}{\partial Y^2} + S\left[\frac{1}{2}\phi_{1s}^2 + X\phi_{1s}\frac{\partial \phi_{1s}}{\partial X} + \left(\phi_{2s} - \frac{1}{4}Y\phi_{1s}\right)\frac{\partial \phi_{1s}}{\partial Y}\right] \tag{23}$$

$$\frac{1}{2}\phi_{1s} + X\frac{\partial \phi_{1s}}{\partial X} - \frac{1}{4}Y\frac{\partial \phi_{1s}}{\partial Y} + \frac{\partial \phi_{2s}}{\partial Y} = 0 \tag{24}$$

$$XU_0\frac{\partial \phi_{1s}}{\partial X} + \left(V_0 - \frac{1}{4}YU_0\right)\frac{\partial \phi_{1s}}{\partial Y} - X\phi_{1s}\frac{\partial U_0}{\partial X} - \left(\phi_{2s} - \frac{1}{4}Y\phi_{1s}\right)\frac{\partial U_0}{\partial Y} = \frac{1}{\mathrm{Pm}}\frac{\partial^2 \phi_{1s}}{\partial Y^2} \tag{25}$$

$$XU_0\frac{\partial \bar{\theta}_0}{\partial X} + \left(V_0 - \frac{1}{4}YU_0\right)\frac{\partial \bar{\theta}_0}{\partial Y} = \frac{1}{\mathrm{Pr}}\left\{1 + \frac{4}{3R_\mathrm{d}}[1 + (\theta_\mathrm{w} - 1)\bar{\theta}_0]^3\right\}\frac{\partial^2 \bar{\theta}_0}{\partial Y^2} + $$
$$+ \frac{4}{\mathrm{Pr}}\frac{1}{R_\mathrm{d}}[1 + (\theta_\mathrm{w} - 1)\bar{\theta}_0]^2\left(\frac{\partial \bar{\theta}_0}{\partial Y}\right)^2 \tag{26}$$

The appropriate boundary conditions to be satisfied previous equations are:

$$\begin{gathered} U_0(X,0) = V_0(X,0) = 0, \quad \phi_{1s}(X,0) = 1, \quad \phi_{2s}(X,0) = 0, \quad \theta_0(X,0) = 1 \\ U_0(X,\infty) = 0, \quad \phi_{1s}(X,\infty) = 0, \quad \theta_0(X,Y) = 0 \end{gathered} \tag{27}$$

and similarly we have transformation for unsteady case:

$$\begin{gathered} u_1 = \sqrt{x}U_1(X,Y), \quad v_1 = \frac{1}{\sqrt[4]{x}}V_1(X,Y), \quad Y = \frac{1}{2}\frac{1}{\sqrt[4]{x}}y \\ H_{\mathrm{x}_1} = \sqrt{x}\phi_{1\mathrm{us}}(X,Y), \quad H_{\mathrm{y}_1} = \frac{1}{\sqrt[4]{x}}\phi_{2\mathrm{u}}(X,Y), \quad \theta = \bar{\theta}_1(X,Y) \end{gathered} \tag{28}$$

By using conditions (28) into eqs. (15)-(19) with boundary conditions (20), we have the system of equations:

$$\frac{1}{2}U_1 + X\frac{\partial U_1}{\partial X} - \frac{1}{4}Y\frac{\partial U_1}{\partial Y} + \frac{\partial V_1}{\partial Y} = 0 \tag{29}$$

$$\frac{1}{2}U_1^2 + XU_0\frac{\partial U_1}{\partial X} + \left(V_0 - \frac{1}{4}YU_0\right)\frac{\partial U_1}{\partial Y} + XU_1\frac{\partial U_0}{\partial X} + \left(V_1 - \frac{1}{4}YU_1\right)\frac{\partial U_0}{\partial Y} + iU_1 = $$
$$= \bar{\theta}_1 + \frac{\partial^2 U_1}{\partial Y^2} + S\left[\phi_{1s}\phi_{1\mathrm{u}} + X\phi_{1s}\frac{\partial \phi_{1\mathrm{u}}}{\partial X} + \left(\phi_{2s} - \frac{1}{4}Y\phi_{1s}\right)\frac{\partial \phi_{1\mathrm{u}}}{\partial Y} + \right.$$
$$\left. + X\phi_{1\mathrm{u}}\frac{\partial \phi_{1s}}{\partial X} + \left(\phi_{2\mathrm{u}} - \frac{1}{4}Y\phi_{1\mathrm{u}}\right)\frac{\partial \phi_{1s}}{\partial Y}\right] \tag{30}$$

$$\frac{1}{2}\phi_{1\mathrm{u}} + X\frac{\partial \phi_{1\mathrm{u}}}{\partial X} - \frac{1}{4}Y\frac{\partial \phi_{1\mathrm{u}}}{\partial Y} + \frac{\partial \phi_{2\mathrm{u}}}{\partial Y} = 0 \tag{31}$$

$$XU_0\frac{\partial\phi_{1\mathrm{u}}}{\partial X}+\left(V_0-\frac{1}{4}YU_0\right)\frac{\partial\phi_{1\mathrm{u}}}{\partial Y}+XU_1\frac{\partial\phi_{1\mathrm{s}}}{\partial X}+\left(V_1-\frac{1}{4}YU_1\right)\frac{\partial\phi_{1\mathrm{s}}}{\partial Y}-$$
$$-\left[X\phi_{1\mathrm{s}}\frac{\partial U_1}{\partial X}+\left(\phi_{2\mathrm{s}}-\frac{1}{4}Y\phi_{1\mathrm{s}}\right)\frac{\partial U_1}{\partial Y}+X\phi_{1\mathrm{u}}\frac{\partial U_0}{\partial X}+\left(\phi_{2\mathrm{u}}-\frac{1}{4}Y\phi_{1\mathrm{u}}\right)\frac{\partial U_0}{\partial Y}\right]+i\phi_{1\mathrm{u}}=\frac{1}{\mathrm{Pm}}\frac{\partial^2\phi_{2\mathrm{u}}}{\partial Y^2} \tag{32}$$

$$XU_1\frac{\partial\bar{\theta}_0}{\partial X}+U_0\frac{\partial\bar{\theta}_1}{\partial X}+\left(V_0-\frac{1}{4}YU_0\right)\frac{\partial\bar{\theta}_1}{\partial Y}+\left(V_1-\frac{1}{4}YU_1\right)\frac{\partial\bar{\theta}_0}{\partial Y}+i\bar{\theta}_1=$$
$$=\frac{1}{\mathrm{Pr}}\left[1+\frac{4}{3R_d}(1+\Delta\bar{\theta}_1)^3\right]\frac{\partial^2\bar{\theta}_1}{\partial Y^2}+\frac{8}{\mathrm{Pr}}\frac{1}{Rd}\Delta(1+\Delta\bar{\theta}_0)^2\frac{\partial\bar{\theta}_0}{\partial Y}\frac{\partial\bar{\theta}_1}{\partial Y}+$$
$$+\frac{4\Delta}{\mathrm{Pr}\,Rd}\left\{2\Delta^3(1+\Delta\bar{\theta}_0)\left[\frac{\partial\bar{\theta}_0}{\partial Y}\right]^2+\Delta(1+\Delta\bar{\theta}_0)^2\frac{\partial^2\bar{\theta}_0}{\partial Y^2}\right\}\bar{\theta}_1 \tag{33}$$

Here, $\Delta=\theta_\mathrm{w}-1$

The appropriate boundary conditions to be satisfied previos equations are:

$$U_1(X,0)=V_1(X,0)=0,\quad \phi_{1\mathrm{u}}(X,0)=1,\quad \phi_{2\mathrm{u}}(X,0)=0,\quad \bar{\theta}_1(X,0)=1$$
$$U_1(X,\infty)=0,\quad \phi_{1\mathrm{u}}(X,\infty)=0,\quad \bar{\theta}_1(X,\infty)=0 \tag{34}$$

System of transformed equations given in (22)-(27) and (29)-(34) along with their boundary conditions are discretize by using finite difference method, backward difference for x-direction and central difference for y-direction. By this way, we get the system of tri-diagonal equations. These set of tri-diagonal equations is solved by using Gaussian elimination technique. To represent the available solution in terms of amplitude and phase of coefficient of skin friction, rate of heat transfer and current density, we proceed as:

$$A_\mathrm{s}=\sqrt{\tau_r^2+\tau_i^2},\quad A_m=\sqrt{J_r^2+J_i^2},\quad A_t=\sqrt{Q_r^2+Q_i^2}$$
$$\phi_\mathrm{s}=\mathrm{tg}^{-1}\left(\frac{\tau_i}{\tau_r}\right),\quad \phi_\mathrm{m}=\mathrm{tg}^{-1}\left(\frac{J_i}{J_r}\right),\quad \phi_\mathrm{t}=\mathrm{tg}^{-1}\left(\frac{Q_i}{Q_r}\right), \tag{35}$$

Here, (τ_r, τ_i), (Jr, J_i), and (Q_r, Q_i) are the real and imaginary parts of the coefficients of skin friction, rate of heat transfer and current density. The numerical solutions obtained by using formulations given in (35) are shown in figs. 2-10 graphically for the different values of different parameters in terms of amplitude and phase angle against ξ.

Stream function formulation

To get the numerical solutions for small and large values of ξ we have the following cases.

When ξ is small

For small values of ξ we have two cases, for steady and unsteady flow.

The case for steady and unsteady flow

For the *steady flow* we have the following group of transformations:

$$\psi=\sqrt[4]{x^3}f_0(Y),\quad g=\sqrt[4]{x^3}\phi_0(Y),\quad Y=\frac{1}{\sqrt[4]{x}}y,\quad \theta_0=\bar\theta_0(Y),\quad \xi=x \tag{36}$$

By using transformations (36) into eqs. (9)-(13), we have the reduced set of equations:

$$f_0'''+\frac{3}{4}f_0f_0''-\frac{1}{2}f_{0'}'^2+\bar\theta_0-S\left(\frac{3}{4}\phi_0\phi_0''-\frac{1}{2}\phi_0'^2\right)=0 \tag{37}$$

$$\frac{1}{\mathrm{Pm}}\phi_0''+\frac{3}{4}f_0\phi_0'-\frac{3}{4}f_0'\phi_0=0 \tag{38}$$

$$\frac{1}{\mathrm{Pr}}\left\{1+\frac{4}{3R_\mathrm{d}}[1+(\theta_\mathrm{w}-1)\bar\theta_0]^3\right\}\bar\theta_{0''}+\frac{3}{4}f_0\bar\theta_{0'}=0 \tag{39}$$

Boundary conditions to be satisfied by the above equations are:

$$\begin{gathered}f_0(Y=0)=f_{0'}(Y=0)=0,\quad \phi_0(Y=0)=0,\quad \phi_{0'}(Y=0)=1,\\ \bar\theta_0(Y=0)=1,\quad f_{0'}(Y=\infty)=0,\quad \phi_{0'}(Y=\infty)=0,\quad \bar\theta_0(Y=\infty)\end{gathered} \tag{40}$$

The case when the *flow is unsteady*, for this we have the following group of transformations:

$$\psi=\sqrt[4]{x^3}F(\xi,Y),\quad G=\sqrt[4]{x^3}\Phi(\xi,Y),\quad Y=\frac{1}{\sqrt[4]{x}}y,\quad \theta=\bar\theta(\xi,Y),\quad x=\xi \tag{41}$$

By using transformations (41) into eqs. (15)-(19), we have the following system of equations and having new system of equation then by collecting like power of ξ we have:

$O(\xi^0)$

$$F_0'''+\frac{3}{4}(f_0F_0''+f_0''F_0)-f_0'F_0'+\theta_{1_0}+S\left[\frac{3}{4}(\phi_0\Phi_0''+\phi_0''\Phi_0)\phi_0'\Phi_0'\right]=0 \tag{42}$$

$$\frac{1}{\mathrm{Pm}}\phi_0''+\frac{3}{4}f_0\phi_0''+\frac{3}{4}F_0\phi_0''-\frac{3}{4}F_0''\phi_0=0 \tag{43}$$

$$\frac{1}{\mathrm{Pm}}\Phi_0''+\frac{3}{4}f_0\phi_0''+\frac{3}{4}F_0\phi_0''-\frac{3}{4}f_0''\Phi_0-\frac{3}{4}F_0''\phi_0=0 \tag{44}$$

The corresponding boundary conditions are:

$$\begin{gathered}F_0(Y=0)=F_0'(Y=0)=0,\quad \Phi_0(Y=0)=0,\quad \Phi_0'(Y=0)=1,\quad \theta_{10}(Y=0)=1\\ F_0'(Y=\infty)=0,\quad \Phi_0'(Y=\infty)=0,\quad \theta_{10}(Y=\infty)=0\end{gathered} \tag{45}$$

$O(\xi^1)$

$$F+\frac{3}{4}(f_0F''+f_0F_1'')-f_0'F_1'+\theta_{11}-F_0'+S\left(\frac{3}{4}\phi_0\Phi_1'+\frac{3}{4}\phi_0''\Phi_1-\phi_0'\Phi_1'\right)=0 \tag{46}$$

$$\frac{1}{\mathrm{Pm}}\Phi_0''+\frac{3}{4}(f_0\Phi_1''-\phi_0F_1'')+\frac{5}{4}(\phi_0''F_1-f_0''\Phi_1)-\frac{1}{2}(f_0'\Phi_{1'}-\phi_0'F_{1'})-\Phi_0'=0 \tag{47}$$

$$[1+\alpha(1+\Delta\theta_0)^3]\theta_{11''} + 3\alpha_1\Delta(1+\Delta\theta_0)^2(\theta_{11}\theta_{0''} + 2\theta_{0'}\theta_{11'}) + +6\alpha_1\Delta^2\theta_{11}(1+\Delta\theta_0)\theta_0'^2 + \Pr\frac{3}{4}f_0\theta_{11}' + \frac{5}{4}F_1\theta_0' - \frac{1}{2}f_0'\theta_{11} - \theta_0 = 0 \quad (48)$$

Here $\alpha_1 = (4/3)R_d$

The related order boundary conditions are:

$$F_1(Y=0) = F_1'(Y=0) = 0, \quad \Phi_1(Y=0)=0, \quad \Phi_1'(Y=0)=0, \quad \theta_{11}(Y=0)=0$$
$$F_{1'}(Y=\infty)=0, \quad \Phi_1'(Y=\infty)=0, \quad \theta_{11}(Y=\infty)=0 \quad (49)$$

The solutions of these equations are obtained by Nactsheim-Swigert iteration technique together with six order implicit Runge-Kutta-Butcher initial value solver. The results obtained with the help of the equations are given in tabs. 1-3 for small values of dimensionless distance from leading edge to downstream.

When ξ is large

To find the solution for ($\xi \geq 1$) in downstream regime, we introduced the following transformations:

$$\eta = \frac{1}{\sqrt{\xi}}Y, \quad F = \sqrt{\xi^3}\tilde{F}(\xi,\eta), \quad \Phi = \sqrt{\xi^3}\tilde{\Phi}(\xi,\eta), \quad \theta = \Theta(\xi,\eta) \quad (50)$$

by using (50) into (1)-(6), we obtained the following set of equations:

$$\tilde{F}''' - i\tilde{F}' + \Theta + \frac{3}{4}\frac{1}{\sqrt{\xi}}f_0\tilde{F}'' - \frac{1}{2}\frac{1}{\xi}f'\tilde{F} - S\left(\frac{3}{4}\frac{1}{\sqrt{\xi}}\phi_0\tilde{\Phi}'' + \frac{3}{4}\frac{3}{\sqrt{\xi}}\phi_{0''}\tilde{\Phi} - \frac{1}{\xi}\phi_{0'}\overline{\Phi'}\right) =$$
$$= \frac{1}{2}\left[f_{0'}\left(\frac{\partial\tilde{F}'}{\partial\xi} + \frac{\eta}{2\xi}\tilde{F}''\right) - \frac{1}{\sqrt{\xi}}f_0\left(\frac{\partial\tilde{F}}{\partial\xi} + \frac{\eta}{2\xi}\tilde{F}'\right)\right] -$$
$$-\frac{1}{2}S\left[\phi_{0'}\left(\frac{\partial\tilde{\Phi}'}{\partial\xi}\right) + \frac{1}{2}\frac{1}{\xi}\phi_0\left(\frac{\partial\tilde{\Phi}'}{\partial\xi} + \frac{\eta}{2\xi}\tilde{\Phi}'\right)\right] \quad (51)$$

$$\frac{1}{\text{Pm}}\tilde{\Phi}''' - i\tilde{\Phi}' + \frac{3}{4}\frac{1}{\sqrt{\xi}}(f_0\tilde{\Phi}'' + \phi_{0''}\tilde{F}) + \frac{1}{2}\xi^{-1}(f_0\tilde{\Phi}' + \phi_{0''}\tilde{F}') =$$
$$= \frac{1}{2}f_{0'}\left(\frac{\partial\tilde{\Phi}}{\partial\xi} + \frac{\eta}{2\xi}\tilde{\Phi}'\right) - \frac{1}{2}\phi_{0'}\left(\frac{\partial\tilde{F}}{\partial\xi}\tilde{F}''\right) + \frac{1}{2}\frac{1}{\sqrt{\xi}}\left[f_0''\left(\frac{\partial\tilde{\Phi}}{\partial\xi} + \frac{\eta}{2\xi}\tilde{\Phi}'\right) - \phi_{0''}\left(\frac{\partial\tilde{F}}{\partial\xi} + \frac{\eta}{2\xi}\tilde{F}'\right)\right] \quad (52)$$

$$\frac{1}{\Pr}\left\{1 + \frac{4}{3R_d}[1+(\theta_w - 1)\Theta]^3\right\}\Theta'' - i\Theta + \frac{3}{4}\frac{1}{\sqrt{\xi}}f_0\Theta' - \frac{1}{2}\xi^{-1}f_{0'}\Theta =$$
$$= \frac{1}{2}\left[f'\left(\frac{\partial\Theta}{\partial\xi} + \frac{\eta}{2\xi}\Theta'\right) - \frac{1}{\sqrt{\xi}}f_0\left(\frac{\partial\tilde{F}}{\partial\xi} + \frac{\eta}{2\xi}\tilde{F}'\right)\right] \quad (53)$$

Boundary equations to be satisfied by the previous equations are:

$$\tilde{F}(\xi,0) = \tilde{F}'(\xi,0) = 0, \quad \tilde{\Phi}(\xi,0)=0, \quad \tilde{\Phi}'(\xi,0)=1, \quad \Theta(\xi,0)=1$$
$$\tilde{F}'(\xi,Y=\infty)=0, \quad \tilde{\Phi}'(\xi,Y=\infty)=0, \quad \Theta(\xi,Y=\infty)=0 \quad (54)$$

Table 1. Numerical values of amplitude and phase angle of heat transfer obtained for S = 0.0 and 0.1 when Pm = 0.1, R_d = 50.0, θ_w= 2.0, Pr = 0.1, against ξ by two methods

ξ	S = 0.0				S = 0.1			
	FDM		Asymptotic		FDM		Asymptotic	
	A_t	ϕ_t	A_t	ϕ_t	A_t	ϕ_t	A_t	ϕ_t
0.0	0.4141	0.0000	0.4048*	0.0000*	0.4045	0.0000	0.4065*	0.0000*
0.2	0.4057	2.2562	0.4092*	2.4936*	0.4048	2.2728	0.4109*	2.5115*
0.4	0.4068	4.5102	0.4221*	4.6780*	0.4060	4.5420	0.4241*	4.7039*
0.6	0.4087	6.7549	0.4430*	6.3065*	0.4080	6.8997	0.4454*	6.3240*
0.8	0.4115	8.9827	0.4714*	8.2285*	0.4109	9.0377	0.4742*	9.0668*
1.0	0.4152	11.1802	0.4825*	11.6054*	0.4146	11.2419	0.4859*	21.5600*
2.0	0.4463	21.2891	–	–	0.4461	21.3537	–	–
4.0	0.5544	35.0198	–	–	0.5544	35.0555	–	–
6.0	0.6870	41.7353	0.6360**	44.3310**	0.6870	41.7557	0.6360**	44.3310**
8.0	0.8200	44.6930	0.7344**	44.7323**	0.8201	44.7066	0.7334**	44.7321**
10.0	0.9428	45.0000	0.8202**	44.9999**	0.9429	45.0000	0.8202**	44.9999**

Here, * and ** are stands for small and large values of ξ respectively

Table 2. Numerical values of amplitude and phase angle of coefficient of skin friction obtained for S = 0.0 and 0.1 when Pm = 0.1, R_d = 50.0, θ_w = 2.0, Pr = 0.1, against by two methods

ξ	S = 0.0				S = 0.1			
	FDM		Asymptotic		FDM		Asymptotic	
	A_t	ϕ_t	A_t	ϕ_t	A_t	ϕ_t	A_t	ϕ_t
0.0	0.8303	0.0000	0.8240*	0.0000*	0.7387	0.0000	0.7811*	0.0000*
0.2	0.8259	4.8921	0.8538*	4.1152*	0.7348	4.8618	0.7538*	4.3383*
0.4	0.8125	9.6693	0.8873*	9.5277*	0.7229	9.6104	0.7309*	9.7034*
0.6	0.7918	14.5824	0.8490*	14.5099*	0.7046	14.0623	0.7191*	14.9031*
0.8	0.7655	18.2740	0.7849*	18.2187*	0.6813	18.1588	0.7099*	18.5839*
1.0	0.7363	21.9605	0.7589*	21.9199*	0.6555	21.8104	0.6893*	21.5093*
2.0	0.4090	34.3274	–	–	0.5223	34.0844	–	–
4.0	0.3258	42.5114	–	–	0.3668	42.3844	–	–
6.0	0.6870	44.6935	0.3345**	45.0000**	0.2917	44.6151	0.3165**	45.0000**
8.0	0.2773	44.7741	0.2944**	45.0000**	0.2479	44.8137	0.2843**	45.0000**
10.0	0.2452	45.0000	0.2658**	45.0000**	0.2192	45.0000	0.2461**	45.0000**

Here, * and ** are stands for small and large values of ξ respectively from which we see that for large ξ:

$$F_0''(0) = \frac{1-i}{1+\sqrt{\Pr}\sqrt{2\xi}}$$
$$g_0''(0) = -(1+i)\sqrt{\frac{\xi \mathrm{Pm}}{2}} \tag{55}$$
$$\Theta_0''(0) = -\frac{(1+i)\sqrt{\dfrac{\xi \Pr}{2}}}{1+\alpha_1(1+\Delta\Theta_0)^3}$$

The results obtained by relations (55) are given in tabs. 1-3 for large values of dimensionless parameter and compared with the solution that obtained by finite difference method and found to be in reasonable agreement.

Table 3. Numerical values of amplitude and phase angle of coefficient of current density obtained for S = 0.0 and 0.1 when Pm = 0.1, R_d = 50.0, θ_w = 2.0, Pr = 0.1, against by two methods

ξ	S = 0.0				S = 0.1			
	FDM		Asymptotic		FDM		Asymptotic	
	A_t	ϕ_t	A_t	ϕ_t	A_t	ϕ_t	A_t	ϕ_t
0.0	0.4289	0.0000	0.3977*	0.0000*	0.4193	0.0000	0.4017*	0.0000*
0.2	0.4205	2.2042	0.4076*	2.6726*	0.4197	2.2185	0.4068*	2.6944*
0.4	0.4216	4.4053	0.4172*	4.9846*	0.4209	4.4329	0.4220*	4.0047*
0.6	0.4236	6.5960	0.4410*	6.9519*	0.4229	6.6350	0.4466*	6.6337*
0.8	0.4264	8.7687	0.4733*	8.5056*	0.4258	8.8168	0.4799*	8.4105*
1.0	0.4301	10.9106	0.4091*	10.6271*	0.4295	10.9649	0.5067*	10.4374*
2.0	0.4613	20.7669	–	–	0.4610	20.8261	–	–
4.0	0.5691	34.2562	–	–	0.4690	34.2922	–	–
6.0	0.7041	41.0385	0.7754**	44.3000**	0.7014	40.9730	0.7754**	44.3000**
8.0	0.8342	43.9551	0.8953**	44.3000**	0.8342	43.9714	0.8953**	44.3000**
10.0	0.9568	45.0000	1.0000**	44.3000**	0.9568	45.0000	1.0000**	44.3000**

Here, * and ** are stands for small and large values of ξ, respectively

Results and discussion

In this section we briefly explain the physical behavior of different physical parameters on coefficients of skin friction, rate of heat transfer and current density in terms of amplitude and phase angle. Moreover the effect of these parameters is also exhibit in terms of transient coefficients of skin friction, rate of heat transfer and current density.

Effects upon amplitude and phase angle

Figures 2(a)-2(d) shows the effect of different values of radiation parameter R_d = 1.0, 2.5, 5.0, and 10.0 when magnetic Prandtl number Pm = 0.5, magnetic force parameter S = 0.3, Prandtl number Pr = 0.71, and ratio of wall temperature to ambient fluid temperature is chosen θ_w = 0.5. From this it is concluded that with the increase of radiation parameter R_d the amplitude and phase angle of heat transfer increases where amplitude and phase angle of coefficient of skin friction decreases. To explain this phenomena physically, we mark this trend with the understanding that when radiation parameter R_d increases, the ambient fluid temperature decreases and according to the Fourier law of heat transfer the flow of heat is towards the ambient fluid and at the surface the fluid motion is slowdown. From figs. 3(a)-3(d), we can see the effects of different values of magnetic Prandtl number Pm. From these figures it is noticed that with the increases of Pm the amplitude and phase angle of skin friction decreases where amplitude and phase angle of current density increases prominently. The reason is that with the increase of Pm the induced current with in the boundary layer tends to spread away from the surface and this re-

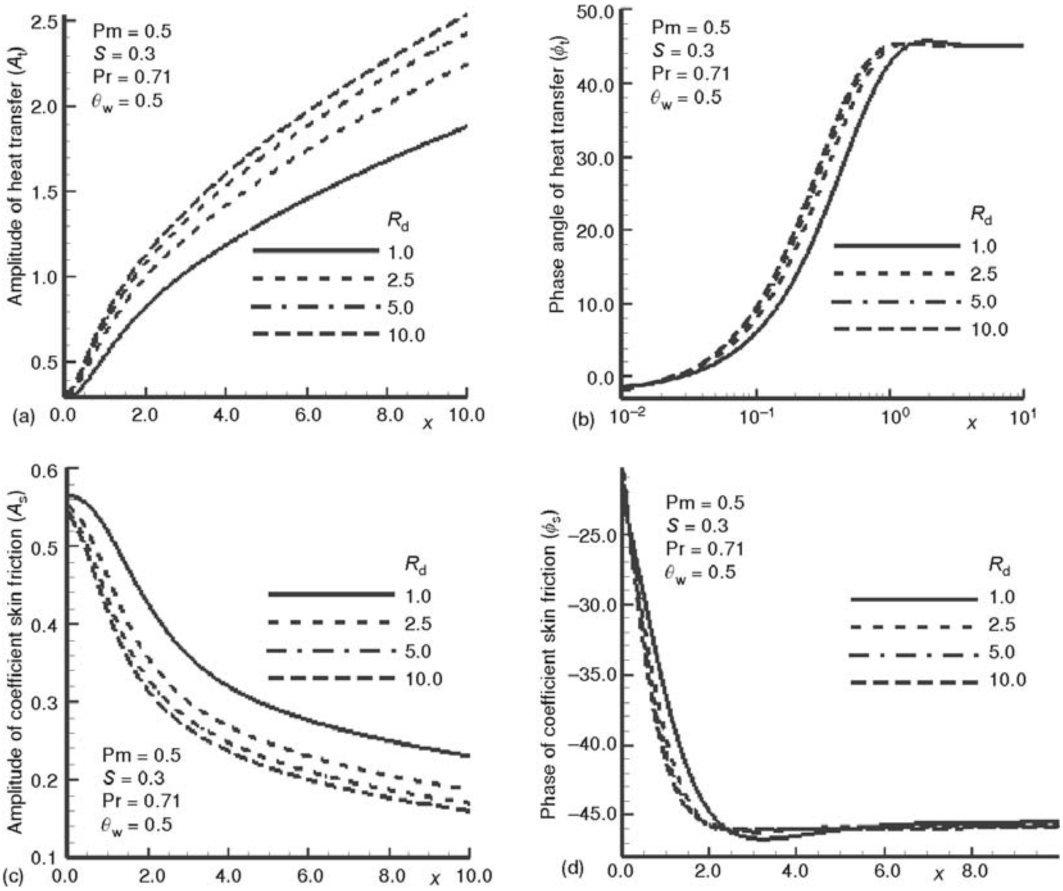

Figure 2. Numerical solution of amplitude and phase angle of heat transfer and skin friction for different values of R_d = 1.0, 2.5, 5.0, and 10.0 while Pr = 0.71, Pm = 0.5, θ_w = 0.5, and S = 0.3

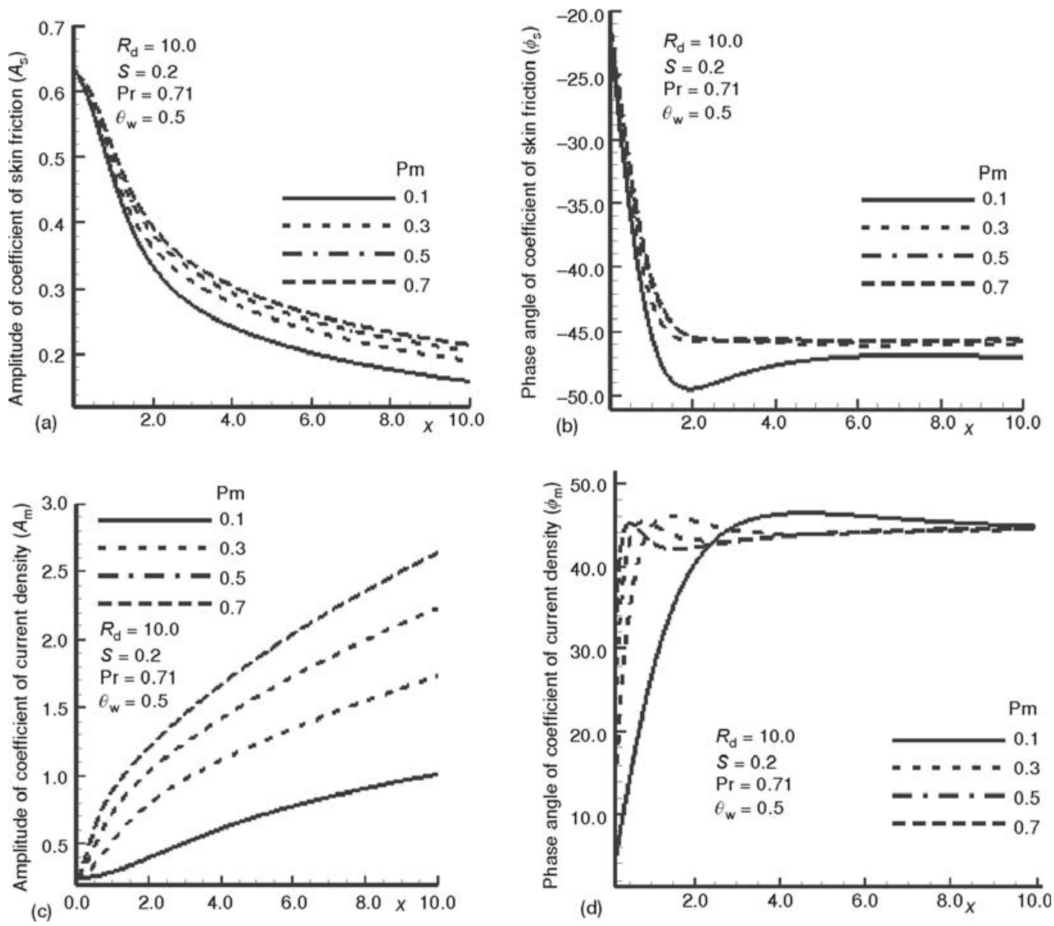

Figure 3. Numerical solution of amplitude and phase angle of coefficient of skin friction and current density for different values of Pm = 0.1, 0.3, 0.5, 0.7 while Pr = 0.71, R_d = 10.0, θ_w = 0.5, and S = 0.2

sult in thickening of the boundary layer, thus the amplitude and phase of current density increases for the case of natural convection. From figs. 4(a)-4(d), it is observed that with the increase of the ratio of wall temperature to ambient fluid temperature θ_w the amplitude and phase of rate of heat transfer decreases where the amplitude and phase of coefficient of skin friction increases.

The amplitude and phase angle of coefficients of rate of heat transfer, skin friction and current density for different values of magnetic force parameter S is given in tabs. 1-3. From these tables, it is found that the amplitude and phase angle of heat transfer decreases and similarly the amplitude and phase angle of the coefficient of skin friction is also decreases. It is also evident from tab. 3 that the amplitude and phase angle of coefficient of current density increases. This situation happen because the imposition of magnetic field parameter decelerates the motion of the fluid that thicken the boundary layer thickness and generate the magnetic current, for this reason the amplitude and the phase angle of coefficients of rate of heat transfer and skin friction decreases and the amplitude and phase angle of current density increases.

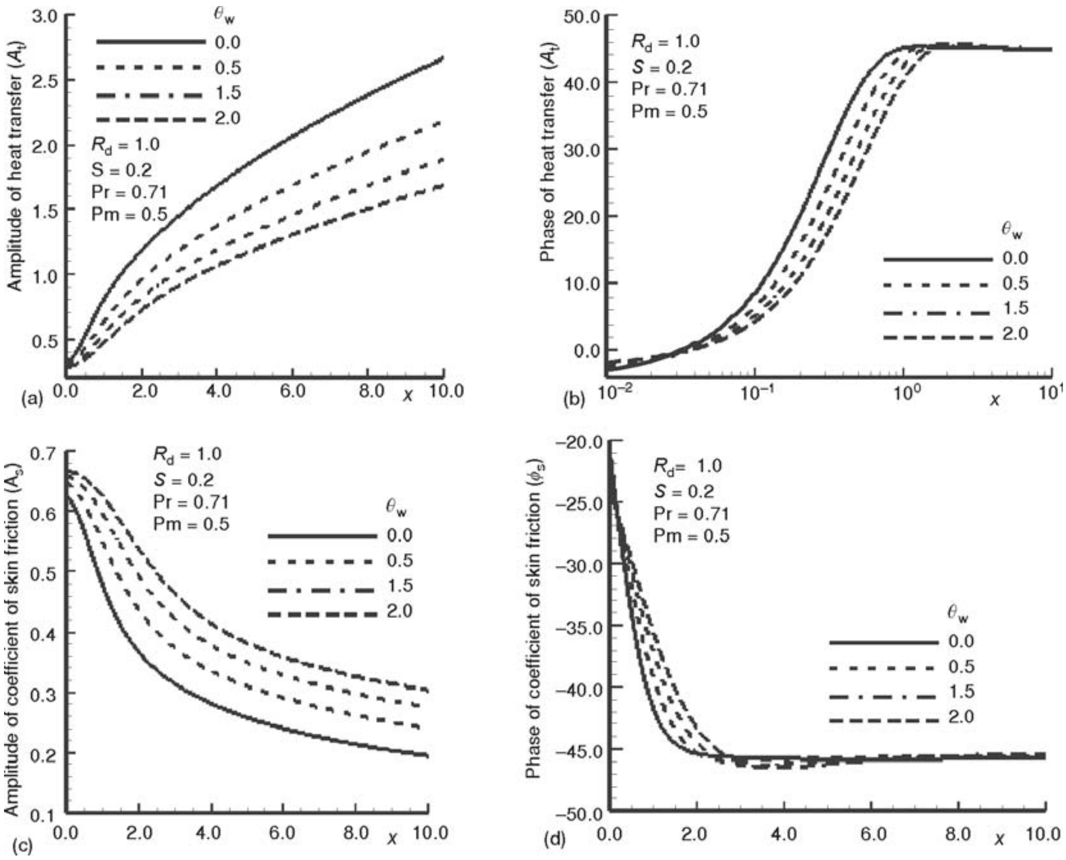

Figure 4. Numerical solution of amplitude and phase angle of heat transfer and skin friction for different values of θ_w = 0.0, 0.5, 1.5, and 2.0 while Pr = 0.71, Pm = 0.5, R_d = 1.0, and S = 0.2

Effects upon transient heat transfer, shear stress, and current density

In the present section we are going to explain the physical profiles of transient rate of heat transfer, skin friction and current density at the surface of vertical plate, for this purpose we define the following relations:

$$\begin{aligned}\tau_t &= \tau_{t0} + \varepsilon A_t \cos(\tau + \phi_t) \\ \tau_s &= \tau_{s0} + \varepsilon A_s \cos(\tau + \phi_s) \\ \tau_m &= \tau_{m0} + \varepsilon A_m \cos(\tau + \phi_m)\end{aligned} \tag{56}$$

it is necessary to mention that τ_{t0}, τ_{s0}, and τ_{m0} are rate of heat transfer, skin friction, and current density comes from steady part, and similarly (A_t, A_s, A_m) and (ϕ_t, ϕ_s, ϕ_m) are amplitudes and phases angle of rate of heat transfer, skin friction and current density comes from fluctuating part and ε is small amplitude oscillation. The effect of radiation parameter R_d on coefficients of transient rate of heat transfer and skin friction are shown in figs. 5(a) and 5(b) with other parameter fixed. It is observed that, the transient rate of heat transfer increases and transient coefficient of skin friction reduces with the increase of radiation parameter R_d. Figures 5(c) and 5(d) illustrated that the coefficients of transient rate of heat transfer and skin friction reduces very prominently against dimensionless time τ with the increase of magnetic force parameter S. The effect of magnetic Prandtl number Pm on the coefficients of skin friction and current density have been exhibited in figs. 5(e) and 5(f). It is observed that the increase in parameter Pm increases the transient coefficient of skin friction and reduces the transient coefficient of current density.

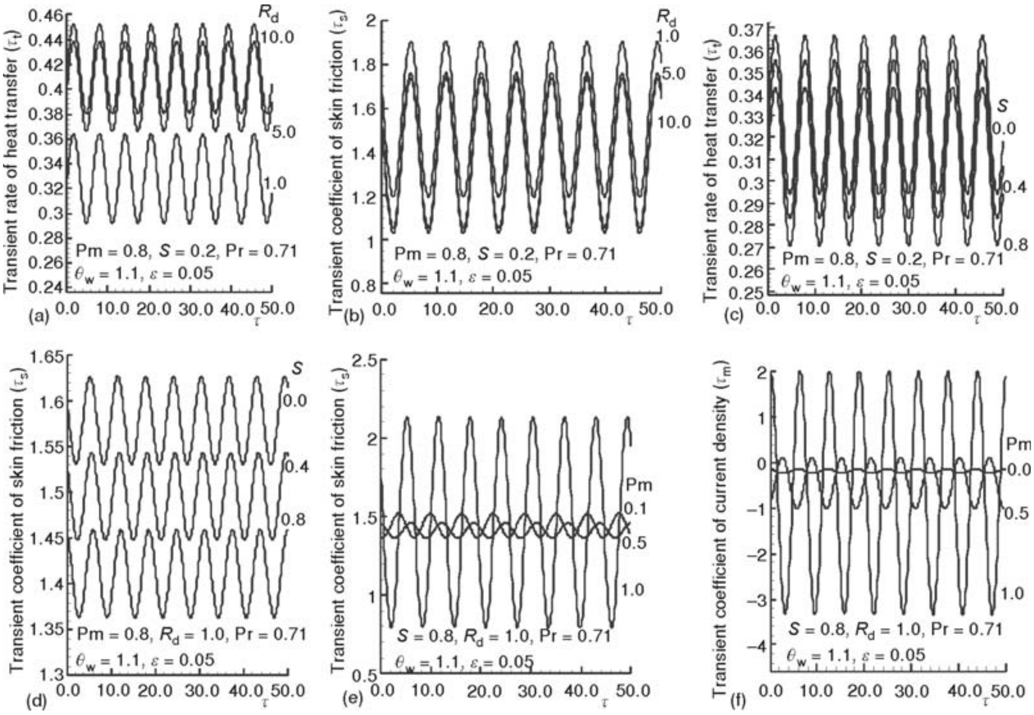

Figure 5. Numerical solution of transient: rate of heat transfer, coefficient of skin friction, and coefficient of current density for different values of R_d = 1.0, 5.0, 10.0, magnetic force parameter S = 0.0, 0.4, 0.8, and Pm = 0.1, 0.5, 1.0 while Pr = 0.71, Pm = 0.8, θ_w = 1.1, S = 0.2, ξ = 10.0, and ε = 0.05

Conclusions

In this study emphasis was given on the effect of different physical parameters on chief physical quantities those are very important in the field of mechanical engineering such as coefficients of rate of heat transfer, skin friction, and current density. From the brief study of figures and tables our findings are given.

It is observed that with the increase of conduction-radiation parameter R_d the amplitude and phase angle of heat transfer increases but coefficient of skin friction decreases. It is also noted that the transient rate of heat transfer increases and skin friction in terms of amplitude and phase angle reduces as the radiation-conduction parameter increases. It is concluded that the amplitude and phase angle of rate of heat transfer have no significance changes with the increase of magnetic Prandtl number Pm. There is very active increase for the case of amplitude and phase angle of coefficients of skin friction and current density is noted with the increase magnetic Prandtl number Pm. The transient coefficient of skin friction increases and coefficient of current density decreases with the increase of Pm. It is also observed that the amplitude and phase of heat transfer decreases with the increase of parameter θ_w and amplitude and phase angle of coefficient of skin friction increases with the increase of ratio of the surface temperature to the ambient fluid temperature θ_w. The coefficients of the rate of heat transfer and skin friction in terms of amplitude and phase angle decreases and current density increases with the increase of magnetic force parameter S. The asymptotic solutions for small and large values of dimensionless stream wise co-ordinate, ξ for different values of magnetic force parameter S when other parameter are fixed are to be found in reasonable agreement those obtained by finite difference method for entire value of ξ.

Nomenclature

B_x – dimensionless magnetic field along the surface
B_y – dimensionless magnetic field normal to the surface
Cf_x – coefficient of skin friction
F – transformed stream function
Gr_L – local Grashof number
Nu_x – local Nusselt number
Pm – magnetic Prandtl number
Pr – Prandtl number
R_d – radiation parameter
Re_x – local Reynolds number
S – magnetic force parameter

Greek symbols

α – thermal diffusivity, [m^2s^{-1}]
β – coefficient of cubical expansion
γ – magnetic diffusion
ψ – fluid steam function, [m^2s^{-1}]
ϕ – transformed stream function for magnetic field
θ – temperature, [K]
θ_w – surface temperature ratio to the ambient fluid
η – similarity transformation
μ – dynamical viscosity, [$kgm^{-1}s^{-1}$]
$\overline{\mu}$ – magnetic permeability
ν – effective kinematics viscosity, [$\mu\rho^{-1}$]
ξ – local mixed convection parameter

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